©2019 International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies

International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies

http://TuEngr.com

SYSTEM DYNAMICS MODELING OF OLIGOPOLY MARKET BASED ON GAME THEORY

Ali Mohammadi [a], Ehsan Javanmardi [a,b*]

[a] *School of Economics, Management and Social Sciences, Shiraz University, Shiraz, IRAN*
[b] *Institute for Grey Systems Studies, Nanjing University of Aeronautics and Astronautics, Nanjing, CHINA*

ARTICLEINFO

Article history:
Received 14 December 2018
Received in revised form 29 March 2019
Accepted 15 April 2019
Available online
26 April 2019

Keywords:
Game theories; System dynamics; Oligopoly markets; Dynamical system game theories.

ABSTRACT

This study proposes a theoretical framework that can describe the dynamic dimensions of the game environment and the factors affecting it and can model the impacts of decisions on the environment and vice versa, through an application of system dynamics. At first, a conceptual model was formulated for oligopoly markets, based on the dynamical system game theory. Following that, corporates' decision-making mechanisms were formulated in an oligopoly market, and the observations showed that the Structure of the demand function served as the central element in determining the Nash equilibrium and players' optimal decisions. Then, three variables, namely cost, quality, and advertisement were considered as decision variables of the problem, and ultimately the final causal game model was designed. The sample under study was the Iranian white cement export market.

Results of simulation revealed that the important parameters of the model, including demand volume, profit rate, and cost, tended toward equilibrium after a fluctuating transition period. Solving the model through common game theory methods would not make it possible to investigate the behaviors of the system in the long run, nor to observe the transition period. The key innovation of this research is to propose a conceptual model of game in dynamical systems, by taking into account multiple decision variables.

1. INTRODUCTION

The world of business is constantly regulated by players' games. In traditional game theories, the strategies and priorities of every player are well-defined and remain stable during the game. In reality, however, any decision made by players could leave a particular, and even transformative, impact on the game environment. In Table (1) a comparison is made between Dynamical Systems (DS) and game theory. There are several advantages of using DS in game theory (Shahanaghi and Hajihosseini, 2011), as DS can help identify interactive relations between factors affecting the game; predict the final behavior of the system in the future and allow for appropriate decisions in the game;

*Corresponding author (Ehsan Javanmardi) E-mail: Ejavanmardi@nuaa.edu.cn,

analyze sensitivity in the case of each factor; and identify the interactive effects of factors.

This study proposes a theoretical framework that can (1) describe the dynamic dimensions of a game environment and the factors affecting it; and (2) model the impacts of the environment on decisions and vice versa, by drawing on studies concerned with DS and feedback loops modeling. The focal concern of this study is to unravel players' dependencies and impacts on one another in the game environment. Generally, discussion questions of this study are: (1) From the perspective of DS, what are the relations governing oligopoly games in which various strategies are used? And (2) What is the structure regulating a game with complete information and infinite recursions under oligopolistic conditions? And (3)What are the impacts of decisions made by each player on the game environment and on the policies of other players over time? The method used in this study is an integration of qualitative and quantitative methods including descriptive research frameworks for formulating the structure of systems. Primarily, the study relies on mathematical methods to identify the Nash equilibrium in an oligopoly game. Furthermore, part of the research method used draws on DS techniques for analyzing and modeling systems. The sample under study is the Iranian white cement export market.

Table 1: A comparison between DS and game theory.

Row	Issue of concern	Dynamical systems	Game theory
1	Type of problem	Soft	Hard
2	Type of system	Dynamic & continuous	Discrete & static
3	Dynamicity	Involves dynamicity	Involves dynamicity in some games
4	Feedback	Recursive loops	Retribution; reaction
5	Delay calculation	Delay function	-
6	Output presentation	All system changes over the period	Mere results
7	Main assumption	Logical	Logical
8	Type of decision	Medium-term; long-term	Short-term; long-term
9	Nature of results	Qualitative/quantitative	Quantitative
10	Decision-maker	An individual in the majority of cases	Teamwork

Some researchers have considered the DS and game theory combination model. Kim and Kim (1997) first introduced the idea of integrating DS with game theory. They used the DS model in a mixed-strategy game between police and drivers. To model the interactions between two competitors in a competitive, duopoly, non-cooperative game, Sice *et al*. (2000) drew on DS and simulated the competitors' behaviors over time. They showed that the model, instead of reaching equilibrium around an optimal value, experienced complex, unintuitive behaviors in different time periods (Sice *et al*., 2000). Akiyama and Kaneko (2000), concerned with game theory, DS, and game dynamics, introduced dynamical systems game theory. In their study, they probed into and modeled dynamics in lumberjacks' games. Integrating neural networks, DS, and game theory, West and Lebiere (2001) tried to find a way of predicting the behavior of dynamic game models. Adamides *et al*. (2004) formalized an evolutionary prisoner's dilemma game, using a DS-based model. Ficici *et al*. (2005), too, investigated game theories and DS analyses. They observed that Nash equilibrium on its own failed to represent game results and it needed DS and feedback loops to display more accurate scrutiny. Elettreby and Hassan (2006) proposed two different types of dynamic multiple-team models.

Gary et al. (2008), in a study concerned with DS and strategy, stated that game theory analyses highlighted the significance of information and ideas in understanding equilibrium and the consequences of decisions made by competitors in competitive interactions. Utilizing evolutionary game theory, Weiguo *et al*. (2011) investigated firms' strategies selection under the penalty mechanism as stipulated in the technology alliance's contracts. Koh *et al*. (2013), investigated city competition in a small network regulated by static equilibrium. Elettreby and Mansour (2012) introduced a model analyzing the Cournot game through DS, specifying how DS could be applied to this type of game.

Broadly speaking, as the literature suggests, studying the structure and optimal policies adopted in oligopolistic markets represents an important area of investigation. Furthermore, existing mathematical models do not specifically address the various dimensions of game theory in oligopolistic markets. Considering the importance of recognizing a system's behavior in the long term, especially for the sake of building trust and engaging numerous effective variables, DS methodologies have to be incorporated into mathematical models of game theory.

2. MATERIAL AND METHODS

This study relied on an integrated version of the DS and game theory to investigate the oligopoly market. To analyze and simulate the system status, Matlab®, Vensim®, and Minitab® were used. The verification and sensitivity analysis of the model proposed were tested against all of the variables used. In doing so, the behaviors of the variables in the system were examined by measuring various values for the parameters in the model, and ultimately the compatibility of the variables to the assumptions of economic theories was verified.

