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PERFORMANCE EVALUATION USING NETWORK DATA ENVELOPMENT ANALYSIS APPROACH WITH GAME THEORY UNDER MIXED GREY-FUZZY UNCERTAINTY IN IRAN KHODRO COMPANY

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ABSTRACT

Standard Data Envelopment Analysis (DEA) is a method for performance evaluation which does not consider the internal relations between decision making units (DMUs). Evaluating DMUs as black-boxes makes the evaluation unreal. Multistage DEA considers the internal structure of DMUs. In this article, all the outputs of first stage will be the only inputs of second stage which are also named intermediate measures. Both cooperative and non-cooperative games can be useful to obtain efficiency scores of DMUs. This study have used Nash bargaining game, Centralized and Stackelberg game to obtain efficiency of each DMU and the result show that efficiencies obtained from centralized game are the same as efficiencies obtained from Stackelberg game and they are greater than efficiencies obtained from Nash game. Also in real world the data is not always certain. We use both fuzzy and grey theory to widely manage the real situation. The case study of this article is Iran Khodro Company which is one of the most important companies at automobile industry and has a wide process of delivering for automobiles.

1. INTRODUCTION

Performance evaluation always is one of the most important activities to survey current situation and to discover improvement opportunities. Among lots of methods, Data Envelopment Analysis (DEA) is one of the best, because of its ability to spot multiple inputs and outputs. Network DEA considers the internal relation between inputs and outputs. In many cases outputs from the first stage become the inputs to the second stage. Outputs from the first stage are referred to as intermediate measures. For example, Seiford and Zhu [1] use a two- stage network structure to measure the profitability and marketability of US commercial banks. Profitability is measured relative to labor and assets as inputs, and the profits and revenues are outputs. In the second stage, for marketability, the profits and revenue are then used as inputs, while the outputs are market value, returns and earnings per share. Chilingerian and Sherman [2] evaluate measuring physician care with a two-stage process. The first stage with inputs including registered nurses, medical supplies, capital and fixed costs is a manager-controlled process. The outputs are patient days, quality of treatment, drugs dispensed among others. These outputs are also the inputs of the second stage. The second stage is physician-controlled. Research grants, quality of patients, and quantity of individuals trained by specialty are the outputs of the second stage.

These DEA approaches with two-stage network structure use the standard DEA approach which does not consider the potential conflicts between the two stages arising from the intermediate measures. For example, in order to achieve an efficient status, the second stage may have to reduce its inputs (intermediate measures). So the outputs of the first stage will reduce, thereby the efficiency of first stage will reduce.

To solve such conflict, Kao and Hwang [3] combine the efficiency scores of the two stages in a geometric manner, and Chen et al. [4] aggregate the two stages using weighted additive model. Liang et al. [5] using game theory concept developed some DEA models. Specifically, Liang et al. [5] develop a Stackelberg game model of a centralized or cooperative game Model.

This paper applies both cooperative and non-cooperative game to obtain the efficiency score of stages in the existence of mixed uncertainties.

2. LITERATURE REVIEW

The overall efficiency of network DEA models is the product of the efficiency of the different processes so alternative efficiency decompositions are possible. Therefore there can be different efficiency scores that correspond to the same level of overall efficiency. So there can be multiple alternative efficiency decompositions and the problem is which efficiency decomposition is better to use. In two-stage systems, there are different approaches of solving the uncertainty about how the processes efficiencies should be computed. One approach is to compute the best and worst possible efficiency scores of each process, by choosing the best score for one process and the worst for the other process, depending on which process efficiency is the decision maker more concerned with [5],[6]. The problem with this approach is that the analyst has to establish an order or ranking of the importance of the processes, something which is neither easy nor practical in the case of more than two stages [8].

Despotis et al. propose a model for computing an Ideal Point with the largest possible efficiency scores of each stage and then determine the process efficiencies using the lexicographic weighted Chebycheff method. This approach can be applied to general multistage networks. Another approach in the case of two-stage systems is to look for efficiency decompositions based on game theory [8]. DMUs are viewed as players in a game, payoffs are cross-efficiency scores, and each DMU may choose to take a game to maximize its payoff [9].

A Stackelberg game was proposed by Liang et al. [8]. This type of leader-follower game is difficult to extend and would imply an ordering of the importance of the different processes. For the case of two-stage systems, Du et al. [25] have proposed the Nash bargaining solution. This is a cooperative game approach and can be used to the multistage systems.

3. APPROACH

In this part the approach using three different methods of game theory considering the uncertainty of both fuzzy and grey theory is presented.

3.1 FUZZY SETS THEORY

Fuzzy numbers and linguistic variables are presented as following:

Variables whose values are words or sentences in natural or artificial languages are named linguistic variables [10].

The values of linguistic variables can be quantified. For instance, performance ratings of alternatives can be described using linguistic variable, such as very bad, bad, partly bad, mediate, partly good, good and very good, given by the decision makers (DMs). Normal interval grey numbers can represent these linguistic positive values, including [0.0, 0.1], [0.1, 0.3], [0.3, 0.4], [0.4, 0.6], [0.6, 0.7], [0.7, 0.9], and [0.9, 1.0], respectively.

3.2 GREY RELATIONAL ANALYSIS

Grey relational analysis provides a mathematical way to evaluate the correlation between the series that compose a set space [11], [12], [13]. Each alternative contains a set of criteria. A grey relational space is the set of values of all alternatives together. The grey relational analysis can capture the correlations between the reference level and other compared factors of a system [14]. The grey relational analysis can recognize both qualitative and quantitative relationships among complex factors in a system [15]. Various normalization methods can be employed to express criteria in dimensionless units in order to be comparable [16]. Following are some concepts [17], [18], [19].

Suppose X is a decision set of grey relations, $x0 \in X$ the referential sequence and $xi \in X$ the comparative sequence with x0(k) and xi (k) representing, respectively, the numerals at point k for x0 and xi. The relation $\gamma(x0(k), xi(k))$ is the grey relational coefficient of these factors in point k if $\gamma(x0(k), xi(k))$ and $\gamma(x0 , xk)$ are real numbers and satisfy the following four grey axioms, and the average value of $\gamma(x0(k),xi(k))$ is the grade of grey relation $\gamma(x0 ,xk)$.

(1) Norm interval

$0 \leq \gamma(x_0 ,x_k) \leq 1, \forall k$; if $x_0 = x_i$ then $\gamma(x_0 ,x_k)=1$,; if $x_0 = x_i \in \phi$, where ϕ is an empty set then$\gamma(x_0 ,x_k)=0$,.

(2) Duality symmetric

If $x, y \in X$ and $X=\{x,y\} \Rightarrow \gamma(x,y)=\gamma(y,x)$,

(3) Wholeness

If $X=\{x_i|i=0,1,\dots,n\}$, $n>2$ then $\gamma(x_i ,x_j) \neq \gamma(x_j ,x_i)$.

(4) Approachability

When $|x_0(k)-x_i(k)|$ is increasing then $\gamma(x_0(k),x_i(k))$ is decreasing.

