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AN ALGORITHM FOR DETERMINING THE STRUCTURAL PARTS OF PETRI MODELS-BASED COMPLEX SYSTEMS

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ABSTRACT

Mathematical modeling is a common tool for the study of complex systems. As a formal model for obtaining the most complete information about the system, the formalism of Petri nets is productively used. In doing so, simulated systems can relate to various application areas. Almost any real complex system, as a rule, consists of several or many objects interacting with each other. Therefore, when constructing Petri models of systems and large objects, it is necessary to solve the problem of the exponential growth of the space of the model state. This problem can be solved by developing compression algorithms for the test set of states while maintaining the correctness and adequacy of the model. This work is devoted to the development of a mathematical basis for the software implementation of the determination of structural parts – constituent components in Petri models of complex systems. Thus the paper considers the transformation of Petri models, which results in a reduction of the original model of a complex system. It has been established that this transformation is an epimorphism. An algorithm has been developed that allows us to determine structural parts in Petri models of complex systems.

Disciplinary: Multidisciplinary (Mathematics, Simulation Software, Computer Programming).

1. INTRODUCTION

The theory of Petri nets is a well-known and extremely convenient formalism for describing and modeling systems with parallel and asynchronous processes [1, 2, 3, 4, 5]. It allows us to describe the device and the behavior of the simulated object most accurately. An important issue when using the mathematical apparatus of Petri nets for modeling systems with parallelism is the question of the correctness and adequacy of the model and the possibility of performing an analysis of its properties. The scope of Petri nets is currently very extensive. These are studies of network protocols, telecommunication networks, computer systems, production and organizational systems [6, 7, 8, 9], as well as studies of the properties and dynamics of biological systems, reconstruction of molecular network complexes, mapping of biochemical metabolic reactions, presentation of neural processes [10, 11, 12].

The possibilities of using the apparatus of Petri nets in both practical and theoretical studies allow practitioners to receive information from theoreticians on how to create more realistic models, and a mathematically rigorous description of the model allows it to be analyzed using modern computer technology. But when constructing Petri models of systems and large objects, it is necessary to solve the problem of the exponential growth of the space of the model states. In this regard, various reduction methods [13, 14, 15, 16, 17] are used to make the transition from the original model to a suitable one, with dimensions that allow its analysis.

This work is devoted to the results of investigations of the reduction of the Petri model of a complex system, which consists in determining of the structural parts of the model of interest to us – the constituent components [18]. Depending on the size of the systems under study, we can talk about constituent components as system-forming clusters.

2. PRELIMINARY INFORMATION

Definition 1. A constituent component N_C is a triple $N_C = (N, X, Y)$, where N is a Petri net with a finite set C of vertices of one selected type, $X \subseteq C$, $Y \subseteq C$ are respectively its input (initial) and output (final) vertices of the selected type, and $X \cap Y = \varnothing$, $C \setminus (X \cup Y)$ is the set of its internal vertices of the selected type. The input and output vertices of the selected type do not have incoming and outgoing arcs, respectively: $\forall c \in X : {}^{\bullet}c = \varnothing$, $\forall c \in Y : c^{\bullet} = \varnothing$. No internal vertex of a constituent component has arcs entering from outside the component and leaving the component. Moreover, the component N_C itself, as an element (vertex of the corresponding type) of the reduced Petri model of the system under study, has incoming and outgoing arcs (initiators and resultants).

Constituent components permit to simulate parallel, asynchronous processes of the Petri model in the system under study.

In [19], the concept of a component relation was introduced. The constituent components of the Petri model N are denoted by N_{C_k} $(k = 1, 2, ..., n)$, obviously $N_{C_k} \subset N$.

Definition 2. A component relation is a relation χ, considered on the set of vertices of the Petri model N which satisfies the following conditions:

1) for each vertex a of the Petri model N $(a \in N)$ and the relation χ, $a \chi a$ is satisfied;

2) for two vertices $a \in N$, $b \in N$, and the relation χ, $a \chi b$ is satisfied if a and $b \in N_{C_k}$ are the vertices of one constituent component N_{C_k}.

