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EVAPOTRANSPIRATION PREDICTION BASED ON CHEBYSHEV INTERPOLATION

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ABSTRACT

This paper considers the possibility of applying the methods of the Chebyshev interpolation theory in predicting evapotranspiration, which allows obtaining more accurate predicted values. As a result, it optimizes the parameters that provide significant savings of resources. Conducted theoretical studies let us propose a method for constructing a predictive indicator of evapotranspiration, based on the problem of Chebyshev approximation, and also construct the indicative curves for alfalfa. Comparing the results of traditional procedures based on classical regression methods with those calculated by the proposed method gives a good match. It confirms the accuracy of both the adopted method and the reliability of the dependencies obtained.

1. INTRODUCTION

Evapotranspiration (total water consumption) is the most important characteristic of water management. Calculation of the evapotranspiration of irrigated crops is the most significant element of irrigated agriculture management at all its stages, including the planning of irrigation of new lands, design of irrigation systems, and their operation. Its value, along with other parameters, is the input information for most hydrological and water-balance models. Inaccuracies in the definition of evapotranspiration in systems design can lead to the death of major capital investments in the construction of excess capacity, the shortage of irrigated crop production, and failure to obtain the design yield due to the inability to ensure optimal water regime.

The development of evapotranspiration models is widespread in most developed countries. The most common method is that of Penman-Monteith [1]. The method is recommended as the standard by the International Organization for Food, and Agriculture of the United Nations (FAO UN). In Russia, with significant soil-climatic diversity, the Penman-Monteith method is difficult to apply, as it requires long-term data on the solar radiation entering the earth's surface in a particular area (these data are not always available). It also does not sufficiently take into account the biological

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characteristics of cultivated plants at different periods of their growth and development. Therefore, for most regions of Russia, empirical methods of determining evapotranspiration, developed by various research organizations, have become widespread along with deterministic methods [2, 3].

2. ESTIMATION OF EVAPOTRANSPIRATION

Regardless of the method of calculation of evapotranspiration, its prediction, occurs using the classical regression methods.

The emergence and development of modern tools and data collection methods, along with the development of the mathematical apparatus for their analysis, allows the use of a more sophisticated mathematical apparatus for predicting the quantitative characteristics of evapotranspiration. The application of the methods of the Chebyshev interpolation theory that are proposed by us, allows us to obtain more accurate predicted values of evapotranspiration. As a result, it allows us to optimize the parameters and yields significant savings of resources. The aim of the research is the prediction of evapotranspiration based on the method of constructing its predictive polynomial using the Chebyshev interpolation method of a discretely given function.

The International Organization for Food and Agriculture of the United Nations (FAO UN) recommended the Penman-Monteith method as a standard method of calculation of evapotranspiration [1]:

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma\frac{900}{\mathrm{T}+273}u_2 d_a}{\Delta + \gamma(1 + 0.34\mathrm{u}_2)}, \qquad (1),$$

where ET_0 – evapotranspiration, mm/day; R_n – solar radiation on the surface of plants, Joule/m^{2}day; G – heat flow from the soil, Joule /m^{2}day; T – the average daily temperature at altitude 2 m, °C; u_2 – wind speed at altitude 2 m, m/s; d_a – lack of air humidity Pascal; Δ – vapor pressure gradient curve, Pascal/°C; γ – psychrometric constant, Pascal /°C.

In Russia, along with the deterministic methods of evapotranspiration calculations, empirical methods are widespread, especially those based on the functional dependence of evapotranspiration on temperature or humidity [4]. Among them are:

- Bioclimatic method of Alpat’eva and Alpat'eva [5], according to which evapotranspiration is directly proportional to the deficit of air humidity, given as

$$ET = k_{\text{ба}} \sum d \qquad (2).$$

Bioclimatic method of G.K. Lgov [6], according to which evapotranspiration is directly proportional to the average daily air temperature as

$$ET = k_{\text{бл}} \sum t, \qquad (3)$$

where ET – evapotranspiration, mm or m^3/ga; $k_{\text{бл}}$ – bioclimatic coefficient, mm/°C or m3/ga·°C; d – the amount of average daily air humidity deficit, t – the amount of average daily air temperatures °C.