According to the above four axioms, the grey relational coefficient is computed by

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$$\xi_i(k)=\frac{\min_i \min_k |[u_0^-(k),u_0^+(k)]-[r_{ik}^-,r_{ik}^+]\ | + \rho \max_i \max_k |[u_0^-(k),u_0^+(k)]-[r_{ik}^-,r_{ik}^+]|}{|[u_0^-(k),u_0^+(k)]-[r_{ik}^-,r_{ik}^+]\ | + \rho \max_i \max_k |[u_0^-(k),u_0^+(k)]-[r_{ik}^-,r_{ik}^+]|} \quad (1)$$

Where $|x_0(k)-x_i(k)|=\Delta_i(k)$, and ζ is the distinguished coefficient. The distinguished coefficient lies between 0 and 1. A value of 0.5 has been employed in the real-life situations [19], [20].

3.3 PROPOSED FUZZY GREY RANKING METHOD

The procedure of the fuzzy grey ranking method is consist of the below issues.

If a multiple attribute decision-making problem with interval numbers has m feasible plans $X_1, X_2, \ldots, X_m$ and n indexes $G_1, G_2, \ldots, G_n$ and the index value of j-th index Gj of alternative Xi is an interval number $[a_{ij}^-, a_{ij}^+]$, $i = 1,2, \ldots, m$, $j = 1,2, \ldots, n$.

Briefly in our approach, first and second stage prepare the data, third stage provides an ideal vector, fourth stage calculates connection coefficients based on decision maker selection of the distinguishing coefficient.

Calculating Grey Relational Analysis is consisting of following steps:

Step1. Convert linguistic or fuzzy variables to normal interval grey numbers according Table 1.

Obviously this method does not calculate positive criterion the same as negative criterion. Positive criterion or benefit criterion is the criterion that, the greater Gj is, better its performance. For example job experience or output of a decision maker unit.

And negative criterion or cost criterion is the criterion that, the smaller Gj is, better its performance. For example transportation cost or input of a decision maker unit. Linguistic variables are related to normal interval grey numbers in Table1.

Table 1.Linguistic variables related to normal interval grey numbers

Linguistic variable		normal interval grey number
for positive criterion	for negative criterion	
Very bad	Very high	[0.0,0.1]
bad	high	[0.1,0.3]
partly bad	partly high	[0.3,0.4]
mediate	mediate	[0.4,0.6]
partly good	partly low	[0.6,0.7]
good	low	[0.7,0.9]
Very good	Very low	[0.9,1.0]

Step2. Construct decision matrix A with normal interval grey numbers.

$$A=\begin{bmatrix} [r_{11}^- \ , \ r_{11}^+] & [r_{12}^- \ , \ r_{12}^+] & \cdots & [r_{1n}^- \ , \ r_{1n}^+] \\ [r_{21}^- \ , \ r_{21}^+] & [r_{22}^- \ , \ r_{22}^+] & \cdots & [r_{2n}^- \ , \ r_{2n}^+] \\ \cdots & \cdots & \cdots & \cdots \\ [r_{m1}^- \ , \ r_{m1}^+] & [r_{m2}^- \ , \ r_{m2}^+] & \cdots & [r_{mn}^- \ , \ r_{mn}^+] \end{bmatrix} \quad (2)$$

Step3. Determine reference number sequence.

The element of reference number sequence is composed of the optimal weighted interval number index value of every plan

$$U_0 = ([u_0^-(1), u_0^+(1)], [u_0^-(2), u_0^+(2)], \ldots, [u_0^-(n), u_0^+(n)]) \quad (3)$$

is called a reference number sequence if

$$u_0^+(j) = \max_{1\le i\le m} r_{ij}^+, j=1,2,\ldots,n \quad u_0^-(j) = \max_{1\le i\le m} r_{ij}^-$$

Step4. Calculate the connection between the sequences composed of interval number standardizing index value of every plan and reference number sequence.

First, calculate the connection coefficient $\xi_i(k)$ with formula (1) between the sequence composed of interval number standardizing index value of every plan $U_i = ([r_{i1}^-, r_{i1}^+], [r_{i2}^-, r_{i2}^+], \ldots, [r_{in}^-(n), r_{in}^+(n)])$ and reference number sequence with formula(3)

Here $\rho \in [0,1]$ is called a distinguishing coefficient. The smaller ρ is, the greater it's distinguishing power. In general, the value of ρ may change according to the practical situation. The classical grey-related parameter ρ is equivalent to the proportion of emphasis given to the max function. Our study follows most existing work and sets a classical grey-related parameter to 0.5.

3.4 NETWORK DATA ENVELOPMENT ANALYSIS APPROACH

In the standard DEA, the units are treated as black-boxes and internal structures of DMUs are ignored. So Network Data Envelopment Analysis is suggested.

Two-stage network structures are processes where outputs from the first stage become inputs to the second stage. The outputs from the first stage are called intermediate measures.

The standard DEA approach does not address potential conflicts between the two stages arising from the intermediate measures. For example, in order to achieve an efficient status, the second stage may have to reduce its inputs (intermediate measures). Such an action would reduce the first stage outputs, so the efficiency of that stage will reduce.

Liang et al. [5] solved such conflict, they developed a number of DEA models using game theory concept. Specifically, they developed a leader–follower model, and a centralized or cooperative game model.

4. GAME THEORY

Game theory helps NDEA (Network Data Envelopment Analysis) to obtain the efficiencies of DMUs. First we explain about three different models, and then we use and compare them.

4.1 NASH BARGAINING GAMe MODEL

Two stages are two individuals bargaining with each other for a better payoff, which is the efficiency of each individual stage. This paper shows that non-linear Nash bargaining model can be converted into a linear programming problem which has one parameter whose lower and upper bounds can be determined. In this model, the standard DEA model determines the breakdown or status quo point. The selection of the breakdown point will affect the bargaining efficiency scores of the two stages.

Figure 1 shows a two-stage process. We suppose there are n DMUs and each DMUj(j = 1, 2, ..., n) has *m* inputs to the first stage, denoted by x_{ij}(i = 1, 2,..., *m*), and d outputs from this stage, denoted by z_{dj}(d = 1, 2, ..., D). Then these d outputs become the inputs to the second stage, which are called intermediate measures.y_{rj}(r =1, 2,... , s) shows the s outputs from the second stage.

Based upon the constant returns to scale (CRS) model, the (CRS) efficiency scores for each DMUj(j = 1,2,. . . ,n) in the first and second stages can be defined by e_j^1 and e_j^2, respectively,

$$e_j^1 = \frac{\sum_{d=1}^{D} w_d^1 z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \quad e_j^2 = \frac{\sum_{r=1}^{s} u_r\, y_{rj}}{\sum_{d=1}^{D} w_d^2 z_{dj}} \leq 1 \qquad (4)$$

where v_i, w_d^1, w_d^2 and u_r are unknown non-negative weights. Then in a linear fractional programming problem which can be converted into a linear CRS DEA model, these ratios are optimized [22].

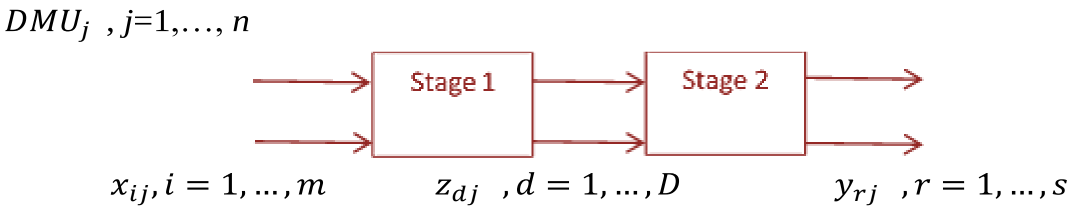

Figure1: The two-stage process.