Each of the vertices of the Petri model N, not included in any of the component components N_{C_k}, can be considered as a component consisting of one vertex. Such a component is called a single component and is denoted as $N_{C_{e_i}}$.

A relation χ is an equivalence relation that splits the set of vertices of the model N into equivalence classes. The corresponding equivalence class of the component relation χ, considered on the set of vertices of the Petri model N, is called the region of the component relation N_{C_k}.

The set of vertices of the Petri model N with the equivalence relation χ is the union of disjoint regions of the relation of components that form a partition of the set of vertices of the net N. For the component relation area $N_{C_k} \subset N$, the following conditions are satisfied:

1) $\forall a,b \in N_{C_k}$: $a\chi b$, wherein it is obvious that if the component N_{C_k} is unity ($k = e_i$), then $b = a$;

2) $\forall a \in N : a \notin N_{C_k} \Rightarrow \exists b \in N_{C_k} : \neg(a\chi b)$.

As a result of the assignment, on the set of vertices of the Petri model N of the component relation χ, a transformation γ of the model N takes place that satisfies the following conditions:

1) $a\chi b \Leftrightarrow \gamma(a) = \gamma(b) = V_k^*$ is satisfied for any vertices $a,b \in N_{C_k}$;

2) $N_{C_{e_i}}$ is the top of the net $N \Leftrightarrow \gamma(N_{C_{e_i}}) = v$,

where V_k^* is the vertex (of the corresponding type) of the reduced model N^* (a component of the model N), and v *is* the vertex (of the corresponding type) of the reduced model N^*.

Transformation γ is epimorphism.

Theorem. If the ratio of the component χ is given on the set of vertices of the Petri model N, then the following is fulfilled for reduction γ:

$$\gamma(aFb) \Rightarrow \gamma(a)F'\gamma(b) \quad (1),$$

where F and F', respectively, are the incidence relation of models N and N^*.

If the vertices of the net N belong to one constituent component N_{C_k} $(k = 1, 2, ..., n)$, the following (1) takes the form

$$\gamma(a\chi b) \Rightarrow \gamma(a)\chi'\gamma(b) \quad (2),$$

where χ, χ', respectively, are the relations of the components of the models N and N^*.

3. EXAMPLES OF OBJECTIVES FOR THE REALIZATION OF REDUCTION γ: CONSTRUCTION OF A REDUCED MODEL N^*

Example 1: Figure 1 shows a reduced Petri model of the famous cryptographic protocol of

Elgamal [20]. This algorithmic scheme ensures two users to receive a shared secret key necessary for the exchange of confidential information when the communication channel is not protected from listening. In doing so, both users are aware of prime numbers p and q. The first user, with the private key x which he created, calculates the public key $y = q^x \bmod p$. The second user, with the private key k which he created, and public key y, calculates a pair of numbers (a,b), which he passes to the first user, who calculates the final key $M = ba^{p-1-x} \bmod p$ ($a = q^k \bmod p$, $y = q^x \bmod p$).

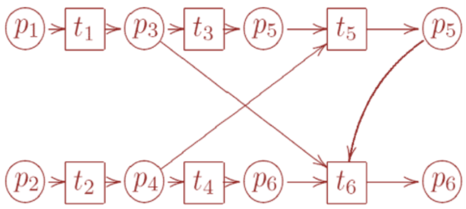

Figure 1: Reduced Petri Model of Elgamal Protocol

The reduction of the model was carried out by identifying the structural parts of the model – constituent components, the type of which, as the vertices of the reduced model, are transitions, transition-vertices t_3, t_4, t_5, t_6. These four vertices are the same constituent components of the reduced model. They model the process of computing $q^x \bmod p$, where p and q are primes. The Petri net, corresponding to this calculation, is shown in Figure 2.

