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APPLICATION OF FRACTAL ANALYSIS METHOD FOR STUDYING STOCK MARKET

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ABSTRACT

The study of financial markets behaviour is an important part of the financial investments theory. The methods for analyzing the financial markets which have been established in the sixties and seventies of the last century are valid only during periods of stable market conditions. They are based on the assumption that the financial markets behaviour is subject to the normal distribution law. In the nineties of the last century, they began to look at this problem from the point of view of fractal analysis. It was observed that financial time series has the property of self-similarity. In the works of Mandelbrot (1983, 2006), the founder of fractal geometry, the behaviour of financial indicators in the market was considered as fractals. The book by E. Peters “Fractal analysis of financial markets” and “Chaos and order in the capital markets” are devoted to the study of this problem. The presented work is devoted to the study of financial time series in the stock market in the current situation. Financial time series in this paper are treated as fractals. The study of the series for persistence and volatility using R / S analysis were carried out. For the persistent series, the persistence hypothesis was again tested by mixing the series. The average lengths of non-periodic cycles were also found for these series.

1. INTRODUCTION

Currently, stock markets are attracting more and more people from those who deal with financial analytics and from ordinary traders to analysts of global corporations and government agencies.

There are many ways to analyze events occurring in stock markets. One of the techniques is the use of fractal analysis for researching financial time series [1–8].

Since the beginning of the 90s of the last century, the study of financial markets began to take place in terms of fractal analysis. Financial time series with the property of self-similarity began to be regarded as fractals [1–8].

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This paper focuses on persistent financial time series, i.e., series with long-term memory. Such series are most interesting from the point of view of investment. They are more predictable since they contain the memory of previous data for the analysis of the indicators following them. The authors investigated several financial series, two of which turned out to be persistent. The average lengths of non-periodic cycles were found for them; they are the important components for the analysis of a financial series to invest in them.

2. METHOD

To determine the type of memory of financial time series, R / S analysis was used, which consists of performing the following steps [5].

1) The source series with length M is converted using logarithmic ratios. The result is a time series of length $N = M - 1$ with the following values:

$$N_i = \log\left(\frac{M_{i+1}}{M_i}\right), \quad i = 1,2,...,M - 1. \tag{1}$$

The necessary requirements for the sample are that its volume N must be large enough and be a multiple of 2.

2) Further, this series is divided into A adjacent subperiods of length n such that $An = N$. Each of them is denoted by I_a, where $a = 1, 2, \ldots, A$. We denote every item in Ia through $N_{k,a}$ at $k = 1,2, \ldots n$. The average value $N_{k,a}$ is determined in each sub-period according to the formula

$$e_a = \frac{1}{n}\sum_{k=1}^{n} N_{k,a}, \tag{2}$$

where $a = 1,2, \ldots, A$..

3) Next, a series of accumulated deviations $(X_{k,a})$ are compiled for each sub-period I_a. It is defined as follows:

$$X_{k,a} = \sum_{i=1}^{k}(N_{i,a} - e_a),\ k = 1,2, \ldots \text{n}. \tag{3}$$

4) In the next step, the range of the accumulated frequency of each sub-period I_a is determined

$$R_a = \max_{1 \le k \le n} X_{k,a} - \min_{1 \le k \le n} X_{k,a} \tag{4}$$

5) Then we calculate the sample standard deviation for each sub-period I_a according to the formula:

$$S_a = \sqrt{\frac{1}{n}\sum_{i=1}^{k}(N_{i,a} - e_a)^2}. \tag{5}$$

6) The average value R/S is determined for length n according to the following formula:

$$(R/S)_n = \frac{1}{A}\sum_{a=1}^{A}\frac{R_a}{S_a}. \quad (6)$$

7) The last step is to increase the length n to the next higher value. Steps 1-6 are repeated until $n = N/2$. Finally, linear regression is constructed, where the variable $\log(n)$ is taken as an argument, and the dependent value is $\log\left(\frac{R}{S}\right)$. The slope of the equation is an estimate of the Hurst index, H. The values of the Hurst index can take the following values.

- $H = 0.5$. In this case, the sample is random.

- $0.5 < H \leq 1$. In this situation, the process is characterized by long-term memory, that is, persistence. This means that subsequent indicators are highly dependent on previous ones. This is close to the sensitivity to the initial conditions which is characteristic to chaos.

- $0 \leq H < 0.5$. Here, the Hurst indicator means an antipersistent process. The system is changing faster than random.