3. CHEBYSHEV INTERPOLATION

Along with the above methods for calculating evapotranspiration, there are others developed by scientists from different countries (for example, [7-10] and others). Regardless of the method of calculation of evapotranspiration, its prediction occurs using the classical regression methods [11]. We suggest using the more advanced mathematical apparatus of the Chebyshev interpolation theory as a basis for predicting evapotranspiration. Compared to the regression and mean square approximation methods, the Chebyshev interpolation method is the most efficient and versatile. It also has a special property not only to obtain high approximation accuracy at discrete representation points of functional dependencies but also to provide the required guaranteed approximation accuracy.

Consider the data of quantitative assessment of evapotranspiration of ET, calculated by one of the methods, and the course of the vegetation period. For the beginning of the period, we take the third decade of April, for the end of the second decade of October.

Thus, the values of evapotranspiration of ET can be specified in the form of a discrete series:

Table 1: Evapotranspiration ET values

t	t_1	t_2	……	t_N
y	y_1	y_2	…….	y_N

Here, t is the decades, numbered in ascending order from 1 - the third decade of April, 2 - the first decade of May, etc. N - the second decade of October, a $y_k = f(t_k)$, $k = 0,1,..N$ accordingly, the value of ET in the corresponding period.

3.1 POLYNOMIAL FUNCTION FOR CHEBYSHEV APPROXIMATION

The task of the Chebyshev approximation of a function by a polynomial

Let the function f (t) be given by the table of its values $y_k = f(t_k)$, $k = 1,2,..N$ (Table 1).

Without loss of the generality, we can assume that $t_1 < t_2 \cdots < t_N$. Any polynomial (for example, trigonometric) $P_n(\vec{a}, t) = a_0 + \sum_{i=1}^{n} a_i \sin i * t$, the order of which $n \leq N$, in relation to a given table $\{t_k, y_k\}$ has a natural characteristic of proximity - maximum dodge:

$$\varphi(\vec{a}) = \max_{0 \leq k \leq N} |y_k - P_n(\vec{a}, t_k)| \tag{4}$$

Consider a fixed value of the order of the polynomial (n)

$$\min_{\vec{a}} \varphi(\vec{a}) = \rho \tag{5}$$

Then, polynomial $P_n(\vec{a}^*, t)$, for which the condition is met:

$$\phi(\vec{a}^*) = \rho \tag{6}$$

This is called **the best approximation polynomial of the table$\{t_k, y_k\}$**.

The solution of the best approximation problem (the construction of a polynomial $P_n(\vec{a}^*, t)$) significantly depends on the ratio between the values of the order of the polynomial (n) and the "length" of the table (N). Therefore, *if* $n = N$, then the task of constructing the polynomial of the best uniform approximation is transformed into the classical interpolation problem, for which $\rho = 0$. The first nontrivial version of the best approximation problem occurs when $N = n+1$. This task is known as **the Chebyshev interpolation problem**. Known methods can be applied to solve the problem (5) (see, for example, [12-14]). In particular, it can be solved through reduction to the linear programming problem [15]. We introduce the notation:

$\vec{a} = (a_0, a_1, \ldots, a_n) \in R^{n+1}$, $B_i = (1, \sin t_i, \sin 2t_i, \cdots, \sin n\, t_i) \in R^{n+1}$, $b_i = -y_i, i \in [1:N]$. Given that the polynomial $P_n(\vec{a}, t)$ can be represented as a scalar product $P_n(\vec{a}, t) = \langle \vec{a}, (1, \sin t, \sin 2t, \cdots, \sin n\, t) \rangle$, task (6) can be written in the form:

$$\max_{i\in[1:2N]} \{\langle B_i, \vec{a} \rangle + b_i\} \underset{\vec{a}\in R^{n+1}}{\rightarrow} \min \tag{7}$$

Then, according to [10], the problem (5) is equivalent to the linear programming problem of the following form:

$$\begin{cases} a_{n+1} \rightarrow \min \\ a_{n+1} - \langle B_i, \vec{a} \rangle - b_i \geq 0, i \in [1:N] \end{cases} \tag{8}$$

4. THE METHOD OF CONSTRUCTING A PREDICTIVE POLYNOMIAL BASED ON THE CHEBYSHEV APPROXIMATION PROBLEM

We believe that we are given the tabular values of evapotranspiration *of* ET (in the table is y) in accordance with the decades of the growing season t (Table 1).