As noted both in Kao and Hwang [3] and Liang et al. [5], it is reasonable to set w_d^1 equal to w_d^2, since the value assigned to the intermediate measures should be the same regardless of whether they are viewed as inputs to the second stage or outputs from the first stage. Then in this case, given the individual efficiency scores e_j^1 and e_j^2, we define the overall efficiency of the entire two-stage process for DMUj (j = 1,. . . ,n) as $e_j = e_j^1 . e_j^2$ since

$$e_j = \frac{\sum_{r=1}^{s} u_r\, y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} = \frac{\sum_{d=1}^{D} w_d^1 z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} . \frac{\sum_{r=1}^{s} u_r\, y_{rj}}{\sum_{d=1}^{D} w_d^2 z_{dj}} = e_j^1 . e_j^2 \qquad (5)$$

The definition (Equation (5)) ensures that $e_j \leq 1$ from $e_j^1 \leq 1$ and $e_j^2 \leq 1$, also the overall process is efficient if and only if $e_j^1 = e_j^2 = 1$.

The Nash approach is to regard the process as a centralized model, where the overall efficiency given in (5) is maximized, and by finding a set of multipliers, a decomposition of the overall efficiency is obtained. This decomposition produces the largest first (or second) stage efficiency score while maintaining the overall efficiency score.

We first briefly introduce the Nash bargaining game approach.

The set of two players in the bargaining is denoted by N = {1, 2}, and a payoff vector is an element of the space R^2. We assume S as a feasible subset of the payoff space, and a breakdown point $\overrightarrow{b}$ is an element of the payoff space. A bargaining problem can be specified as the triple (N, S, $\overrightarrow{b}$) consisting of participating individuals, feasible set, and breakdown point. The feasible set should be compact, convex, and contain some payoff vector such that each individual's payoff is at least as large as the individual's breakdown payoff [23]. The solution is a function F (N, S,$\overrightarrow{b}$) that is associated with each bargaining problem (N, S, $\overrightarrow{b}$). A reasonable solution should satisfy the four properties: (i) Pareto efficiency (PE), (ii) invariance with respect to affine transformation (IAT), (iii) independence of irrelevant alternatives (IIA), and (iv) symmetry(SYM) [23], [24]. These properties are extensively discussed in the literature.

We regard the two individual stages as two players, the efficiency ratios as the payoffs, and weights chosen for efficiency scores as strategies. To proceed, one needs to find a breakdown point for stages 1 and 2. If one decides not to bargain with the other player, the breakdown point represents possible payoff pairs obtained. A number of elements can be natural candidates for this role.

If the two stages do not negotiate, their efficiency scores will be the worst. Note that such a DMU may not exist, however, its inputs and outputs are observed. Let $x_i^{max} = \max_j\{x_{ij}\}$, $y_r^{min} = \min_j\{y_{rj}\}$, $z_d^{min} = \min_j\{z_{dj}\}$ and $z_d^{max} = \max_j\{z_{dj}\}$ then $(x_i^{max} \ , z_d^{min} \ )$ (i=1,…,m , d=1,…,D) represents the least ideal DMU in the first stage, they consume the maximum amount of input values, and produce the least amount of intermediate measures. Similarly in the second stage, we denote $(z_d^{max} \ , y_j^{min})$ (d=1,… , D, r=1,…,s) as the least ideal DMU, which consumes the maximum amount of intermediate measures while producing the least output.

The worst CRS efficiency is the above two least ideal DMUs. The (CRS) efficiency scores of the two least ideal DMUs in the first and second stage are denoted as θ_{min}^1 and θ_{min}^2, respectively.We use θ_{min}^1 and θ_{min}^2 as our breakdown point. Then our (input-oriented) DEA bargaining model for a specific DMU_0 can be expressed as [6]

$$
\begin{aligned}
&\max \quad \alpha \times \sum_{r=1}^{s} \mu_{r2} y_{r0} - \theta_{min}^1 \sum_{r=1}^{s} \mu_{r2} y_{r0} - \theta_{min}^2 \sum_{d=1}^{D} \omega_d z_{d0} + \theta_{min}^1 \theta_{min}^2 \\
&s.t. \sum_{d=1}^{D} \omega_d z_{d0} \geq \theta_{min}^1 \\
&\sum_{r=1}^{s} \mu_{r2} y_{r0} \geq \theta_{min}^2 \\
&\sum_{i=1}^{m} \gamma_i x_{i0} = 1 \\
&\sum_{d=1}^{D} \omega_d z_{d0} = \alpha \\
&\sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \leq 0 \qquad j = 1, \dots, n \\
&\alpha \times \sum_{r=1}^{s} \mu_{r2} y_{rj} - \sum_{d=1}^{D} \omega_d z_{dj} \leq 0 \qquad j = 1, \dots, n \\
&\mu_{r1} = \alpha \mu_{r2} \qquad r = 1, .., s \\
&\alpha, \gamma_i, \ \omega_d, \mu_{r1}, \mu_{r2} > 0 \qquad r = 1, .., s, i = 1, \dots, m, d = 1, \dots, D \qquad (6)
\end{aligned}
$$

Note the constraints in model (6) that $\sum_{d=1}^{D} \omega_d z_{d0} \geq \theta_{min}^1$, $\sum_{i=1}^{m} \gamma_i x_{i0} = 1$, $\sum_{d=1}^{D} \omega_d z_{d0} = \alpha$, and for any j = 1,…, n, $\sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \leq 0$. Then we have $\theta_{min}^1 \leq \alpha = \sum_{d=1}^{D} \omega_d z_{d0} \leq \sum_{i=1}^{m} \gamma_i x_{i0} = 1$, therefore we have both upper and lower bounds on α, and indicates that the first-stage efficiency score for each DMU is the optimal value of α.

Thus α will be a parameter within $[\theta_{min}^1, 1]$. Then model (6) can be solved as a parametric linear program via the possible α values within $[\theta_{min}^1, 1]$.

We set the initial value for α as the upper bound one, and solve the correspondinglinear program. Then we begin to decrease α by a positive number ε(=0.0001 for example) for each step t,α_t= 1-$\varepsilon \times t$, t = 1,2,… until the lower bound θ_{min}^1 is reached, and solve each linear program of model (6) corresponding to α_t and the corresponding optimal objective value is denoted by Ω_t.

Note that not all values taken by α within $[\theta_{min}^1, 1]$ lead to feasible solutions for program (6).

Let $\Omega^* = \max_t \Omega_t$ and denote the specific α_t associated with Ω^* as α^*. Note that Ω^*which is our solution to model (6), associated with several α^* values.

The relations $e_0^{1*} = \alpha^*(\sum_{d=1}^{D} w_d^* z_{d0})$, $e_0^{2*} = (\sum_{r=1}^{s} \mu_{r2}^* y_{r0})$, and $e_0^* = e_0^{1*}.e_0^{2*}$ are denoted as DMUo's bargaining efficiency scores for the first and second stages and the overall process, respectively.