It can be said that the higher is the Hurst index, the smaller is the number of "notches" in the time series [5]. Then, the financial time series was investigated concerning chaotic cycles. For this, the V-statistic was calculated, which gives a more accurate measurement of the cycle length. This indicator can be used to get good performance in the presence of noise. It is defined as follows [5]:

$$V_n = \frac{(R/S)_n}{\sqrt{n}}. \quad (7)$$

This ratio will lead to a horizontal line if the R/S statistics changes the scale in proportion to the square root of time, i.e. function graph V will be flat if the process is independent probabilistic. If the process is persistent and R/S changes its scale faster than the square root of time ($H > 0{,}5$), the graph has a slope directed upwards. If the process is antipersistent ($H < 0{,}5$), the graph has a slope directed down [5]. Plotting V-statistics is as follows: values V_n are put on the Oy axis, and $\log(n)$ are put along axis Ox. At the points in which the graph becomes straightforward, the process with long-term memory dissipates.

To test the null hypothesis which consists in the fact that the system is an independent process, these values must be compared with the theoretical values $E((R/S)_n)$. These values are calculated by the formula [5]:

$$E((R/S)_n) = \frac{(n-0{,}5)}{n}\sqrt{\frac{2}{\pi n}} \cdot \sum_{r=1}^{n-1}\sqrt{\frac{n-r}{r}}. \quad (8)$$

Further, the same series of financial indicators were checked for volatility [5]. For this, the values of the original series were transformed into a series of logarithmic differences:

$$S_i = \ln\left(\frac{M_i}{M_{i-1}}\right), \quad i = 2, \dots, M. \quad (9)$$

Volatility is the deviation of adjacent increments S_i. These increments are disjoint and independent:

$$V_n = \frac{\sum_{i=1}^{n}(S_i - \bar{S})^2}{n-1} \quad (10),$$

where V_n is dispersion for n days, $\bar{S}$ is an average value for S_i $(i = 1, 2, \dots n)$. Change in volatility over time n is calculated as

$$L_n = \ln\left(\frac{V_n}{V_{n-1}}\right). \quad (11)$$

Then, R/S analysis is applied to the series obtained as described above.

For persistent rows, a study was conducted in the presence of cycles. For this, the entire period was divided into subperiods according to the schedule of V-statistics of the financial series. The separation criterion was the slope of the V- statistics curve. For each sub-period, the Hurst index was calculated and the significance of the regression equations was determined. After interpreting the result obtained, the cycles for the investigated persistent financial series were determined.

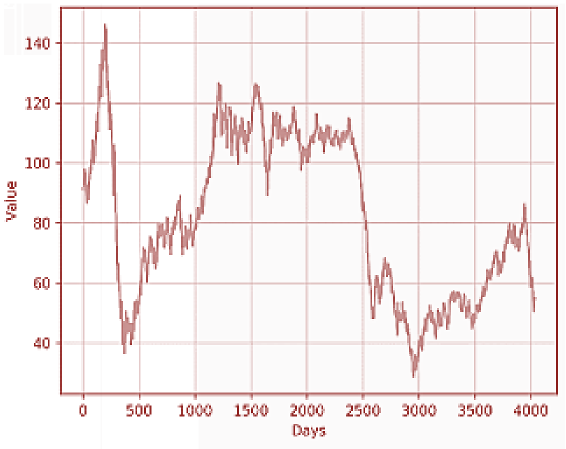

Figure 1: Quotations of Brent oil (19.12. 2007 - 01.01.2019)

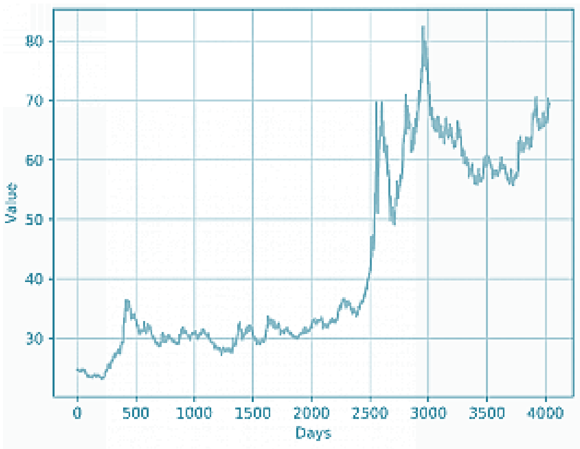

Figure 2: USD / RUB quotations (19.12. 2007 - 01.01.2019)