Stages of construction include:

Block 1- "The degree of the polynomial ": Set the degree of the polynomial

$n < N, m \geq n+2$.

Block 2- "Constructing a polynomial":

For a given degree of a polynomial, choose the number of nodes used $m \geq n+2$ to solve the auxiliary problem.

Suppose $i = 0$.

Solve the problem:

$$\max_{i+1\leq k\leq i+m} |y_k - P_n(\vec{a}, t_k)| \underset{\vec{a}\in R^{n+1}}{\rightarrow} \min \tag{9}.$$

Assume that the vector of coefficients $\vec{a}_i^*$ is a solution to the problem (9), where $P_n(\vec{a}, t) = a_0 + \sum_{i=1}^{n} a_i \sin i * t$.

Block 3- "Building a series of polynomials": we are conducting a series of experiments for $n = 1,2,3$ и $m = n+2, n+3, n+4$.

Block 4- "Optimization of the polynomial": Analyze the results obtained and choose the optimal polynomial.

Implementation of the proposed interpolation algorithm will be carried out by using the example of alfalfa. Alfalfa has a high potential for increasing yields with sufficient irrigation.

We carry out computational experiments using a program that implements obtaining the values of the coefficients of polynomials based on solving the problem of the Chebyshev approximation. This program was implemented in the Matlab environment [16], and used to build indicators of commodity markets (for example, [17]). The program code element has the following form:

```
Function program = program
n=input ('Enter value n='); {Enter the degree of a polynomial}
m=input ('Enter value m=');
maxi= load('max.txt'); { Array of values }
k = 0;
….
y = linprog(q, G, H);{ Finding optimal coefficients for the Chebyshev approximation}
u(l,l) = l;
for j = 2:(n+l)
u(j,l) =sin(k*(j-l));
end
…
time = ((m+l):18);        {Plotting graphs}
plot(time,maxi((m+l):18),'b',time,mini((m+l):18),'k',time,v,'ro-',time,w,'g',time,z,'y');
grid on;
end
```

5. CONSTRUCTION OF A PREDICTIVE POLYNOMIAL OF EVAPOTRANSPIRATION OF ALFALFA BASED ON THE PROBLEM OF THE CHEBYSHEV APPROXIMATION

We set the table of values $y_к = y(t_k), k \in [1:N]$ evapotranspiration of ET alfalfa during the t_k decades of the growing season, numbered in ascending order. Where t_1 - the third decade of April, t_{18} - the second decade of October. The data are given for the Saratov region [18].

Table 2: Numerical values of *ET* evapotranspiration

t	1	2	3	4	5	6	7	8	9
y	11,78	15,8	28,1	54,3	67,1	22,5	31,8	33,1	37,3
t	10	11	12	13	14	15	16	17	18
y	44,2	18,2	24	25,2	28,6	39	18,4	7,8	7,5

The discrete graph is shown in Figure 1.