The industry under study in our research is white cement export market of Iran. The price for this product in Iran, compared to neighboring countries, is considerably lower, and cement in terms of price is considered to be a coarse material. Given the huge volume of cement produced in the country, the costs of cement transportation are remarkably higher than the product's economic value. In Iran, there are 8 white cement-producing companies. As the situation suggests, foreign companies are not actively engaged in Iran's export market mainly because of two reasons: first such companies normally work according to high prices, and second, transporting Iranian cement to international borders would not be possible because of extensive transportation costs. As a result of this condition, the market is currently dominated by oligopolistic competition in which a number of companies govern the whole market.

2.1 CONCEPTUAL MODELING OF DS-BASED GAME THEORY

One of the major parameters in dynamic games is the "game environment." Another parameter is the status of each firm (player) at every given moment of time. In dynamic games, the conditions and state of each player may undergo transformation at any given moment. On this account, games in a DS could be represented as follows:

$$G:\left(E(t),S(t)\right)\rightarrow\left(E(t+1),S(t+1)\right)$$
$$G:\left(E(t),S(t)\right)\rightarrow\left(E(t+1),S(t+1)\right) \qquad (1)$$

In Relation 1, G is the dynamic system game, E is the environment status, S is the players' status, and t is time. On this account, changes in the game could be re-formulated via:

$$u:\left(E(t),S(t)\right)\to\left(E(t)',S(t)'\right)$$
$$v:\left(E(t)',S(t)',O(t)\right)\to\left(E(t+1),S(t+1)\right) \quad (2)$$
$$G:u\circ v$$

In Relation 2, u is rules of nature, v is the impacts of players' decisions, and O is decisions and measures adopted by players. In Relation 2, the order in which v and u appear in the formula is not important. Another parameter in DS games is the mechanism of players' decision-making formulated as follows:

$$Z^{i}:\left(E(t),S(t)\right)\to\left(O^{i}(t)\right) \quad (3),$$

where i is the number of players, and Z is the mechanism of decision-making. In this configuration, the game, as a dynamic system, is represented by decision-making process Z within dynamical system G (Akiyama and Kaneko., 2002). Every firm in the market normally tries to ultimately maximize its profit. Under normal conditions, the profit function could be formulated as follows:

$$\pi=D.P-C \quad (4)$$

In Relation 4, π is the profit function for each firm, D is the demand function or the volume of sales for each firm, P is the sales price for each product, and C is the cost function for each firm. Demand models are usually divided into several types: linear, multiplicative competitive interaction, Logit, exponential, and other types (Cooper and Nakanishi, 1988). Therefore the objective function of the problem (profit maximization) for each firm was configured as follows:

$$D=f\,(P,Q,A,D_0) \quad (5)$$

$$C=f\,(A,D,\alpha,\beta,\mu) \quad (6)$$

$$\pi=D.P-C=\left[D_{(P,Q,A)}.P\right]-\left[C_{(A,D,\alpha,\beta,\mu)}\right] \quad (7)$$

Relation 5 demonstrates that demand is a function of prices (P), quality (Q), advertisement (A), and potential market demand (D_0). Relation 6 represents the total cost function for each firm; α is the variable cost for producing every product unit, β is the fixed cost of production, A is the costs of the advertisements, μ is the growing costs for every product unit, and Q is the quality of the current product after its quality was improved.

Additionally, market share (the ratio of a firm's profit to expected profit) was used to represent the players' status. As a result, the vector of players' status could be configured via:

$$S_i^t=\left\{R_i^t,\frac{\pi_i^t}{\pi_i^e}\right\} \quad , \quad i=\{1,2,\dots,n\} \quad (8),$$

where S_i^t is the status of i-th firm at moment t, R_i^t is the market share of i-th firm at moment t, π_i^t is the total profit of i-th firm at moment t. The total number of firms existing in the market is represented by n. Figure 1, incorporating all these details, illustrates the conceptual diagram of the game (Mohammadi et al. 2016a).

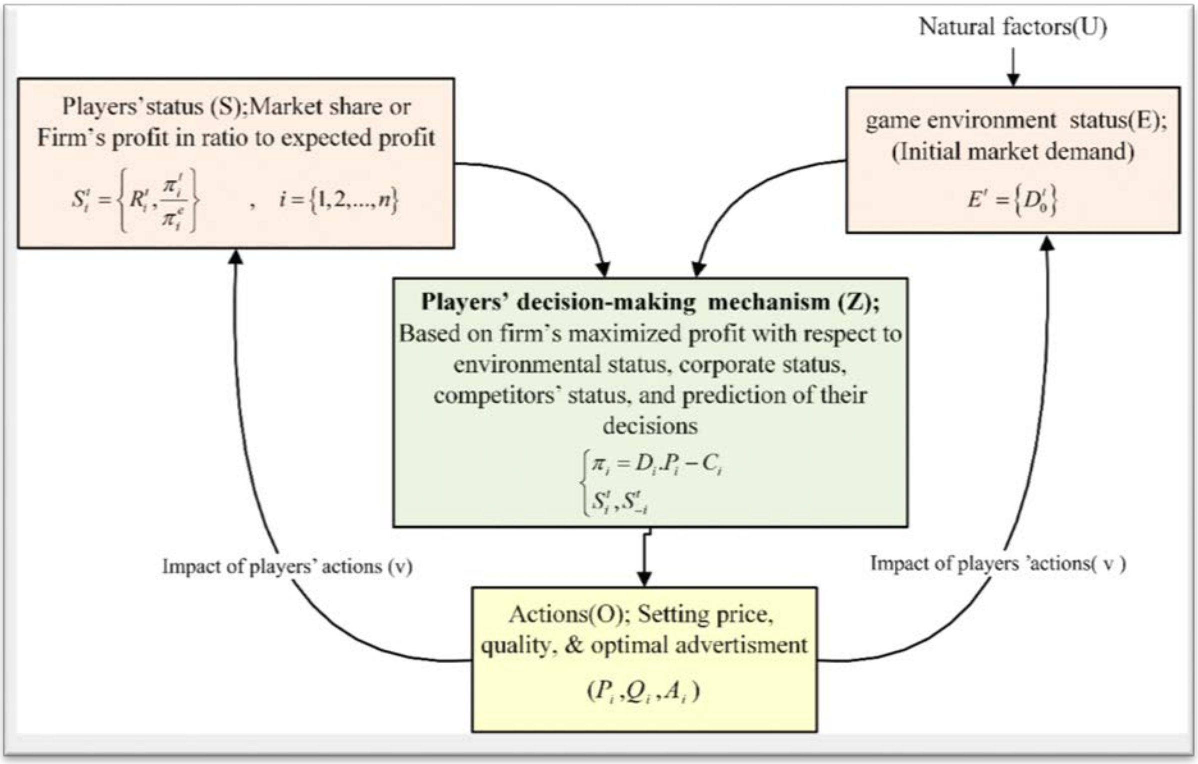

Figure 1: The conceptual model of decision-making in an oligopoly market
(after Akiyama and Kaneko (2002)).