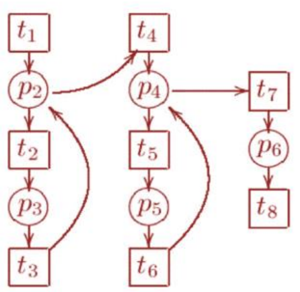

Figure 2: Petri net modeling the process of computing $q^x \bmod p$

Example 2: Figure 3 shows a temporary Petri model for the task of extracting and systematizing information stored in three distributed databases for decision making [9].

Figure 4 shows a reduced exemplary of the model in Figure 3. The places P_2^* and P_4^* in it are components, which simulate the process of extracting the necessary information, being structural parts of the original model, from three distributed databases p_2^1, p_2^2, p_2^3 and p_4^1, p_4^2, p_4^3, possibly for different requests-programs. The place p_3 is additional computing resources, the transition t_2 is the development of the program, the transition t_3 is systematization of information according to the conditions p_1 .

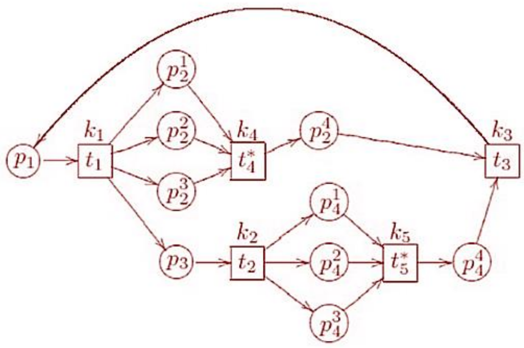

Figure 3: Petri model with time for the task of extracting and systematizing information.

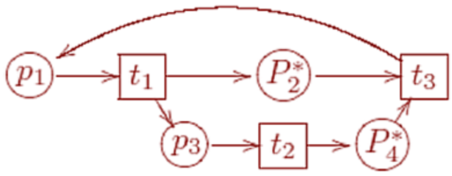

Figure 4: The reduced Petri model of the problem of extracting and systematizing information

4. SEARCH ALGORITHM FOR COMPONENTS OF THE PETRI MODEL

According to the definition of the constituent components of the Petri model (Definition 1), there are two types of constituent components. These are components-transitions and components-places.

4.1 DETERMINING COMPONENT-TRANSITIONS

To determine components-transitions in the Petri model, it is necessary to simultaneously test two conditions for any two vertex transitions:

1) For any two vertices-transitions t_i and t_j of the bipartite graph under consideration, one of which, let it be t_i is the beginning, and the other, respectively t_j, the end, the possibility of the existence of a path from the beginning to the beginning is checked, not passing through the end.

The algorithm creates visits to all reachable vertices of the graph from the vertex t_i . If a path from a vertex t_i to a vertex t_i is detected that does not pass through t_j, then these vertices t_i and t_j are not the vertices of the component-transition;

2) The possibility of the existence of a path is considered that goes along the inverse edges along the vertices of the graph inside the pair t_i and t_j being checked, to enter the component-transition, which leads to the end without passing through the beginning.

The component-transition includes all those vertices that were visited by traversing the graph in depth along the inverse edges.

4.2 DETERMINING COMPONENTS-PLACES

To determine components-places in the Petri model, after the first two steps of checking conditions 1-2 from 4 *A*, it is necessary to add one step of attachment of the corresponding places for the selected component-transition and thereby obtain the component-places.

The constructed algorithm allows you:

1) to determine successively possible components-transitions in the model, from minimum to maximum by looking at all pairs of transitions;

2) to determine sequentially possible components-places in the model, from minimum to maximum, with the addition of one step of attaching the top of the place for the selected component-transition;

3) to determine successively possible components-places in the model, from minimum to maximum, in which there are transitions that synchronize flows;

4) to determine possible components-places in the model, from minimum to maximum, looking through all pairs of places;

5) to determine possible components-places in the model, the number of vertices in which is the number closest to the square root of the number of all vertices of the model in question.