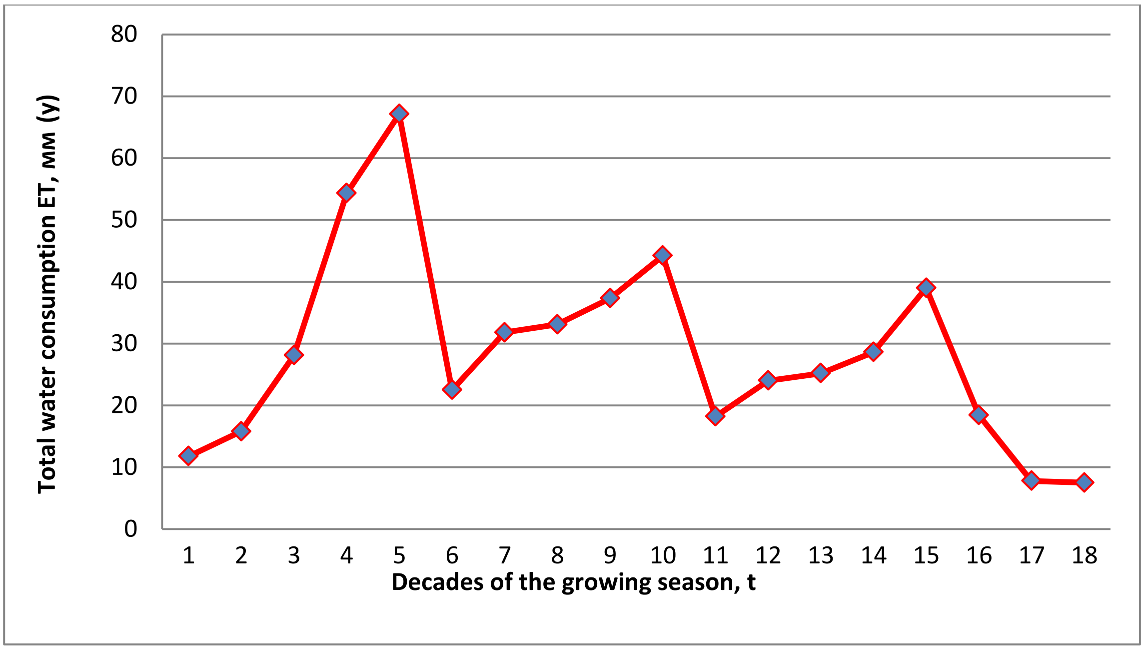

Figure 1: Discrete graph of the evapotranspiration function of the $ET(y)$

Based on the above method, it is possible to build a forecast polynomial over the entire growing season. However, as it is known that [19], the average daily water consumption of alfalfa increases with the development of plants: in the “regrowth – branching” period, it is in the range of 10–30 m^3/ha, “Branching – budding” 30–50 m^3/ha and in the “budding – flowering” period 50–70 m^3/ha. Besides, the magnitude of the total water consumption of alfalfa has certain dynamics in mowing: the highest water consumption is the first mowing (36.3 - 38.7%) of the total water consumption; slightly lower water consumption was the second (31.1 - 33.1%) and the third cut was characterized by a decrease in crop productivity (27.5 - 28.9%) of the total water consumption [18]. That is, in alfalfa, the value of evapotranspiration substantially depends on the stages of development and cutting. So, for greater accuracy, it is desirable to consider shorter intervals when constructing a predictive polynomial. In addition, the value of evapotranspiration depends on climatic conditions. In this regard, we have considered the numerical parameters for years with different climatic characteristics: medium-arid (y_{MD}) and moderately humidified (y_{MM}).

We will analyze the two periods. 1-5- $th(t_1 - t_5)$ and 6-10- $th(t_6 - t_{10})$.

First, we construct a graph of the function y (evapotranspiration of the ET) together with the graph of the linear regression equation over the gap (Figure 2).

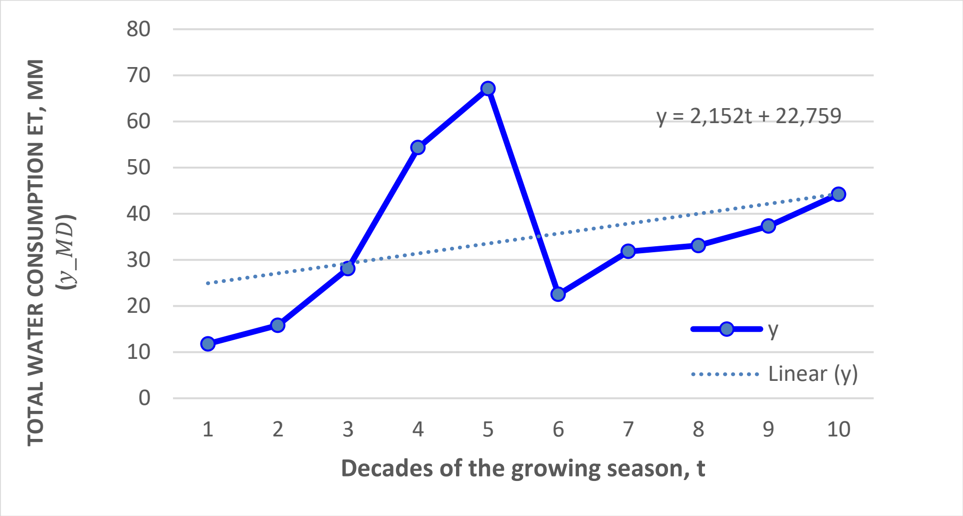

Figure 2: Graph of the evapotranspiration function $ET(y)$ together with the linear regression equation