3. RESULTS

In this section, the white market under study, as an oligopoly market, is analyzed based on the model (see Figure 1), with a specific focus on the behaviors of dominant players in the environment. In every game, the point of Nash equilibrium, or the best game every player displays, is determined against the best game played by competitors as follows:

$$B_i(O_j^*)=O_i^* \tag{9}$$

where O is players' decisions vector, and B_i is i-th player's best decision against j-th player's best decision. The O vector for player i is framed via:

$$O_i=\{P_i,Q_i,A_i\} \tag{10}$$

Normally, to determine B_i based on the objective function, the following calculation should be made first:

$$Max\ \pi_i=D_i.P_i-C_i \Rightarrow B_i(O_j^*)=\frac{d\pi_i}{dO_i}=\nabla\pi_i=0 \tag{11}$$

Normally, the following Relation holds true, regardless of the types of demand and cost functions:

$$\nabla \pi_i = \begin{bmatrix} \frac{d\pi_i}{dP_i} \\ \frac{d\pi_i}{dQ_i} \\ \frac{d\pi_i}{dA_i} \end{bmatrix} = \begin{bmatrix} \frac{dD_i}{dP_i}.P_i + D_i.\frac{dP_i}{dP_i} - \frac{dC_i}{dP_i} \\ \frac{dD_i}{dQ_i}.P_i + D_i.\frac{dP_i}{dQ_i} - \frac{dC_i}{dQ_i} \\ \frac{dD_i}{dA_i}.P_i + D_i.\frac{dP_i}{dA_i} - \frac{dC_i}{dA_i} \end{bmatrix} \tag{12}$$

In cases where Relation 13 is established,

$$\frac{dP_i}{dP_i} = 1 \quad , \quad \frac{dP_i}{dQ_i} = 0 \quad , \quad \frac{dP_i}{dA_i} = 0 \qquad (13).$$

The gradient represented in Relation 12 is configured as

$$\nabla \pi_i = \begin{bmatrix} \frac{d\pi_i}{dP_i} \\ \frac{d\pi_i}{dQ_i} \\ \frac{d\pi_i}{dA_i} \end{bmatrix} = \begin{bmatrix} \frac{dD_i}{dP_i}.P_i + D_i - \frac{dC_i}{dP_i} \\ \frac{dD_i}{dQ_i}.P - \frac{dC_i}{dQ_i} \\ \frac{dD_i}{dA_i}.P - \frac{dC_i}{dA_i} \end{bmatrix} \qquad (14).$$

If the cost function is assumed to be a linear function, the following Relation could be formulated:

$$C = f\,(\beta, \alpha, \mu, A, D) = \beta + \alpha.D + \mu(Q - Q_0) + A \qquad (15).$$

The derivative of the cost function, then, with respect to the decision variables, can be stated as

$$\frac{dC_i}{dP_i} = \frac{d\beta_i}{dP_i} + \frac{dD_i}{dP_i}.\alpha_i + \frac{d\alpha_i}{dP_i}.D_i + \frac{d\mu_i}{dP_i}.(Q_i - Q_{0i}) + \frac{d(Q_i - Q_{0i})}{dP_i}.\mu_i + \frac{dA_i}{dP_i} \qquad (16).$$

If Relation 16 is re-framed based on the following assumption:

$$\frac{d\beta_i}{dP_i} = 0 \quad , \quad \frac{d\alpha_i}{dP_i} = 0 \quad , \quad \frac{d\mu_i}{dP_i} = 0 \quad , \quad \frac{d(Q_i - Q_{0i})}{dP_i} = 0 \quad , \quad \frac{dA_i}{dP_i} = 0$$

then Relation 17 can be formulated:

$$\frac{dC_i}{dP_i} = \frac{dD_i}{dP_i}.\alpha_i \qquad (17).$$

Similarly, we have

$$\frac{dC_i}{dQ_i} = \frac{d\beta_i}{dQ_i} + \frac{dD_i}{dQ_i}.\alpha_i + \frac{d\alpha_i}{dQ_i}.D_i + \frac{d\mu_i}{dQ_i}.(Q_i - Q_{0i}) + \frac{d(Q_i - Q_{0i})}{dQ_i}.\mu_i + \frac{dA_i}{dQ_i} \qquad (18).$$

If Relation 18 is re-framed based on the following assumption:

$$\frac{d\beta_i}{dQ_i} = 0 \quad , \quad \frac{d\alpha_i}{dQ_i} = 0 \quad , \quad \frac{d\mu_i}{dQ_i} = 0 \quad , \quad \frac{d(Q_i - Q_{0i})}{dQ_i} = 1 \quad , \quad \frac{dA_i}{dQ_i} = 0,$$

then Relation 19 can be formulated,

$$\frac{dC_i}{dQ_i} = \frac{dD_i}{dQ_i}.\alpha_i + \mu_i \qquad (19).$$

Furthermore, the derivative of the cost function with respect to the decision variable "advertising" is calculated through

$$\frac{dC_i}{dA_i}=\frac{d\beta_i}{dA_i}+\frac{dD_i}{dA_i}.\alpha_i+\frac{d\alpha_i}{dA_i}.D_i+\frac{d\mu_i}{dA_i}.(Q_i-Q_{0i})+\frac{d(Q_i-Q_{0i})}{dA_i}.\mu_i+\frac{dA_i}{dA_i} \tag{20}$$