The run time of the search algorithm for the maximum component-transition in the Petri net is $O(V^2 \cdot E)$, where V is the number of vertices, E is the number of edges of the bipartite graph.

The run time of the check algorithm for a pair of transitions is $O(E)$.

5. EXAMPLES OF IMPLEMENTATION OF THE SEARCH ALGORITHM FOR COMPONENTS OF THE PETRI MODEL

Let's consider the Petri model from [21], shown in Figure 5. This is a model of the railway traffic scheme of a railway junction with six interconnected specialized stations. Stations on this model are represented by places P_8^*, P_{10}^*, P_{14}^*, P_{16}^*, P_{17}^*, P_{20}^*.

Places P_8^*, P_{17}^* and P_{14}^*, P_{20}^* correspond to through railway stations with four and seven internal tracks, respectively; places P_{10}^* and P_{16}^* correspond to dead-end railway stations with five and three internal tracks, respectively. These locations are component-places. Indeed, consider the Petri model shown in Figure 6. This is the Petri model from Figure 5, but in it, the railway traffic for each station is modeled by the corresponding Petri net.

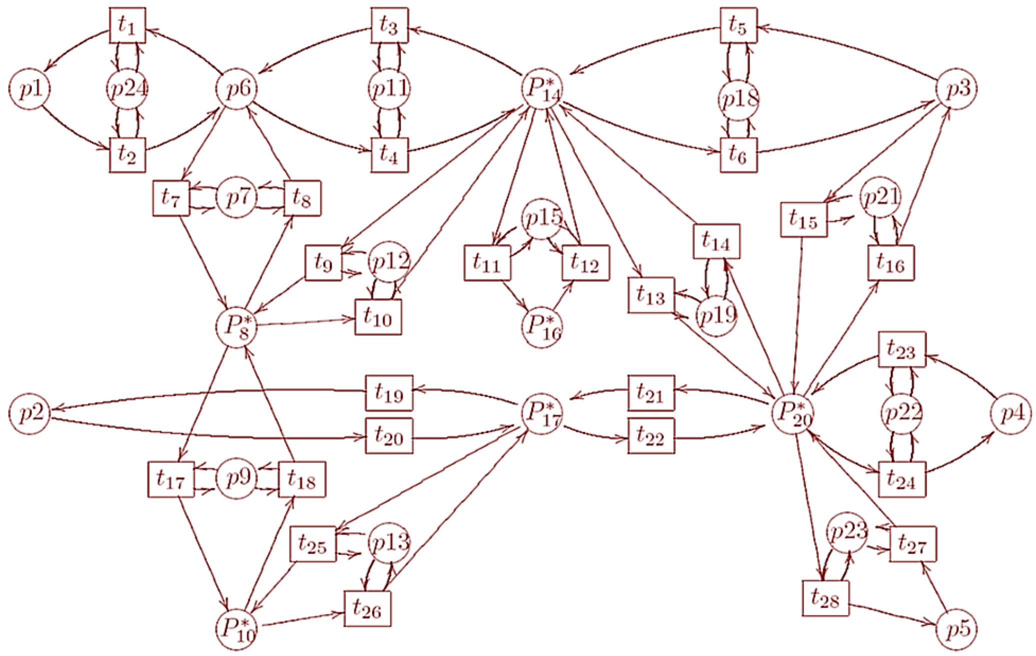

Figure 5: Petri model of a railway transport scheme of a railway junction, where P_8^*, P_{10}^*, P_{14}^*, P_{16}^*, P_{17}^*, P_{20}^* are components-places