Our bargaining model is not about finding the best overall efficiency score, but rather is about finding the best achievable efficiency through negotiation. A breakdown point (0,0) does not necessarily lead to the best achievable efficiency for Stage 1 or 2,but leads to the best overall efficiency score. A breakdown point of (0,0) implies that if the two stages do not negotiate, they will get an efficiency score of zero. This may further indicate that (0, 0) is not a good candidate for a breakdown point in our bargaining model.

The efficiency of DMUs was mentioned according to CRS models by GAMS program as described.

$$\theta_{\min}^1 = \max \sum_{d=1}^{D} \omega_d z_{d0}$$

$$s.t. \sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \quad \leq 0 \qquad j = 1, \dots, n$$

$$\sum_{i=1}^{m} \gamma_i x_{i0} = 1$$

$$\omega_d \geq 0, d = 1, \dots, D; \qquad \gamma_i \geq 0, \qquad i = 1, \dots, m; \qquad (7)$$

Similarly CRS model for stage 2 is as following.

$$\theta_{\min}^2 = \max \sum_{d=1}^{D} \mu_{r2} y_{r0}$$

$$s.t. \sum_{d=1}^{D} \mu_{r2} y_{rj} - \sum_{i=1}^{m} \omega_d z_{dj} \quad \leq 0 \qquad j = 1, \dots, n$$

$$\sum_{i=1}^{m} \omega_d z_{d0} = 1$$

$$\omega_d \geq 0, d = 1, \dots, D; \qquad \mu_{r2} \geq 0, \qquad r = 1, \dots, s; \qquad (8)$$

4.2 CENTRALIZED MODEL

According to cooperative game theory, or centralized control, the two stage process can be viewed as one where the stages jointly determine a set of optimal weights on the intermediate factors to maximize their efficiency scores [5] For example where the manufacturer and retailer jointly determine prices, order quantities, etc. to achieve maximum profit [26]. In other words, the centralized approach lets both stages be optimized simultaneously. As in Liang et al. [27], Kao and Hwang [3], and [5], the optimization can be based upon maximizing the average of $e_0^1$and e_0^2in a non-linear program. However, it is noted that because of the assumption $\omega_d^1 = \omega_d^2$in (6), the result is $e_j^1.e_j^2 = \frac{\sum_{r=1}^{s} \mu_r y_{rj}}{\sum_{i=1}^{m} \gamma_i x_{ij}}$ .Therefore, instead of maximizing the average of e_0^1 , e_0^2 we have

$$e_0^{centralized} = Max\, e_0^1 \,.\, e_0^2 = Max \;\frac{\sum_{r=1}^{s} \mu_r y_{r0}}{\sum_{i=1}^{m} \gamma_i x_{i0}}$$

$$s.t.\, e_j^1 \leq 1 \quad , e_j^2 \leq 1 \quad , \omega_d^1 = \omega_d^2 \qquad (9)$$

Model (9) can be converted into the following linear program format:

$$e_0^{centralized} = Max \quad \sum_{r=1}^{s} \mu_r y_{r0}$$

$$s.t. \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{d=1}^{D} \omega_d z_{dj} \leq 0 \qquad j = 1, \dots, n$$

$$\sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \quad \leq 0 \qquad j = 1, \dots, n$$

$$\sum_{i=1}^{m} \gamma_i x_{i0} = 1$$

$$\omega_d \geq 0, d = 1, \dots, D; \quad \gamma_i \geq 0, \ i = 1, \dots, m; \qquad \mu_r \geq 0, \ r = 1, \dots, s \qquad (10)$$

Model (10) is the centralized model developed in [5] and the Kao and Hwang [3] model.

The unique overall efficiency of the two-stage process is obtained from Model (10). Then we have the efficiencies for the first and second stages as below

$$e_0^{1,centralized} = \frac{\sum_{d=1}^{D} \omega_d^* z_{d0}}{\sum_{i=1}^{m} \gamma_i^* \ x_{i0}} = \sum_{d=1}^{D} \omega_d^* z_{d0}, \ \text{and} \ e_0^{2,centralized} = \frac{\sum_{r=1}^{s} \mu_r^* y_{r0}}{\sum_{d=1}^{D} \omega_d^* z_{d0}} \qquad (11)$$

If we denote the optimal value to model (10) as $e_0^{centralized}$, then we have $e_0^{centralized} = e_0^{1,centralized} . e_0^{2,centralized}$.

Optimal multipliers from model (10) are not unique, as noted in Kao and Hwang [3].Deriving the maximum achievable value of $e_0^{1,centralized}$ or $e_0^{2,centralized}$ is proposed. In fact as shown in [5], they tested whether $\mathrm{e}_0^{1,\mathrm{centralized}}$ and $\mathrm{e}_0^{2,\mathrm{centralized}}$ obtained from model (10), are unique or not. The maximum achievable value of $\mathrm{e}_0^{1,\mathrm{centralized}}$can be calculated via

$$\mathrm{e}_0^{1+} = \mathrm{Max} \ \sum_{d=1}^{D} \omega_d z_{d0}$$

$$s.t. \sum_{r=1}^{s} \mu_r \ y_{r0} = \mathrm{e}_0^{\mathrm{centralized}}$$

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{d=1}^{D} \omega_d z_{dj} \leq 0 \qquad j = 1, \dots, n$$

$$\sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \quad \leq 0 \qquad j = 1, \dots, n$$

$$\sum_{i=1}^{m} \gamma_i x_{i0} = 1$$

$$\omega_d \geq 0, d = 1, \dots, D; \qquad \gamma_i \geq 0, \ i = 1, \dots, m; \ \ \mu_r \geq 0, \ r = 1, \dots, s. \qquad (12)$$

So the minimum of $\mathrm{e}_0^{2,\mathrm{centralized}}$ will be obtained, in other words, $\mathrm{e}_0^{2-} = \frac{\mathrm{e}_0^{\mathrm{centralized}}}{\mathrm{e}_0^{1+}}$. And we have the maximum of $\mathrm{e}_0^{2,\mathrm{centralized}}$;

$$\mathrm{e}_0^{2+} = \mathrm{Max} \ \ \sum_{r=1}^{s} \mu_r y_{r0}$$

$$s.t. \sum_{r=1}^{s} \mu_r y_{r0} - \mathrm{e}_0^{\mathrm{centralized}} \sum_{i=1}^{m} \gamma_i x_{i0} = 0$$

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{d=1}^{D} \omega_d z_{dj} \leq 0 \qquad j = 1, \dots, n$$

$$\sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \quad \leq 0 \qquad j = 1, \dots, n$$

$$\sum_{d=1}^{D} \omega_d z_{d0} = 1$$

$$\omega_d \geq 0, d = 1, \dots, D; \qquad \gamma_i \geq 0, \ i = 1, \dots, m; \qquad \mu_r \geq 0, r = 1, \dots, s, \qquad (13)$$

And the minimum of $\mathrm{e}_0^{1,\mathrm{centralized}}$is $\mathrm{e}_0^{1-} = \frac{\mathrm{e}_0^{\mathrm{centralized}}}{\mathrm{e}_0^{2+}}$. Note that $\mathrm{e}_0^{1+} = \mathrm{e}_0^{1-}$if and only if $\mathrm{e}_0^{2+} = \mathrm{e}_0^{2-}$.It isobvious that if $\mathrm{e}_0^{1+} = \mathrm{e}_0^{1-}$ or $\mathrm{e}_0^{2+} = \mathrm{e}_0^{2-}$then $\mathrm{e}_0^{1,\mathrm{centralized}}$ and $\mathrm{e}_0^{2,\mathrm{centralized}}$ are uniquely determined by model (10).If $e_0^{1+} \neq e_0^{1-}$ or $e_0^{2+} \neq e_0^{2-}$then model (13) will obtain an alternative decomposition of$e_0^{1,centralized}$and $e_0^{2,centralized}$.