Considering the assumption

$$\frac{d\beta_i}{dA_i}=0 \quad , \quad \frac{d\alpha_i}{dA_i}=0 \quad , \quad \frac{d\mu_i}{dA_i}=0 \quad , \quad \frac{d(Q_i-Q_{0i})}{dA_i}=0 \quad , \quad \frac{dA_i}{dA_i}=1,$$

we have

$$\frac{dC_i}{dA_i}=\frac{dD_i}{dA_i}.\alpha_i+1 \tag{21}$$

On this account, the gradient of the payoff function, which is calculated via Relation 14, can be formulated as

$$\nabla\pi_i=\begin{bmatrix}\frac{d\pi_i}{dP_i}\\ \frac{d\pi_i}{dQ_i}\\ \frac{d\pi_i}{dA_i}\end{bmatrix}=\begin{bmatrix}\frac{dD_i}{dP_i}.P_i+D_i-\frac{dD_i}{dP_i}.\alpha_i\\ \frac{dD_i}{dQ_i}.P_i-\frac{dD_i}{dQ_i}.\alpha_i-\mu_i\\ \frac{dD_i}{dA_i}.P_i-\frac{dD_i}{dA_i}.\alpha_i-1\end{bmatrix}=\begin{bmatrix}\frac{dD_i}{dP_i}.(P_i-\alpha_i)+D_i\\ \frac{dD_i}{dQ_i}.(P_i-\alpha_i)-\mu_i\\ \frac{dD_i}{dA_i}.(P_i-\alpha_i)-1\end{bmatrix} \tag{22}$$

Plausibly, to calculate the optimal values of decision variables, the root of the gradient should be first computed ($\nabla\pi_i=0$); therefore, a system of three equations and three unknowns with no constraints should be solved. Moreover, after the roots of the gradient of the objective function ($\nabla\pi_i$) are decided, the Hessian matrix of the payoff function should be calculated to make sure of the maximal values of the roots. To confirm the maximal values of the gradient roots, the analyst should find the conditions that demonstrate the Hessian matrix is a concave matrix. That is, the minors of the Hessian matrix should be alternatively negative and positive:

$$H(\pi_i)=\left[\begin{array}{ccc}\overbrace{\frac{d^2D_i}{d^2P_i}(P_i-\alpha_i)+2\frac{dD_i}{dP_i}}^{\frac{d\nabla\pi_i}{dP_i}} & \overbrace{\frac{d^2D_i}{dP_i.dQ_i}(P_i-\alpha_i)+\frac{dD_i}{dQ_i}}^{\frac{d\nabla\pi_i}{dQ_i}} & \overbrace{\frac{d^2D_i}{dP_i.dA_i}(P_i-\alpha_i)+\frac{dD}{dA}}^{\frac{d\nabla\pi_i}{dA_i}}\\ \frac{d^2D_i}{dQ_i.dP_i}(P_i-\alpha_i)+\frac{dD_i}{dQ_i} & \frac{d^2D_i}{d^2Q_i}(P_i-\alpha_i) & \frac{d^2D_i}{dQ_i.dA_i}(P_i-\alpha_i)\\ \frac{d^2D_i}{dA_i.dP_i}(P_i-\alpha_i)+\frac{dD_i}{dA_i} & \frac{d^2D_i}{dA_i.dQ_i}(P_i-\alpha_i) & \frac{d^2D_i}{d^2A_i}(P_i-\alpha_i)\end{array}\right. \tag{23}$$

Considering the condition of concavity, the following condition too should be observed:

$$\frac{d^2D_i}{d^2P_i}(P_i-\alpha_i)+2\frac{dD_i}{dP_i}<0 \tag{24}$$

Yet, through DS techniques, the uniqueness of the equilibrium can be detected. One of the most

*Corresponding author (Ehsan Javanmardi) E-mail: Ejavanmardi@nuaa.edu.cn,

important non-linear demand models is the interactive-competitive MCI model. The types of this model can be analyzed in a game model according to the following format:

$$D_i = \frac{M_i}{\sum_{j=1}^{n} M_j} \times D_0 = \frac{D_0 e^{\rho_i} . \prod_{k=1}^{K} (X_{ki})^{\nu_k} .\varepsilon_i}{\sum_{j=1}^{n} e^{\rho_j} . \prod_{k=1}^{K} (X_{kj})^{\nu_k} .\varepsilon_j} \quad (25),$$

where X_{ki} is the k-th parameter affecting firm i, ν_k is the k-th parameter affecting the demand for firm i, ε_i is error, ρ_i is the parameter of fixed impact of firm i (or the impact factor of marketing attempts made by firm i), and M_j is the marketing attempts made by firm j (Cooper and Nakanishi, 1988). The value of Relation M_i (marketing attempts made by firm i) is as follows:

$$M_i = e^{\rho_i} . \prod_{k=1}^{K} (X_{ki})^{\nu_k} .\varepsilon_i = e^{\rho_i} .(P_i)^{\gamma} .(A_i)^{\delta} .(Q_i)^{\theta} \quad (26),$$

Where γ , δ and θ are the exogenous variables of the demand function and are the coefficients of price, advertisement, and quality variables. It would not be possible to estimate ν_k coefficients, due to the non-linear format of the function. In addition, if the function is put in Relation 22 for the purpose of taking its derivative and finding the equilibrium value of the decision variables, it would not be possible to solve the three equations and three unknowns system, due to the multiplicative nature of the relations (Mohammadi et al. 2016b). Logarithmic transformation was primarily on Relation 25. If the logarithm of both sides of Relation 25 is taken, then giving result Relation 27

$$D_i = R_i \times D_0 \Rightarrow \log(D_i) = \log(R_i \times D_0) = \log(D_0) + \log(R_i) \quad (27).$$

Then, we have:

$$\begin{aligned} &\Rightarrow \log(D_i) = \log(D_0) + \log\left(\frac{M_i}{\sum_{j=1}^{n} M_j}\right) = \log(D_0) + \log(M_i) - \log\left(\sum_{j=1}^{n} M_j\right) \\ &\Rightarrow \log(D_i) = \log(D_0) + \rho_i + \sum_{k=1}^{K} \nu_k .\log(X_{ki}) + \log(\varepsilon_i) - \log\left(\sum_{j=1}^{n} e^{\rho_j} . \prod_{k=1}^{K} X_{kj}^{\nu_k} .\varepsilon_j\right) \end{aligned} \quad (28).$$

If the above relations are added up for all firms i (i=1, 2, …, n) and divide the result by n, then we have

$$\log(\tilde{D}_i) = \log(D_0) + \bar{\rho} + \sum_{k=1}^{K} \nu_k .\log(\tilde{X}_k) + \log(\tilde{\varepsilon}) - \log\left(\sum_{j=1}^{n} e^{\rho_j} . \prod_{k=1}^{K} X_{kj}^{\nu_k} .\varepsilon_j\right) \quad (29),$$

where $\tilde{D}_i$, $\tilde{X}_k$ and $\tilde{\varepsilon}$ are the geometric means of D_i , X_k and ε, while $\bar{\rho}$ is the arithmetic mean of ρ. Now, by deducing Relation 30 from Relation 29, we have

$$\log\left(\frac{D_i}{\tilde{D}_i}\right) = \rho_i^* + \sum_{k=1}^{K} \nu_k .\log\left(\frac{\tilde{X}_{ki}}{\tilde{X}_k}\right) + \varepsilon_i^* \quad (30).$$