As forecast polynomials, we will consider trigonometric polynomials of the first degree $P = a_0 + a_1 \sin t$ and second degree $Q = a_0 + a_1 \sin t + a_2 \sin 2t$. As practice shows, an increase in the degree of a polynomial only leads to an increase in the number of calculations without improving the accuracy of the forecast.

Having conducted numerical experiments according to the proposed method, we find that for the specified period, the predictive polynomials of the first and second degree, for medium-dry years respectively have the following form:

$P = 34{,}08 - 23{,}3 \sin t$, $Q = 34{,}39 - 33{,}14 \sin t - 0{,}67 \sin 2t$.

For moderately moisturized:

$P = 30{,}29 - 20{,}4 \sin t$, $Q = 30{,}5 - 27 \sin t - 0{,}5 \sin 2t$.

Evapotranspiration ET schedules together with graphs of forecast polynomials of the first (P) and second (Q) degrees for medium-arid (y_{MD}) and moderately moist (y_{MM}) years are shown in Figures 3 and 4. Values are for $1-5th$ periods (decade of the growing season, numbered in ascending order).

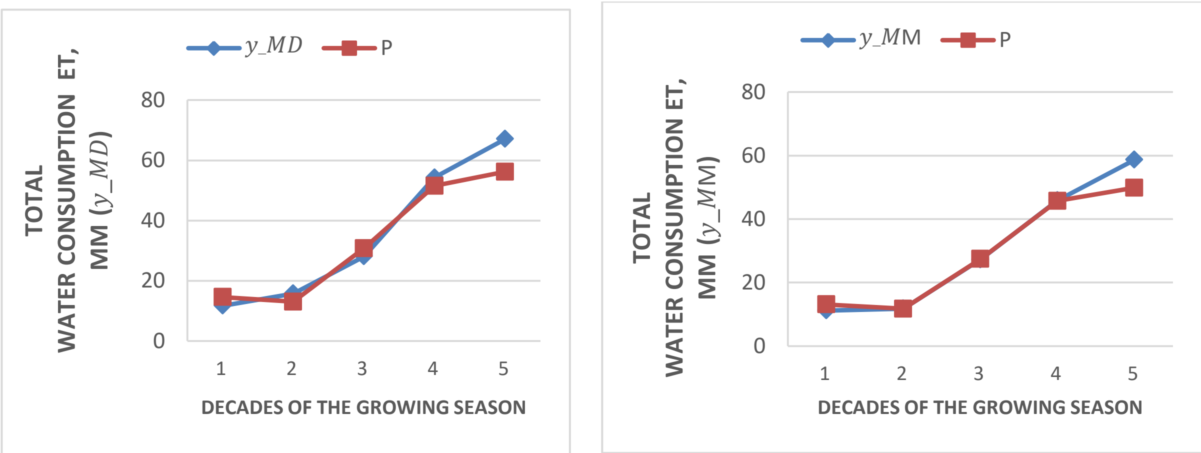

Figure 3: Graphics of evapotranspiration of ET together with graphs of predictive polynomials of the first degree P for medium-arid (y_{MD}) and moderately moisturized (y_{MM}) years, period $1-5th$