4.3 STACKELBERG GAME

Being a non-cooperative game, this game is characterized by the leader–follower, or Stackelberg game. For example, there is Stackelberg game in a supply chain where there is no cooperation between the manufacture (leader) and the retailer (follower). The manufacturer defines its optimal investment based on an estimation of the local advertisement by the retailer to maximize its profit. On the other hand, the optimal local advertisement cost of the retailer, based on the information from the manufacturer, will be determined to maximize retailer's profit [28]. If the first stage is the leader, then the first stage performance is more important, and the efficiency of the second stage is computed subject to the fixed efficiency of the first stage. First the efficiency for the first stage is calculated. The model for a specific DMU0 is written Based upon the CRS model.

$$
\begin{aligned}
& e_0^{1*} = \max \textstyle\sum_{d=1}^{D} \omega_d z_{d0} \\
& s.t. \textstyle\sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \quad \leq 0 \qquad j = 1, \dots, n \\
& \textstyle\sum_{i=1}^{m} \gamma_i x_{i0} = 1 \\
& \omega_d \geq 0, d = 1, \dots, D; \qquad \gamma_i \geq 0, \qquad i = 1, \dots, m; \qquad (14)
\end{aligned}
$$

Model (14) is the standard (CCR) DEA and the regular DEA efficiency score is indicated by e_0^{1*}. We obtain the efficiency for the first stage, so the second stage will only consider ω_d that maintains $e_0^1 = e_0^{1*}$. i.e., the second stage now treats $\sum_{d=1}^{D} \omega_d z_{d0}$ as the ''single'' input as a restriction that the efficiency score of the first stage remains at e_0^{1*}.

To compute e_0^2, the second stage's efficiency, we have [5]

$$
\begin{aligned}
& e_0^{2*} = Max \quad \frac{\sum_{r=1}^{s} U_r \, y_{r0}}{Q \sum_{d=1}^{D} w_d \, z_{d0}} \\
& s.t. \frac{\sum_{r=1}^{s} U_r \, y_{rj}}{Q \sum_{d=1}^{D} w_d \, z_{dj}} \leq 1 \qquad j = 1,2, \dots, n \\
& \textstyle\sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \quad \leq 0 \qquad j = 1, \dots, n \\
& \textstyle\sum_{i=1}^{m} \gamma_i x_{i0} = 1 \\
& \textstyle\sum_{d=1}^{D} \omega_d z_{d0} = e_0^{1*} \\
& U_r \ , Q, \omega_d, \gamma_i \geq 0, \ d = 1, \dots, D; \ \ i = 1, \dots, m; \ \ \mu_r \geq 0, \ r = 1, \dots, s \qquad (15)
\end{aligned}
$$

To make a linear model, let $\mu_r = \frac{U_r}{Q}$

$$
\begin{aligned}
& e_0^{2*} = Max \quad \frac{\sum_{r=1}^{s} \mu_r \, y_{r0}}{e_0^{1*}} \\
& s.t. \textstyle\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{d=1}^{D} \omega_d z_{dj} \leq 0 \qquad j = 1, \dots, n \\
& \textstyle\sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \quad \leq 0 \qquad j = 1, \dots, n
\end{aligned}
$$

$\sum_{i=1}^{m} \gamma_i x_{i0} = 1$

$\sum_{d=1}^{D} \omega_d z_{d0} = e_0^{1*}$

$$\omega_d \geq 0, d = 1, \dots, D; \quad \gamma_i \geq 0,\ i = 1, \dots, m; \quad \mu_r \geq 0,\ r = 1, \dots, s \qquad (16)$$

Using model (16) we have $e_0^{1*} . e_0^{2*} = \sum_{r=1}^{s} \mu_r^* y_{r0}$ at optimality, with $\sum_{i=1}^{m} \gamma^*{}_i x_{i0} = 1$. In other words $e_0^{1*} . e_0^{2*} = \frac{\sum_{r=1}^{s} \mu_r^* y_{r0}}{\sum_{i=1}^{m} \gamma^*{}_i x_{i0}}$ also we have at optimality, $e_0^{1^0} . e_0^{2^0} = \frac{\sum_{r=1}^{s} \mu_r^* y_{r0}}{\sum_{i=1}^{m} \gamma^*{}_i x_{i0}}$ in model (16). So the leader–follower approach also implies efficiency decomposition for the two-stage process. In other words, the overall efficiency is computed by efficiencies of individual stages.

5. CASE STUDY

At this research the delivery department of Iran Khodro Company will be evaluated. This company delivers 21 types of cars to their owners. In fact every car delivery is a DMU which is compared to other DMUs. Car delivery is a two stage process. At the first stage which is called PDI (Pre Delivery Inspection), visual defects are checked. If a car has no defect, it will enter second stage which is sending. Inputs and outputs of the two stages are denoted at Figure 2.

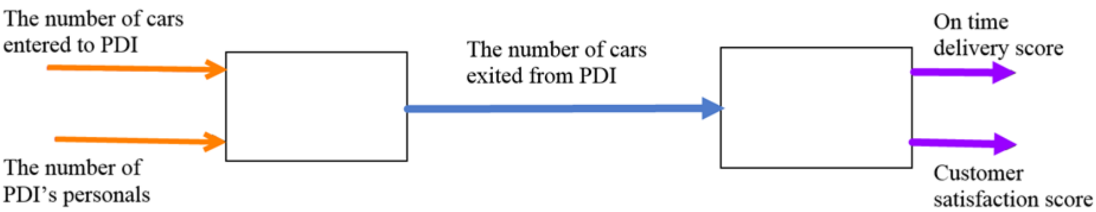

Figure 2: Inputs and outputs of the two stages process of Iran Khodro Company

The number of cars entered to PDI and the number of cars exited from PDI are deterministic digits which become normal digits via dividing the digit by the greatest digit related to their columns. Obviously the normal digit is a digit at [0, 1]. Other three digits; the number of PDI's personals, online delivery score, customer satisfaction score are linguistic variables.

We evaluate performance of 21 DMUs (21 cars) by a three stage algorithm.

Stage1. Calculating Grey Relational Coefficient

In this research there is fuzzy and also grey uncertainty. We use grey relational coefficient to make deterministic digits. There are various methods to obtain grey relational coefficient in different researches, in this research we use the following algorithm:

Step1. Convert linguistic variables to normal interval grey numbers according to the Table1.

It is obvious that this method does not act identically with positive criterion and negative criterion. Positive criterion or benefit criterion is a criterion which is better when it is larger. For example job experience and output of a DMU are positive criterion. Negative criterion or cost criterion is a criterion which is better when it is smaller. For example cost transportation and input of a DMU are negative criterion. Research data is in Table 2.