In Relation 30, there are the following assumptions:

$$\rho_i^* = \rho_i - \bar{\rho}$$

$$\varepsilon_i^* = \log\left(\frac{\varepsilon_i}{\tilde{\varepsilon}}\right)$$

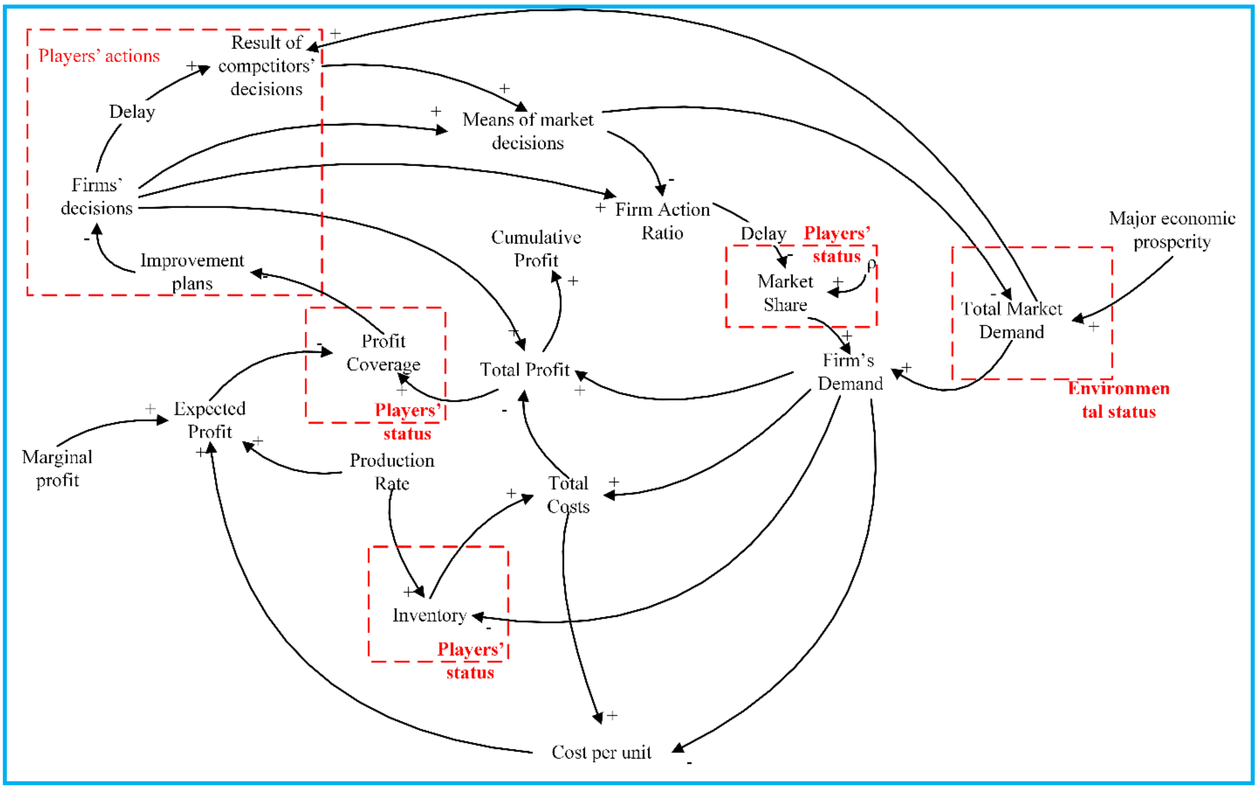

Figure 2: The casual game model which considers the impacts of the environment and players

Considering these relations and such parameters like price, quality, and advertisement, relations governing demand could be re-formulated as follows:

$$\log(\frac{D_i}{\tilde{D}_i}) = \rho_i^* - \gamma.\log(\frac{P_i}{\tilde{P}}) + \delta.\log(\frac{A_i}{\tilde{A}}) + \theta.\log(\frac{Q_i}{\tilde{Q}}) + \varepsilon_i^* \quad (31).$$

The casual game model was constructed based on the relations reviewed above, especially Relation 31. Considering the results of these relations, as well as the conceptual model proposed, the causal model of game in an oligopoly market was framed through DS methods. Figure (2) illustrates the model proposed (Mohammadi *et al*. 2016b).

4. DISCUSSION

The data gathered from the white cement status in the Iranian export market were used to measure the coefficients of three variables, namely price, quality and advertisement, based on the calculations in Relation 30. The coefficients of such variables as price, quality and advertisement were estimated to be 1.186, 0.42 and 0.37, respectively. Following the estimates of coefficients, the final model of flow inventory (see Figure 3) was simulated. The eight modes based on which the base model was simulated were regulated by eight assumptions: (1) change in marginal profit up to 100%; (2) change in the initial status of the firms up to 20%; (3) difference in the impact factor of a firm's marketing in proportion to the mean; (4) constant change in total demand due to the effect of natural factors (extraneous variables) on total demand; (5) constant change and flux in a firm's volume of production; (6) development of the production line and expansion of the production volume up to 100% halfway through the simulation; (7) emergence of a new competitor in the market; and (8) higher fixed costs as a result of a firm's liabilities to banks.