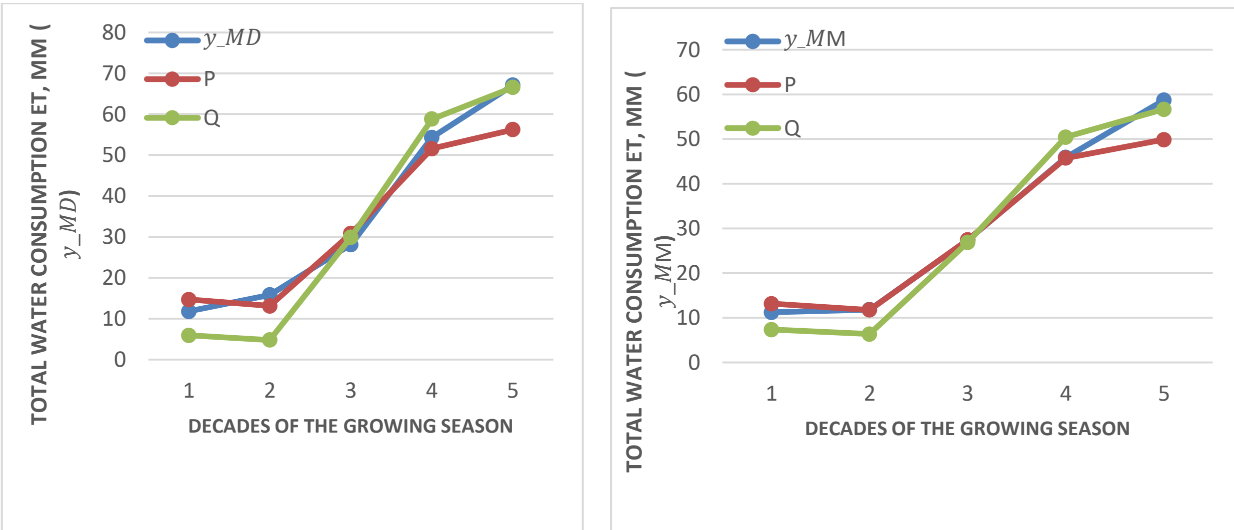

Figure 4: Evapotranspiration ET schedules together with graphs of forecast polynomials of the first (P) and second (Q) degrees for medium-arid (y_{MD}) and moderately humidified (y_{MM}) years, period $1-5^{th}$

Similarly, we will conduct numerical experiments according to the proposed method for the 6th-10th periods. We find that for the specified period, the predictive polynomials of the first and second degree, for medium-dry years respectively have the following form:

P= $50{,}9-22{,}46\sin t$, Q= $40{,}3-8{,}7\sin t+0{,}25\sin 2t$.

For moderately moisturized:

P= $48{,}6-16{,}7\sin t$, Q= $38{,}3-3{,}4\sin t+0{,}28\sin 2t$.

Evapotranspiration ET graphs together with the graphs of the forecast polynomials of the first (P) and second (Q) degrees for medium-arid (y_{MD}) and moderately humidified (y_{MM}) years are shown in Figures 5 and 6. The values are for $6th-10th$ periods.

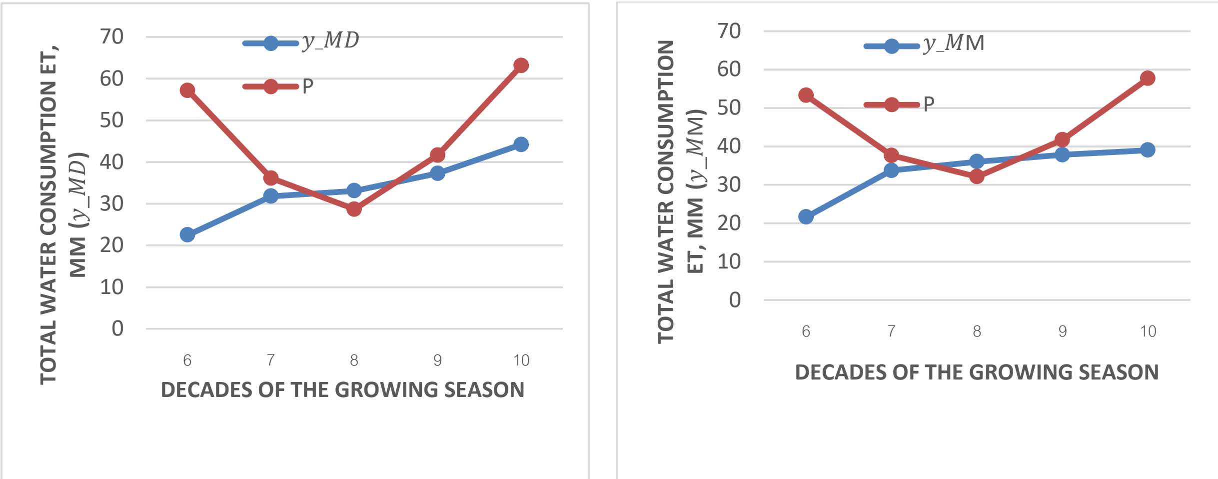

Figure 5: Evapotranspiration of ET charts in conjunction with the graphs of forecast polynomials of the first degree P for medium-arid (y_{MD}) and moderately wetted (y_{MM}) years, period $6-10^{th}$