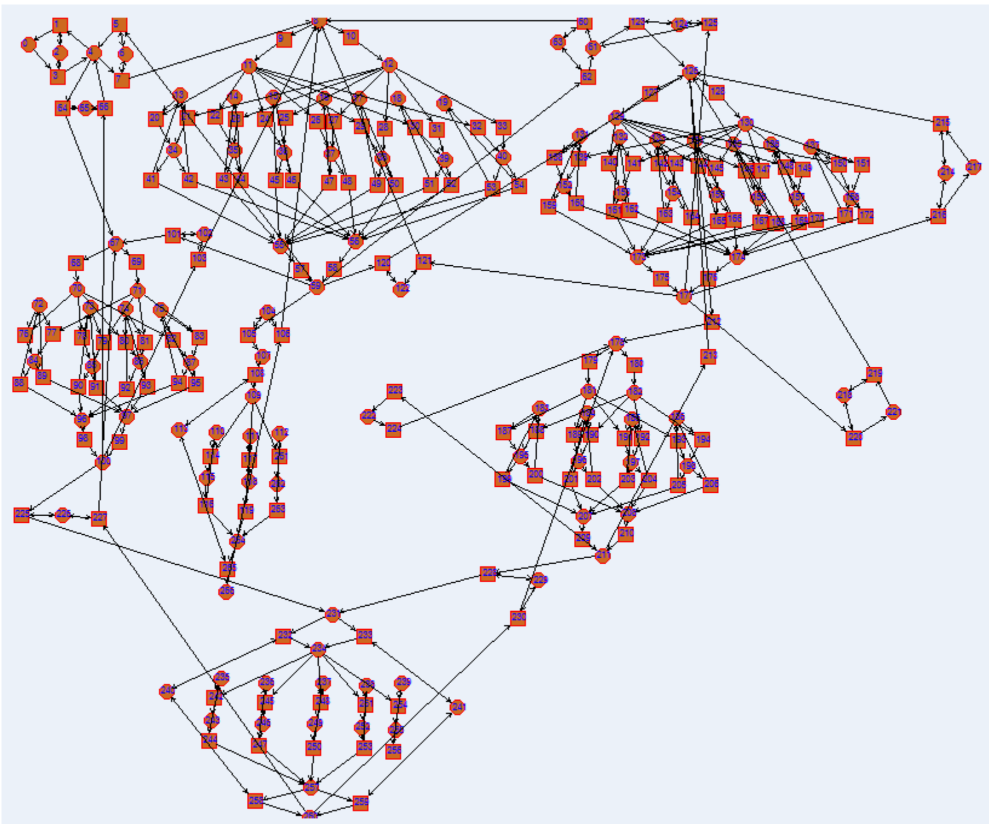

Figure 6: Petri model of railway traffic of some railway junction with six interconnected railway stations

The algorithm for searching for the constituent components of the Petri model, considered in this work, allows us to determine the constituent components in the model from Figure 6. For example, you can determine the components-places shown in Figure 5.

Figures 7, 8, 9 show examples of determining components-places in the Petri model from Figure 6. These are the components-places highlighted in yellow in Figures 7, 8, 9. For the model under consideration, they represent its structural parts – models of some of the interconnected specialized railway stations of the railway junction.

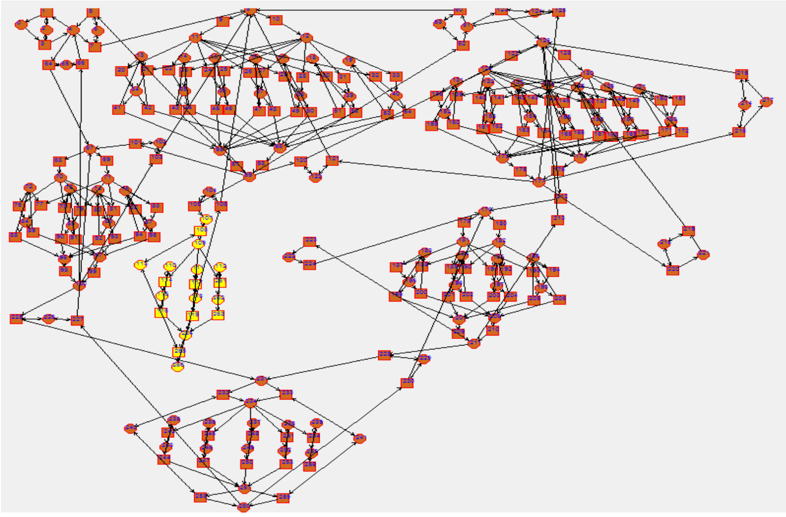

Figure 7: The model from Figure 6 with a highlighted component simulating a rail traffic pattern for a dead-end railway station with three internal tracks