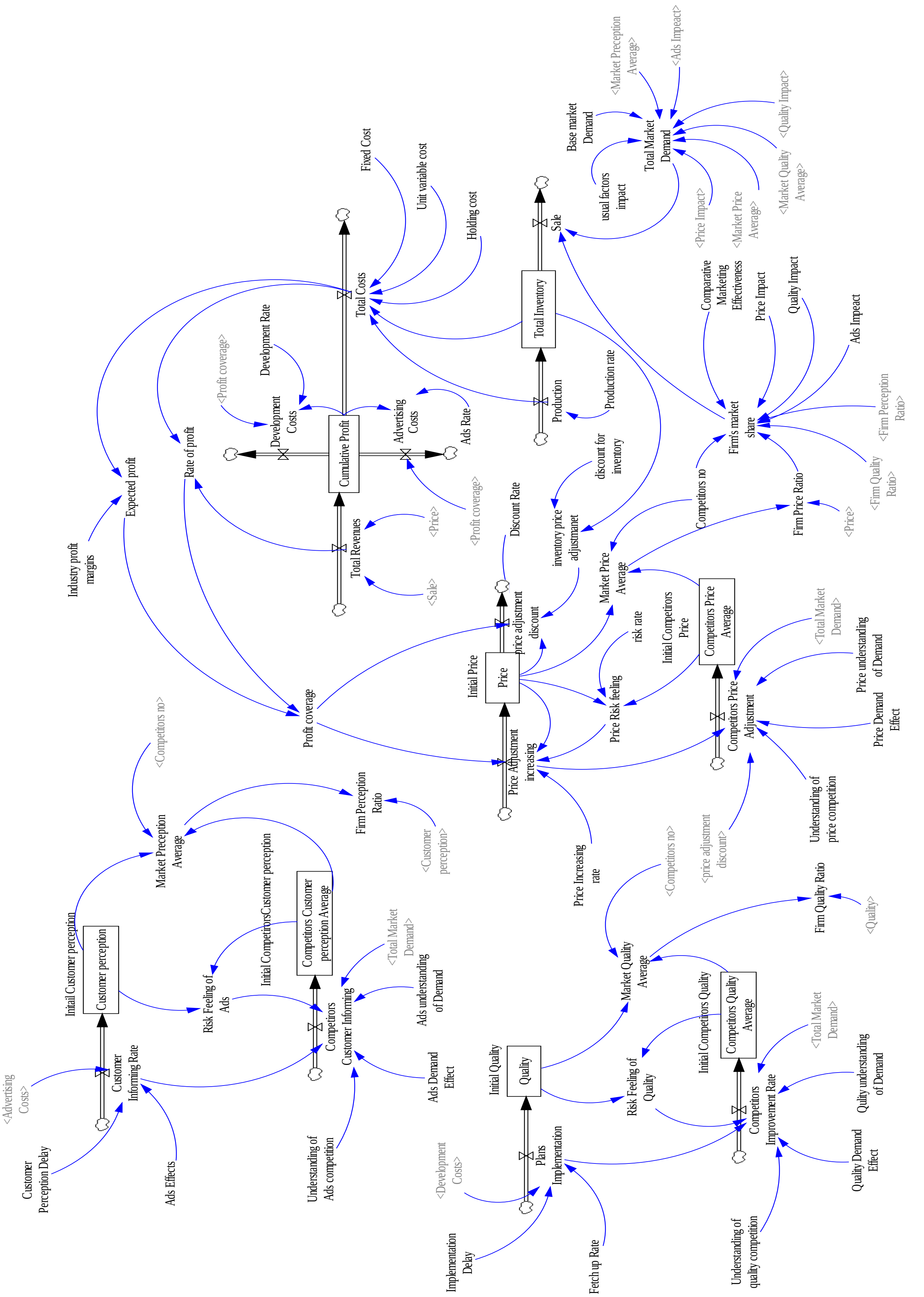

Figure 3: A schematic representation of inventory flow in an oligopoly market as designed through the final model

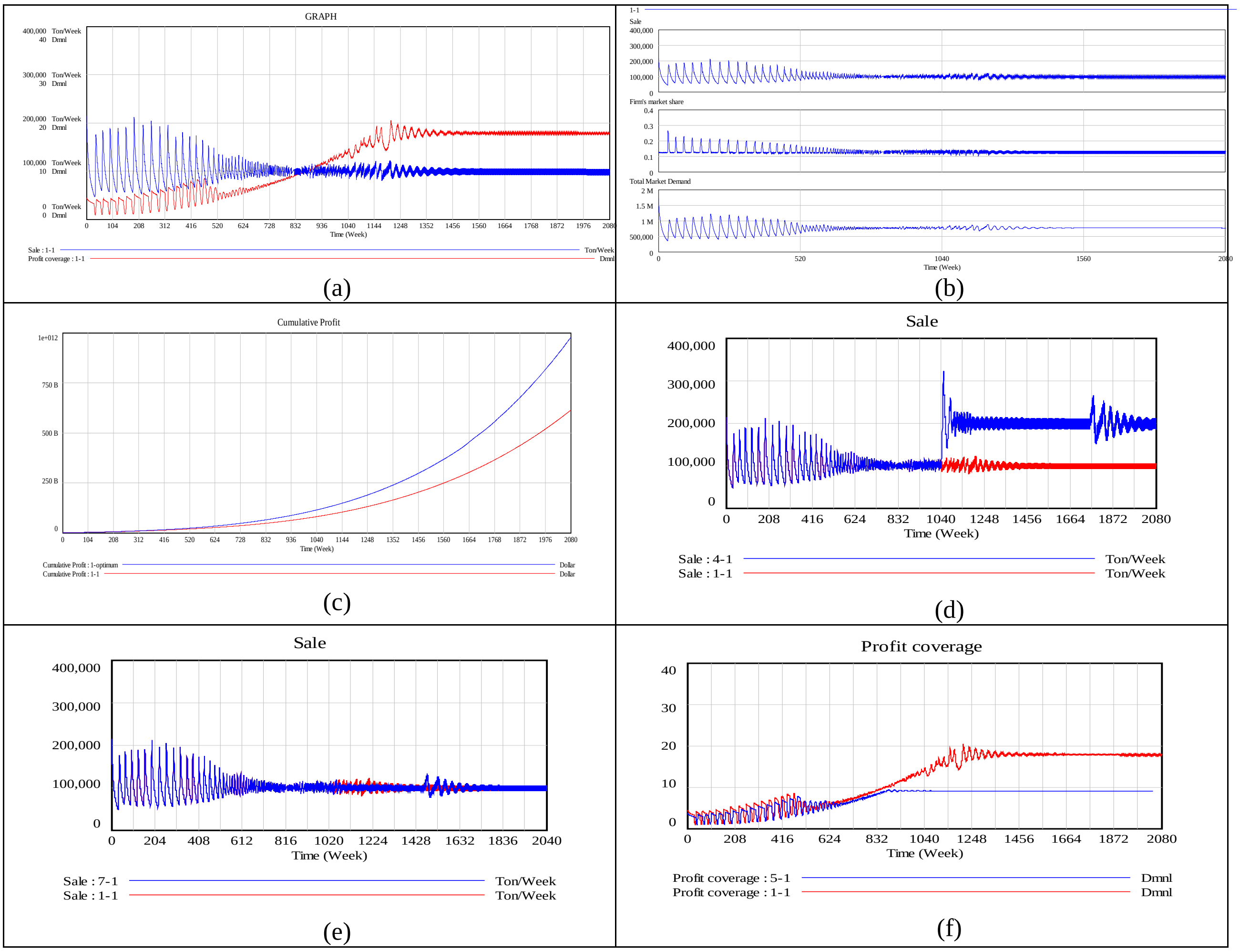

Figure 4: (a) the behavior of profit coverage in the base mode and its comparison with firms' sales; (b) the behavior of firms' market share and total market demand in the base mode simulation; (c) a comparison of cumulative profit in optimal and base modes; (d) the behavior of firms' sales in the mode of increased production capacity in the middle of the simulation period; (e) the behavior of sales in the mode new competitor in the middle of the simulation period; (f) the behavior of profit coverage in every period in the mode of increased fixed costs.