Step2. Construct decision matrix A with normal interval grey numbers.

Table2. The data of Iran Khodro Car Delivery

	Car models	The number of cars entered to PDI	The number of PDI's personals	The number of cars exited from PDI	On time delivery score	Customer satisfaction score
1	Automatic Tondar 90+	16841	low	16807	mediate	very bad
2	Tondar 90+	1443	low	1438	Partly bad	bad
3	H30 CROSS AT	31907	mediate	31522	Partly good	mediate
4	Automatic Peugeot 2008-EP6	2540	low	2426	Partly bad	bad
5	Peugeot 206-1600 cc	38404	high	38285	very good	Partly good
6	Peugeot 206 SD-1600 cc	38659	high	38533	good	good
7	Peugeot 206	59677	Very high	59499	Partly bad	good
8	Automatic Peugeot 207i	17407	low	17177	very good	Partly bad
9	Peugeot207i	26071	low	25881	good	very bad
10	Peugeot pars TU5	34975	Partly low	34953	mediate	mediate
11	Peugeot pars hybrid	9358	Very low	9355	good	mediate
12	Tondar 90	26047	partly high	26024	very good	Partly good
13	Automatic Tondar 90	2539	low	2519	very good	very good
14	Tondar pick up	4203	low	4196	very good	Partly bad
15	Dena	27269	mediate	26986	good	very bad
16	Tourbocharged Dena+	833	low	759	good	bad
17	Dena+	14018	mediate	13744	good	mediate
18	Runna	6900	Partly low	6849	very good	Partly good
19	Samand SE	145	Very low	143	very good	very good
20	Tourbocharged Soren EF7-TC	1752	low	1751	very good	good
21	Soren P2	210	Very low	209	Partly good	very good

Table 3. Normal data of car delivery in Iran Khodro Company

	Car models	The number of cars entered to PDI-normal	The number of PDI's personals-grey normal	The number of cars exited from PDI-normal	On time delivery score-grey normal	Customer satisfaction score-grey normal
1	Automatic Tondar 90+	0.2822	[0.7,0.9]	0.2825	[0.4,0.6]	[0.0,0.1]
2	Tondar 90+	0.0242	[0.7,0.9]	0.0242	[0.3,0.4]	[0.1,0.3]
3	H30 CROSS AT	0.5347	[0.4,0.6]	0.5298	[0.6,0.7]	[0.4,0.6]
4	Automatic Peugeot 2008-EP6	0.0426	[0.7,0.9]	0.0408	[0.3,0.4]	[0.1,0.3]
5	Peugeot 206-1600 cc	0.6435	[0.1,0.3]	0.6435	[0.9,1.0]	[0.6,0.7]
6	Peugeot 206 SD-1600cc	0.6478	[0.1,0.3]	0.6476	[0.7,0.9]	[0.7,0.9]
7	Peugeot 206	1	[0.0,0.1]	1	[0.3,0.4]	[0.7,0.9]
8	Automatic Peugeot 207i	0.2917	[0.7,0.9]	0.2887	[0.9,1.0]	[0.3,0.4]
9	Peugeot 207i	0.4369	[0.7,0.9]	0.435	[0.7,0.9]	[0.0,0.1]
10	Peugeot cars TU5	0.5861	[0.6,0.7]	0.5875	[0.4,0.6]	[0.4,0.6]
11	Peugeot cars hybrid	0.1568	[0.9,1.0]	0.1572	[0.7,0.9]	[0.4,0.6]
12	Tondar 90	0.4365	[0.3,0.4]	0.4374	[0.9,1.0]	[0.6,0.7]
13	Automatic Tondar 90	0.0425	[0.7,0.9]	0.0423	[0.9,1.0]	[0.9,1.0]
14	Tondar pick up	0.0704	[0.7,0.9]	0.0705	[0.9,1.0]	[0.3,0.4]
15	Dena	0.4569	[0.4,0.6]	0.4536	[0.7,0.9]	[0.0,0.1]
16	Tourbocharged Dena+	0.014	[0.7,0.9]	0.0128	[0.7,0.9]	[0.1,0.3]
17	Dena+	0.2349	[0.4,0.6]	0.231	[0.7,0.9]	[0.4,0.6]
18	Runna	0.1156	[0.6,0.7]	0.1151	[0.9,1.0]	[0.6,0.7]
19	Samand SE	0.0024	[0.9,1.0]	0.0024	[0.0,0.1]	[0.9,1.0]
20	Tourbocharged Soren EF7-TC	0.0294	[0.7,0.9]	0.0294	[0.7,0.9]	[0.7,0.9]
21	Soren P2	0.0035	[0.9,1.0]	0.0035	[0.6,0.7]	[0.9,1.0]

Data of Matrix is a part of Table 3. Attributes of this matrix are Iran Khodro cars which should be delivered. And the number of them is 21. The data of each attribute is defined in a row, so the matrix has 21 rows. The criteria of this matrix are uncertain inputs and outputs of DMUs which the number of them is three. The data related to each criterion is defined in columns, so the matrix has three columns. For example interval grey number $[r_{ij}^{-} \ , \ r_{ij}^{+}]$ demonstrates the input or output jth related to ith car.

The number of entered car to PDI is a deterministic digit which became normal via dividing the mentioned digit by 59677 (the greatest digit in related column). In the same way, the number of exited car from PDI is a deterministic digit which became normal via dividing the mentioned digit by 59677 (the greatest digit in related column).

Also linguistic variable "the number of PDI's personals" that is an input of first stage, so is a negative criterion, became normal grey number according to the Table3. Linguistic variables "on time delivery score" and "customer satisfaction score" are outputs of second stage so they are positive criteria, and they became normal grey number according to the Table1. The normal numbers are demonstrated in Table 3.

Step3. Determine reference number sequence.

Reference number sequence of three uncertain numbers is demonstrated at Table 4.

Table 4. Reference number sequence of three uncertain numbers

	The number of PDI's personals	On time delivery score	Customer satisfaction score
Max(Min)	0.9	0.9	0.9
Max(Max)	1	1	1

Table 5: Differences between reference number sequence and interval grey numbers.

	Car models	The number of PDI's personals		On time delivery score		Customer satisfaction score	
		Max(min)-min	Max(max)-max	Max(min)-min	Max(max)-max	Max(min)-min	Max(max)-max
1	Automatic Tondar 90+	0.2	0.1	0.5	0.4	0.9	0.9
2	Tondar 90+	0.2	0.1	0.6	0.6	0.8	0.7
3	H30 CROSS AT	0.5	0.4	0.3	0.3	0.5	0.4
4	Automatic Peugeot 2008-EP6	0.2	0.1	0.6	0.6	0.8	0.7
5	Peugeot 206-1600 cc	0.8	0.7	0	0	0.3	0.3
6	Peugeot 206 SD-1600cc	0.8	0.7	0.2	0.1	0.2	0.1
7	Peugeot 206	0.9	0.9	0.6	0.6	0.2	0.1
8	Automatic Peugeot 207i	0.2	0.1	0	0	0.6	0.6
9	Peugeot 207i	0.2	0.1	0.2	0.1	0.9	0.9
10	Peugeot cars TU5	0.3	0.3	0.5	0.4	0.5	0.4
11	Peugeot cars hybrid	0	0	0.2	0.1	0.5	0.4
12	Tondar 90	0.6	0.6	0	0	0.3	0.3
13	Automatic Tondar 90	0.2	0.1	0	0	0	0
14	Tondar pick up	0.2	0.1	0	0	0.6	0.6
15	Dena	0.5	0.4	0.2	0.1	0.9	0.9
16	Tourbocharged Dena+	0.2	0.1	0.2	0.1	0.8	0.7
17	Dena+	0.5	0.4	0.2	0.1	0.5	0.4
18	Runna	0.3	0.3	0	0	0.3	0.3
19	Samand SE	0	0	0.9	0.9	0	0
20	Tourbocharged Soren EF7-TC	0.2	0.1	0.2	0.1	0.2	0.1
21	Soren P2	0	0	0.3	0.3	0	0

Min value is lower limit of an interval number and max is upper limit of an interval number. So max (max) is maximum of all upper limits of interval numbers in a column.