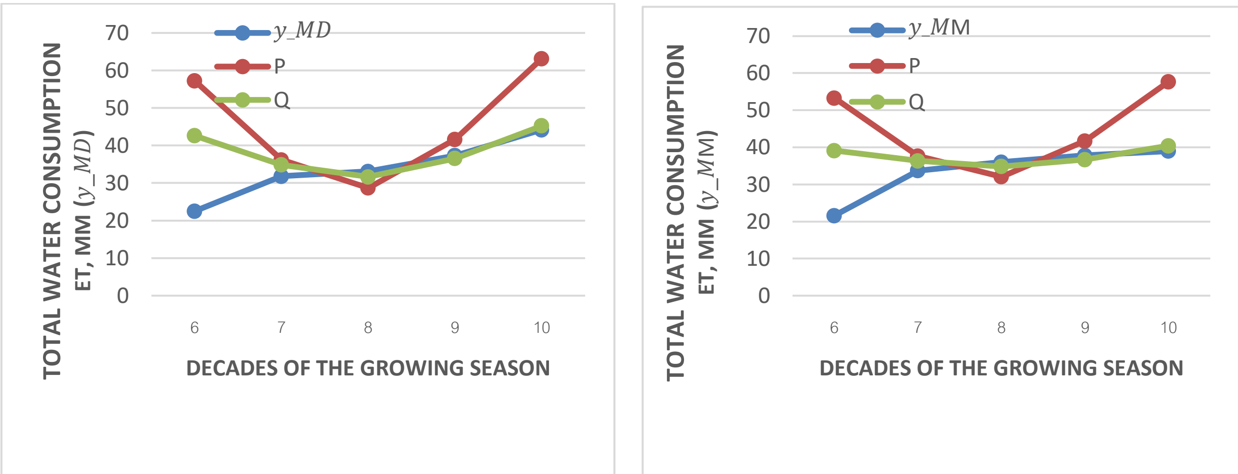

Figure 6: Evapotranspiration ET schedules together with the graphs of forecast polynomials of the first (P) and second (Q) degrees for medium-arid (y_{MD}) and moderately humidified (y_{MM}) years, period $6-10^{th}$

Analyzing the obtained results, we find that for the period of $1^{st}-5^{th}$, the total deviations of the predicted polynomials of the first degree from the actual evapotranspiration values are 4-8% less. Meanwhile, the corresponding total deviations of the predicted polynomials of the second degree are 6-12% less than the total deviations from the calculated evapotranspiration values calculated by other methods. Thus, for this period, it is advisable to choose a polynomial of the first degree as a predictive polynomial, calculating its coefficients by the proposed method.

For the same period, the $6^{th}-10^{th}$ total deviations of the predicted polynomials of the first degree from the actual values of evapotranspiration are one third larger, corresponding to the total deviations of the predicted polynomials of the second degree. The total deviations from the calculated evapotranspiration value calculated by other methods are greater than the total deviations of the predicted polynomials of the second degree and less than those of the first-degree polynomials. Thus, for this period, it is advisable to choose a

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second-degree polynomial as a predictive polynomial. The dependency graphs shown in Figures 5 and 7 demonstrate this well. Moreover, as we can see, the forecast trend is maintained for years of different humidity (in our example, medium-dry (y_{MD}) and moderately wetted (y_{MM}) years, the coefficients of polynomials will be different.

6. CONCLUSION

On the basis of the Chebyshev interpolation theory of a discretely given function, a method for constructing a predictive polynomial for predicting evapotranspiration has been proposed. A program has been developed on the basis of the proposed algorithm (implemented in the Matlab environment) that is implemented, obtaining the values of the coefficients of polynomials.