The time unit for simulation, according to the weekly decisions made by cement companies' managers, was set to be "one week", and the duration for simulation was considered to be 1040-2080 weeks. Figure 4 shows the results of simulation and behaviors of the main variables of the study (including players' states and the environment). Given the findings, it could be argued that the behavior of such variables as sales and market share initially underwent considerable fluctuations, although the fluctuations declined over time in such a way that from the middle of the simulation period the variables reached a stable and balanced status. An interesting issue about balance is that the equilibrium status of firms' sales in the market represented a status in which the entire production would be sold out. In reality, however, the market share of cement companies is decided by their production capacity, and the equilibrium status can be reached when firms can sell their whole production volume. Additionally, the behavior of the variable profit coverage was concentrated on as a player status indicator. The analysis revealed that profit coverage also displayed a purposive behavior. This observation implied that if the players acted logically (reasonably) by investing in quality improvement and customer awareness, their profit coverage would reach an appropriate

*Corresponding author (Ehsan Javanmardi) E-mail: Ejavanmardi@nuaa.edu.cn,

degree of equilibrium after a period of fluctuation.

It could be concluded that players with their behaviors and decisions can, under specific conditions, engender fluctuations, although the system eventually moves toward a stable balance in the long term. This situation leads players to make decisions (e.g. cooperation) that would facilitate the state of equilibrium. Yet, firms cannot establish stable cooperation in the short term, because they have different initial conditions. Such conditions lead firms to break their ties with other dominant firms, despite their tendency for cooperation. For instance, a difference in product quality could make it difficult for firms to stabilize their cooperation in the short term.

Similarly, a difference in fixed production prices could break cooperation in the short term. It could also be concluded that in an oligopoly market, the system after a period of fluctuation will finally reach a balanced status in terms of players' status. The goal-orientedness of the system, as far as players' behaviors and the key variables in the model are concerned, evinces the existence of equilibrium in the system and players' decisions. Moreover, results of the model optimization suggest that offering more discount and increasing/decreasing price rates would not generate more profit for firms; rather, trying to make products known to the market and customers, as well as customers' awareness of the product, could remarkably boost a firm's cumulative profit over time. Additionally, a change in customers' awareness, too, demands advertising, which itself needs time and costs. As a result, the first and most cost-benefit option firms have is to change prices to gain expected marginal profit, as their ultimate purpose. Moreover, varying expectations of profitability might diversify players' decisions only in the medium term, but eventually, the system is expected to reach a state of equilibrium. Furthermore, when a firm was in conditions different from its competitors at the beginning of the simulation, it tried to harmonize itself with the market. This attempt, however, could engender more fluctuations at the beginning of the simulation, although the trend would eventually return to equilibrium. For instance, as far as quality is concerned, a firm, when it lacks standard quality, might make every effort to enhance its quality to meet average market expectations, and the efforts are realized after a given period of time.

In fluctuating markets where demand undergoes constant flux, gaining more profit under optimal conditions would be more likely. Meanwhile, the behavior of the model and that of the major variables considerably tend to be goal-oriented and move toward equilibrium. In fluctuating markers, it is to firms' advantage to invest in long-term activities, instead of focusing on continuous price changes. Long-term activities include developing product quality and raising customer awareness. Additionally, the emergence of a new competitor in the market could lead to a firm's reduction of prices. Yet, a firm's volume of sales may not experience any changes because the firm's capacity is not altered. Of course, under such conditions, the firm's market share may be reduced because a proportion of the firm's current market is taken over by the newly emerged competitor. Similarly, the firm's profit rate may decline as a result of the presence of the new competitor. Generally speaking, if there is demand in the market, the emergence of a new competitor will not affect the firm's sales, but it could reduce the firm's market share and profit rate because the product price is reduced.

Finally, The conceptual model proposed in this study, as a framework composed of DS and game theory, can serve many purposes. Given the investigations conducted in the study, it was revealed that the demand function and its format determined the point of equilibrium in the game. In some cases, it was not possible to identify the Nash equilibrium through the relations of game theories alone.

Overall, one could argue that oligopoly market models which solely rely on game theory suffer from the following shortcomings:

- It does not consider the market and the decision-making process within a feedback loop structure;
- It considers relations and their impact on the game structure without including time delays;
- Every firm will need to assume a definitive demand function to determine game equilibrium, whereas mere mathematical relations fail to represent all of the issues existing in a competitive structure.

Considering these issues, in this study demand functions were converted so that they could be used in DS models. These conversions provided several advantages: first they made it possible to estimate the coefficient of every parameter by linearizing multiplicative demand equations; second, relations 31-32 revealed that what really influenced sales and market share was the deviation of parameters affecting demand with respect to average market conditions.

In other words, according to the model simulations, it was revealed that the behaviors of the main variables of the model and players' states were generally goal-oriented and moved toward a point of equilibrium. This point, as well as the transitory period before equilibrium, directly depended on initial values such as initial market prices, number of competitors, the impact factor of every firm, the conventional value of marginal profit, and final product costs. Furthermore, an investigation of optimal behavior showed that product quality development and advertising for the purpose of increasing customer awareness should be prioritized. Without a doubt, solving the model through common game theory methods could not reflect the behavior of the system in the long term or in the transitory period.

5. CONCLUSION

This study proposed an integrated model of game theory and DS to investigate the behavior of firms in an oligopoly market. In this regard, the first firms' mechanism of decision-making in an oligopoly market was explored and then formalized through methods of game theory. To construct the final causal model of game in an oligopoly market, two major actions were taken: (a) the causal DS models of an oligopoly market were constructed based on relations formulated according to the payoff functions and on the conceptual model; and (b) the three variables, price, quality and advertisement, were considered to be decision variables of the problem.