Step4. Calculate the connection between the sequences composed of interval number of every attribute and reference number sequence.

First, calculate the connection coefficient $\xi_i(k)$ between the sequences composed of interval number of every attribute $U_i = ([r_{i1}^-, r_{i1}^+], [r_{i2}^-, r_{i2}^+], \ldots, [r_{in}^-(n), r_{in}^+(n)])$ and reference number sequence

$U_0 = ([u_0^-(1), u_0^+(1)], [u_0^-(2), u_0^+(2)], \dots, [u_0^-(n), u_0^+(n)])$ according to formula (1).

The interval grey numbers are subtracted from reference number sequence at Table 6.

The maximum of {Max (min) - min, Max (max) - min} are calculated. These numbers are $|[u_0^-(k), u_0^+(k)] - [r_{ik}^-, r_{ik}^+]\ \ |$. Then the minimum of three previous columns was obtained and mentioned in a column, then the minimum of all elements in the minimum column is obtained zero. In fact $\min_i \quad \min_k |[u_0^-(k), u_0^+(k)] - [r_{ik}^-, r_{ik}^+]\ \ |$ is zero. Also the maximum of three previous columns was obtained and mentioned in a column, then the maximum of all elements in the maximum column is obtained 0.9. In fact 0.9 is $\max_i \quad \max_k |[u_0^-(k), u_0^+(k)] - [r_{ik}^-, r_{ik}^+]\ \ |$. The results are mentioned at Table 6.

Table6: The proceed calculations of grey relational coefficient.

	Car models	The number of PDI's personals	On time delivery score	Customer satisfaction score	minimum of columns 3,4,5	maximum of columns 3,4,5
		Max{Max(min)-min, Max(max)-max}	Max{Max(min)-min, Max(max)-max}	Max{Max(min)-min, Max(max)-max}		
1	Automatic Tondar 90+	0.2	0.5	0.9	0.2	0.9
2	Tondar 90+	0.2	0.6	0.8	0.2	0.8
3	H30 CROSS AT	0.5	0.3	0.5	0.3	0.5
4	Automatic Peugeot 2008-EP6	0.2	0.6	0.8	0.2	0.8
5	Peugeot 206-1600cc	0.8	0	0.3	0	0.8
6	Peugeot 206 SD-1600cc	0.8	0.2	0.2	0.2	0.8
7	Peugeot 206	0.9	0.6	0.2	0.2	0.9
8	Automatic Peugeot 207i	0.2	0	0.6	0	0.6
9	Peugeot 207i	0.2	0.2	0.9	0.2	0.9
10	Peugeot cars TU5	0.3	0.5	0.5	0.3	0.5
11	Peugeot cars hybrid	0	0.2	0.5	0	0.5
12	Tondar 90	0.6	0	0.3	0	0.6
13	Automatic Tondar 90	0.2	0	0	0	0.2
14	Tondar pick up	0.2	0	0.6	0	0.6
15	Dena	0.5	0.2	0.9	0.2	0.9
16	Tourbocharged Dena+	0.2	0.2	0.8	0.2	0.8
17	Dena+	0.5	0.2	0.5	0.2	0.5
18	Runna	0.3	0	0.3	0	0.3
19	Samand SE	0	0.9	0	0	0.9
20	Tourbocharged Soren EF7-TC	0.2	0.2	0.2	0.2	0.2
21	Soren P2	0	0.3	0	0	0.3

Then using formula (1) grey relational coefficients are calculated at Table 7.

Calculations of different steps were done by excel program.

Stage2. Using Network Data Envelopment Analysis

We calculate efficiency of two stages Data Envelopment Analysis (DEA) in delivery process of Iran Khodro Company by using three different game theories (Nash game, Centralized game, Stackelberg game).

Grey relational coefficients were obtained in previous part, so here we us them in DEA models and evaluate the efficiencies. Different models were solved by GAMS program.

Table 7: Grey relational coefficients.

	Car models	Grey relational coefficient		
		The number of PDI's personals	On time delivery score	Customer satisfaction score
1	Automatic Tondar 90+	0.6923	0.4737	0.3333
2	Tondar 90+	0.6923	0.4286	0.3600
3	H30 CROSS AT	0.4737	0.6000	0.4737
4	Automatic Peugeot 2008-EP6	0.6923	0.4286	0.3600
5	Peugeot 206-1600 cc	0.3600	1	0.6000
6	Peugeot 206 SD-1600 cc	0.3600	0.6923	0.6923
7	Peugeot 206	0.3333	0.4286	0.6923
8	Automatic Peugeot 207i	0.6923	1	0.4286
9	Peugeot207i	0.6923	0.6923	0.3333
10	Peugeot cars TU5	0.6000	0.4737	0.4737
11	Peugeot cars hybrid	1	0.6923	0.4737
12	Tondar 90	0.4286	1	0.6000
13	Automatic Tondar 90	0.6923	1	1
14	Tondar pick up	0.6923	1	0.4286
15	Dena	0.4737	0.6923	0.3333
16	Tourbocharged Dena+	0.6923	0.6923	0.3600
17	Dena+	0.4737	0.6923	0.4737
18	Runna	0.6000	1	0.6000
19	Samand SE	1	0.3333	1
20	Tourbocharged Soren EF7-TC	0.6923	0.6923	0.6923
21	Soren P2	1	0.6000	1

5.1 NASH BARGAINING GAME MODEL

Two stages are two individuals bargaining with each other for a better payoff, which is the efficiency of each individual stage.

The efficiencies of two previous models are $\theta^1_{\min}$, $\theta^2_{\min}$ and we use them in model (6). After solving model the overall efficiencies are mentioned at Table 8.

Table 8: The result of Nash bargaining game.