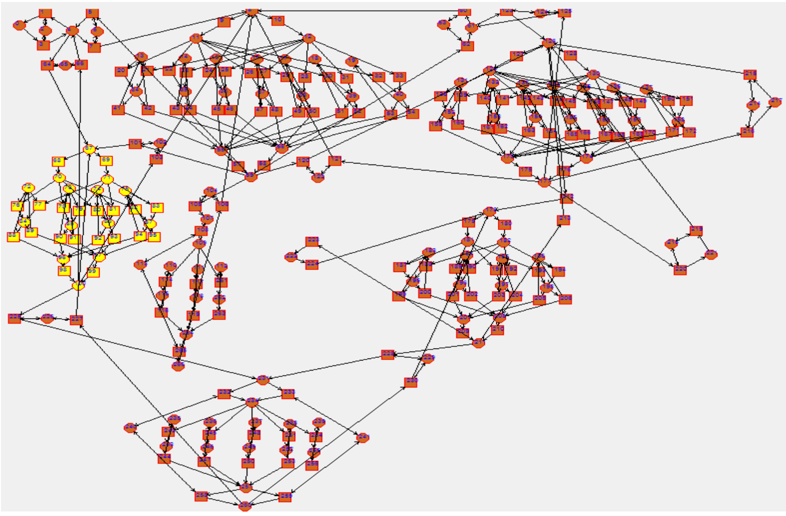

Figure 8: The model from Figure 5 with a highlighted component simulating a railway traffic pattern for a through railway station with four internal tracks.

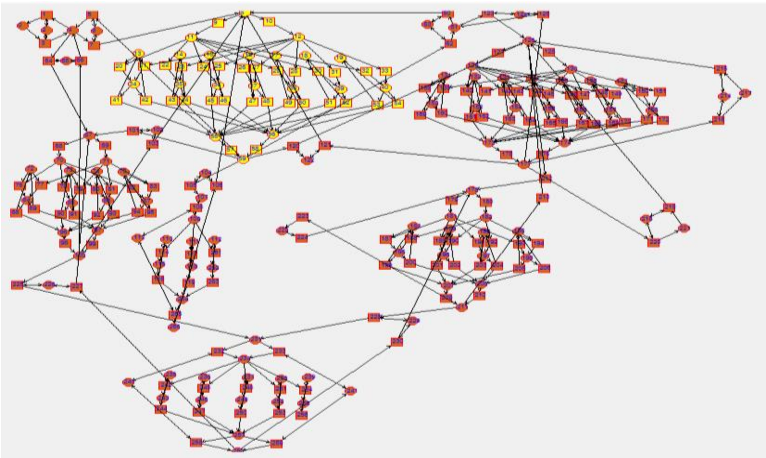

Figure 9: The model from Figure 5 with a highlighted component simulating a railway traffic pattern for a through railway station with seven internal tracks.

6. CONCLUSION

The determination of the structural parts of the Petri model of the studied complex system is extremely useful for the analysis of complex models. This approach allows us to analyze complex models using decomposition, by considering them in parts. A two-aspects approach [18, 22] to the functioning of the structural parts of the Petri model opens up new possibilities for reducing the time for model verification. The implementation of the algorithm for searching for the constituent components of the Petri model is carried out in C # (C-Sharp) language in order to obtain a visual tool for processing data of sequentially synchronized nets.

7. DATA AND MATERIAL AVAILABILITY

This study already includes all the information about this study.

8. ACKNOWLEDGEMENT

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