The model-based simulation was conducted according to eight different modes; an attempt was made to identify and analyze players' optimal decisions in all of the eight modes. The study tried to formulate a mechanism which could (1) identify the behavior of the model over time; (2) track the equilibrium of the game; (3) find the impact of players' decisions on the system; and (4) create a structure for policy-making. The significant innovation of this study was the conceptual model proposed as a foundation for surveying game models based on a DS framework. Another innovative contribution of this study was the mechanism it suggested for exploring oligopoly markets; the mechanism could model the behavior of players and the consequences of their decisions.

The same mechanism also functioned satisfactorily in oligopoly markets with multiple decision markets, and in a case where mathematical models, due to the multiplicative nature of the payoff function, would fail to make calculations and to find the optimal behavior or consequences of decisions. The proposed model demonstrated players' behaviors, emphasized that a game involved a point of equilibrium, and showed the final and output behavior of the system in terms of players' ultimate goals. Mathematical models, however, lack these specifications. In the model proposed, the number of participants in the game could be modified. This specification makes it possible to include the data of newly emerged competitors. The model also takes into account the impact of players' behaviors on the general structure of the market and on total market demand, although the majority of models in the literature failed to include this item. Yet, although DS models *were* sensitive to players' initial states and delays in decision-making, mathematical models suggested in the literature generally ignored these factors. Additionally, inspecting the impact of three variables concomitantly overtime was another important innovation of this study. Future studies can construct conceptual models in other economic or social fields. Moreover, the model proposed for oligopoly markers could be applied to other markers as well. The impact of other parameters in the proposed model could also inspire further research projects.

6. REFERENCES

Adamides, E., Pomonis, N. & Stamboulis, Y. (2004). Embedding game-theoretic concepts into system-dynamics models: the case of complementary products development. *International Conference of System Dynamics*, July 25-29, Oxford.

Akiyama, E., & Kaneko, K. (2000). Dynamical systems game theory and dynamics of games. *Physica D: Nonlinear Phenomena*, 147(3-4), 221-258.

Akiyama, E., & Kaneko, K. (2002). Dynamical systems game theory II. *Physica D: Nonlinear Phenomena*, 167(1-2), 36-71. doi: 10.1016/s0167-2789(02)00402-5

Cooper, L. G., & Nakanishi, M. (1988). Market-Share Analysis. Evaluating Competitive Marketing Effectiveness. *International Series in Quantitative Marketing (ISQM)*. Boston, MA: Kluwer Academic Publishers.

Elettreby, M., & Hassan, S. (2006). Dynamical multi-team Cournot game. *Chaos, Solitons & Fractals*, 27(3), 666-672. doi: 10.1016/j.chaos.2005.04.075

Elettreby, M., & Mansour, M. (2012). On Cournot dynamic multi-team game using incomplete information dynamical system. *Applied Mathematics And Computation*, 218(21), 10691-10696. doi: 10.1016/j.amc.2012.04.038

Ficici, S., Melnik, O., & Pollack, J. (2005). A Game-Theoretic and Dynamical-Systems Analysis of Selection Methods in Coevolution. *IEEE Transactions On Evolutionary Computation*, 9(6), 580-602. doi: 10.1109/tevc.2005.856203

Gary, M. S., Kunc, M., Morecroft, J. D., & Rockart, S. F. (2008). System dynamics and strategy. *System Dynamics Review: The Journal of the System Dynamics Society*, 24(4), 407-429.

Kim, D., & Kim, D. (1997). A system dynamics model for a mixed-strategy game between police and driver. *System Dynamics Review*, 13(1), 33-52.

Koh, A., Shepherd, S., & Watling, D. (2013). Competition between two cities using cordon tolls: an exploration of response surfaces and possibilities for collusion. *Transportmetrica A: Transport Science*, 9(10), 896-924.

Mohammadi, A., Shirazi, A., Talebnezhad, A., Javaheri, A., & Javanmardi, E. (2016). Providing a conceptual model in dynamical system Game theory in the study of cognitive behavior competitors in an oligopoly market. *International Business Management*, 10(10), 1921-1928.

Mohammadi, A., Shirazi, A., Talebnezhad, A., Javaheri, A., & Javanmardi, E. (2016). The combination of system dynamics and game theory in analyzing oligopoly markets. *Management Science Letters*, 6(4), 265-274. doi: 10.5267/j.msl.2016.2.003

Shahanaghi, K., & Hajihosseini, M. (2011). Integrating dynamical systems and game theory in investigating systems. *8th International Conference on Industrial Engineering*. https://www.civilica.com/Paper-IIEC08-IIEC08_299.html. PDF

Sice, P., Mosekilde, E., Moscardini, A., Lawler, K., & French, I. (2000). Using system dynamics to analyse interactions in duopoly competition. *System Dynamics Review*, 16(2), 113-133.

Weiguo, Z., Zhen, L., & Jun, L. (2011, November). Study on Contracts and Penalty Mechanism of Technology Alliance Cooperative Innovation Based on Evolutionary Game. *In Information Management, Innovation Management and Industrial Engineering (ICIII),* 2011 International Conference on (Vol. 3, pp. 382-386). IEEE.

West, R., & Lebiere, C. (2001). Simple games as dynamic, coupled systems: randomness and other emergent properties. *Cognitive Systems Research*, 1(4), 221-239.

Professor Dr. Ali Mohammadi is Professor at School of Economics, Management and Social Sciences, Shiraz University, Shiraz, IRAN. He is currently the Chairman of the School of Economics, Management and Social Sciences of Shiraz University. He obtained his Bachelor's degree from Shiraz University, Iran. He earned his Master's degree and Ph.D. in Industrial Management from Tarbiat Modares University, Iran. His main research activities are in Systems Theory, Operation Research and Mathematical and Statistical Method in Management.

Dr. Ehsan Javanmardi is a Postdoctoral Researcher of Institute for Grey Systems Studies, Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, China. He obtained his Bachelor's degree from Islamic Azad University Shiraz, Iran, and his Master's degree from Iran University of Science and Technology (IUST), Tehran, Iran. Also, He obtained his Ph.D. degree in Systems Management from Shiraz University, Shiraz, Iran, in 2016. His current research interests include System Dynamics, Game Theory, Applied econometric, Grey system, Socio-Economic Systems.