Comparison of the prediction research results with those calculated by the proposed method, based on the example of alfalfa, gives a good match. It also indicates the correctness of the adopted methodology and reliability of the dependences obtained. This technique can be extended to different climatic conditions and crops while maintaining the accuracy of the approximation.

7. AVAILABILITY OF DATA AND MATERIAL

Data used or generated from this study can be requested to the corresponding author.

8. REFERENCES

[1] Chapter 2 - FAO Penman-Monteith equation. - [Electronic resource], access mode: http://www.fao.org/docrep/x0490e/x0490e06.htm

[2] Korsak, V. V. Determination of total water consumption of crops in arid zones / V. V. Korsak, Prokopets R.V., Nikishanov A.N., Arzhanukhina E.V., Yudina M.R. // Scientific life, 2016, №1, - P. 41-51.

[3] Cheremisinov, A.A. Review of computational methods for determining the total evaporation of irrigated agricultural fields / A.A. Cheremisinov, A.Yu. Cheremisinov // Scientific journal of the Russian Research Institute of Land Reclamation Problems, No. 1 (21), 2016, pp. 113–133.

[4] Konstantinov A.P., Subbotin A.S. Methods of irrigation rationing: Tutorial. - L .: LGMI /, 1981, 81 p.

[5] Alpat'ev, A.M. Moisture turns in nature and their transformation. - SPb .: Gidrometeoizdat, 1969. - 324 p.

[6] Lgov, G.K. Irrigated agriculture. - M .: Kolos, 1979. 191 p.

[7] Allen, R. G. Using the FAO - 56 case study report / Journal of Hydrology. - V.229. -2000.- P. 27-41.

[8] Jensen, J.R. The role of evatranspiration models in irrigation scheduling / J.R. Jensen, J.L. Wright // Transactions of the ASAE. –1978. –V. 21. - P. 82–87.

[9] Jensen, M.E. Evapotranspiration and Irrigation Water Requirements / M.E.Ensen, R. D. Burman; edited by R. G. Allen; Am. Soc. Civil Engrs. - N. Y., 1990 - 360 p. - (ASCE Manuals and Reports on Engineering Practices, No. 70).

[10] Penman-Monteith, FAO-24 reference crop evapotranspiration and Australia. F. H. S. Chiew, N. N. Kamadalasa, H. M. Malano, T. A. McMahon // Agric. Water Management. - 1995 - Vol. 28 - p. 9–21.

[11] Khan G.J., Shapiro S.S. Statistical models in engineering problems. - Moscow, Mir, 1969 - 270 p.

[12] Dzyadyk, V.K. Introduction to the theory of uniform approximation of functions by polynomials / V.K. Dzyadyk M .: Publishing house "Science", 1977. -514 p.

[13] Dudov S.I., Zlatorunskaya I.V. Best approximation of a compact arbitrary norm // Advances in Mathematics Research. Nova Science Publishers: 2003 v.2, p.81-114.

[14] Dudov S.I., Romanov R.O. On the external estimation of the segment function by a polynomial strip // Math. Mechanics: Sat. scientific tr. Saratov: Publishing House of Saratov University. 2014. issue 16. p.31-35.

[15] Demyanov V.F., Malozemov V.N. Introduction to minimax. M .: Science, 1972.-368s.

[16] Dyakonov, V.P. MATLAB. Full tutorial / V. P. Dyakonov. M .: DMK Press, 2012.- 768c.

[17] Dudov S.I., Kamyshova G.N., Terekhov P.O. On one approach to forecasting in the grain market on the basis of the price corridor // "Mathematical and computer modeling in economics, insurance, and risk management": a collection of materials from the VI International Youth Scientific and Practical Conference. Saratov: Publisher Sarat. Un-ta, 2017. P. 53-57.

[18] Pankova T.A. Improving the efficiency of water use in the irrigation of alfalfa on the dark chestnut soils of the dry steppe Trans-Volga. - Saratov .: Publishing Center Science, 2017. 110c.

[19] Machado, A. D. B., Souza, M. J., & Catapan, A. H. (2019). Systematic Review: Intersection

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