	Car models	$\theta^1_{\min}$	$\theta^2_{\min}$	e_0^{nash}
1	Automatic Tondar 90+	0.998634	0.009781	0.337662
2	Tondar 90+	0.997455	0.103313	0.032535
3	H30 CROSS AT	0.988945	0.006606	0.001783
4	Automatic Peugeot 2008-EP6	0.955309	0.061279	0.019771
5	Peugeot 206-1600 cc	0.999218	0.009065	0.002726
6	Peugeot 206 SD-1600 cc	0.998923	0.006236	0.001717
7	Peugeot 206	1.000000	0.002500	0.000507
8	Automatic Peugeot 207i	0.987317	0.020206	0.006491
9	Peugeot207i	0.993262	0.009284	0.002800
10	Peugeot cars TU5	1.000000	0.004703	0.001289
11	Peugeot cars hybrid	1.000000	0.025690	0.008839
12	Tondar 90	0.999818	0.013336	0.004046
13	Automatic Tondar 90	0.992862	0.137904	0.046274
14	Tondar pick up	0.998872	0.082742	0.025994
15	Dena	0.990411	0.008903	0.002671
16	Tourbocharged Dena+	0.911959	0.315501	0.103156
17	Dena+	0.981024	0.017482	0.005571
18	Runna	0.993177	0.050681	0.015798
19	Samand SE	0.997455	1.000000	0.327672
20	Tourbocharged Soren EF7-TC	0.997455	0.137361	0.043363
21	Soren P2	0.997455	1.000000	0.337662

5.2 CENTRALIZED GAME

According to cooperative game theory, or centralized control, the two stage process can be viewed as one where the stages jointly determine a set of optimal weights on the intermediate factors to maximize their efficiency scores [5].

After solving models (10), (12), (13) by GAMS, overall efficiency and first stage efficiency and second stage efficiency are at Table 9.

Table9. The result of Centralized game

	Car models	$e_0^{centralized}$	e_0^{1+}	e_0^{2+}
1	Automatic Tondar 90+	0.009768	0.998634	0.009781
2	Tondar 90+	0.103050	0.997455	0.103313
3	H30 CROSS AT	0.006533	0.988945	0.006606
4	Automatic Peugeot 2008-EP6	0.058540	0.955309	0.061279
5	Peugeot 206-1600 cc	0.009058	0.999218	0.009065
6	Peugeot 206 SD-1600 cc	0.006229	0.998923	0.006236
7	Peugeot 206	0.002500	1.000000	0.002500
8	Automatic Peugeot 207i	0.019949	0.987317	0.020206
9	Peugeot207i	0.009221	0.993262	0.009284
10	Peugeot cars TU5	0.004703	1.000000	0.004703
11	Peugeot cars hybrid	0.025690	1.000000	0.025690
12	Tondar 90	0.013334	0.999818	0.013336
13	Automatic Tondar 90	0.136906	0.992762	0.137904
14	Tondar pick up	0.082649	0.998862	0.082742
15	Dena	0.008818	0.990411	0.008903
16	Tourbocharged Dena+	0.287724	0.911959	0.315501
17	Dena+	0.017151	0.981024	0.017482
18	Runna	0.050335	0.993177	0.050681
19	Samand SE	0.997455	0.997455	1.000000
20	Tourbocharged Soren EF7-TC	0.137012	0.997455	0.137361
21	Soren P2	0.997455	0.997455	1.000000

Table10: The result of Stackelberg game

	Cars models	e_0^{1*}	e_0^{2*}	e_0^{s}
1	Automatic Tondar 90+	0.998634	0.009781	0.009768
2	Tondar 90+	0.997455	0.103313	0.103050
3	H30 CROSS AT	0.988945	0.006606	0.006533
4	Automatic Peugeot 2008-EP6	0.955309	0.061279	0.058540
5	Peugeot 206-1600 cc	0.999218	0.009065	0.009058
6	Peugeot 206 SD-1600 cc	0.998923	0.006236	0.006229
7	Peugeot 206	1.000000	0.002500	0.002500
8	Automatic Peugeot 207i	0.987317	0.020206	0.019949
9	Peugeot207i	0.993262	0.009284	0.009221
10	Peugeot cars TU5	1.000000	0.004703	0.004703
11	Peugeot cars hybrid	1.000000	0.025690	0.025690
12	Tondar 90	0.999818	0.013336	0.013334
13	Automatic Tondar 90	0.992762	0.137904	0.136906
14	Tondar pick up	0.998872	0.082742	0.082649
15	Dena	0.990411	0.008903	0.008818
16	Tourbocharged Dena+	0.911959	0.315501	0.287724
17	Dena+	0.981024	0.017482	0.017151
18	Runna	0.993177	0.050681	0.050335
19	Samand SE	0.997455	1.000000	0.997455
20	Tourbocharged Soren EF7-TC	0.997455	0.137361	0.137012
21	Soren P2	0.997455	1.000000	0.997455

5.3 STACKELBERG GAME

This game is a non-cooperative game. It is characterized by the leader–follower, or Stackelberg game. For example, there is Stackelberg game in a supply chain where there is no cooperation between the manufacture (leader) and the retailer (follower). The manufacturer defines its optimal investment based on an estimation of the local advertisement by the retailer to maximize its profit.

On the other hand, the optimal local advertisement cost of the retailer, based on the information from the manufacturer, will be determined to maximize retailer's profit [28].

If the first stage is the leader, then the first stage performance is more important, and the efficiency of the second stage is computed subject to the fixed efficiency of the first stage. After solving models (14), (16) we have data at Table 10.

Table 11 compares the result from three game models.

Table 11: Comparison the result of three game models.

	Car models	e_0^{nash}	$e_0^{centralized}$	e_0^{s}
1	Automatic Tondar 90+	0.337662	0.009768	0.009768
2	Tondar 90+	0.032535	0.103050	0.103050
3	H30 CROSS AT	0.001783	0.006533	0.006533
4	Automatic Peugeot 2008-EP6	0.019771	0.058540	0.058540
5	Peugeot 206-1600 cc	0.002726	0.009058	0.009058
6	Peugeot 206 SD-1600 cc	0.001717	0.006229	0.006229
7	Peugeot 206	0.000507	0.002500	0.002500
8	Automatic Peugeot 207i	0.006491	0.019949	0.019949
9	Peugeot207i	0.002800	0.009221	0.009221
10	Peugeot pars TU5	0.001289	0.004703	0.004703
11	Peugeot pars hybrid	0.008839	0.025690	0.025690
12	Tondar 90	0.004046	0.013334	0.013334
13	Automatic Tondar 90	0.046274	0.136906	0.136906
14	Tondar pick up	0.025994	0.082649	0.082649
15	Dena	0.002671	0.008818	0.008818
16	Tourbocharged Dena+	0.103156	0.287724	0.287724
17	Dena+	0.005571	0.017151	0.017151
18	Runna	0.015798	0.050335	0.050335
19	Samand SE	0.327672	0.997455	0.997455
20	Tourbocharged Soren EF7-TC	0.043363	0.137012	0.137012
21	Soren P2	0.337662	0.997455	0.997455

6. CONCLUSION

Nash bargaining game, Centralized and Stackelberg game are used to obtain efficiency of each DMU and the results show that efficiencies obtained from centralized game are the same as efficiencies obtained from Stackelberg game and they are greater than efficiencies obtained from Nash game. So if the DMUs can cooperate with each other

Also the data is not always certain in real world. We use both fuzzy and grey theory to widely manage the real situation.

Iran Khodro Company which is one of the most important companies at automobile industry is the case study of this article and has a wide process of delivering for automobiles. It has a two stage process of delivering for 21 types of automobiles. Samand SE and Soren P2 have the highest efficiency, whilst Peugeot 206 has the lowest efficiency.

7. AVAILABILITY OF DATA AND MATERIAL

Data can be made available by contacting the corresponding author.

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