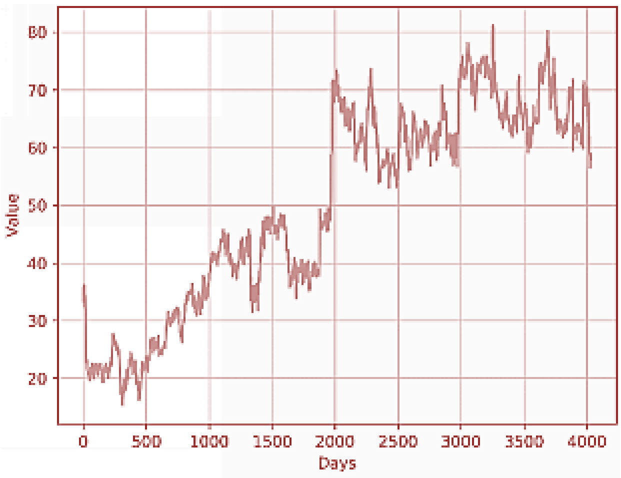

Figure 3: Quotations of ViaSat IT-company (19.12. 2007 - 01.01.2019)

3. RESULT

We investigated the following financial time series for the period from 19/12/2007 to 18/01/2019: prices for Brent oil (Figure 1), dollar/ruble rates (Figure 2), prices for shares of the American IT-company ViaSat (Figure 3). The data source was open sources on the Internet containing databases of stock quotes.

From this data, an R/S analysis was performed and the Hirst index was calculated. As an apparatus for the study, a program written in the programming language Python developed by the authors was used (The use of language for data analysis can be found in [9], [10]). The result of the regression analysis are shown in Table 1.

To check the significance of the regression equation, we can use Fisher statistics which is compared with the table value for the corresponding significance level, the number of factors (in our case, one) and the number of elements in the sample. We should also check the significance of the parameter H using Student's statistics, which is also compared with the table value. In our case, they turned out to be equal. $F_{0.05;1;31} = 4.17$. Therefore, the regression equation and its parameters are significant. In our cases, the correlation coefficient shows a high closeness of the relationship. The coefficient of determination in each of these cases is quite large, i.e. in more than 99% of cases, changes in the factor trait lead to a change in the resulting trait.

Table 1: Result of R/S-analysis.

No	Financial time series	Hurst's indicator	Correlation	Determination	F-statistics
	Quotations of Brent	0.61	0.998	0.996	7664
	USD / RUB quotations	0.6	0.999	0.998	14075
	Quotations of ViaSat IT-company	0.501	0.995	0.990	3170

The Hurst index for the financial series of the Brent crude oil price and the dollar against the ruble ratios turned out to be greater than or equal to 0.6, which allows us to conclude that these series are persistent, i.e. possesses long-term memory. The financial time series of stock quotes of the IT-company ViaSat has a Hurst index value of about 0.5, which gives grounds to say that the indicators of this time series are random in nature.

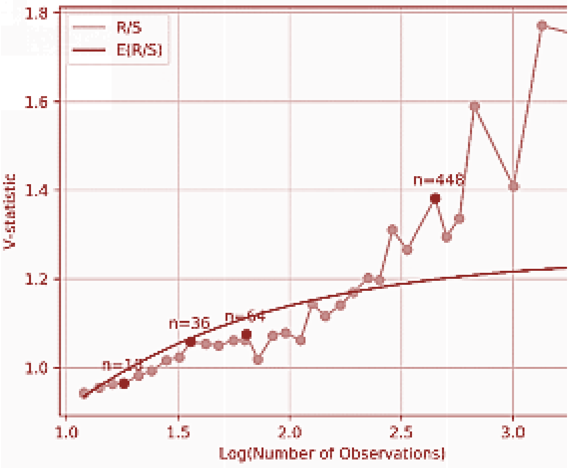

Figure 4: V-statistics of Brent oil (19.12. 2007 - 01.01.2019)

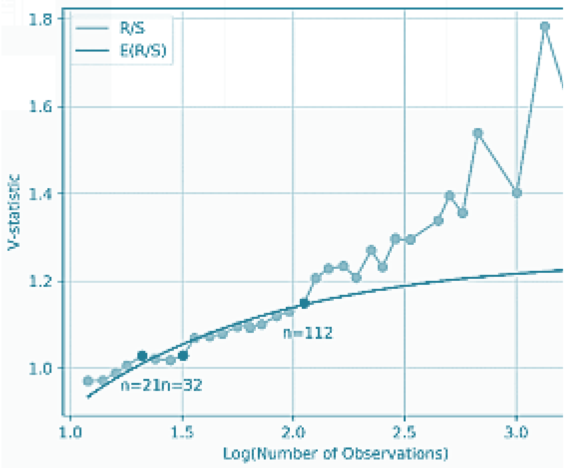

Figure 5: V-statistics of USD / RUB ratios (19.12. 2007 - 01.01.2019)

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Let us check the persistence hypothesis of the two series mentioned above as follows. Obviously, if a financial series has long-term memory, then the order of the data in this series is very important. While mixing the data and re-calculating the Hirst index it should be low. We randomly mixed the levels of indicators and calculated the Hirst index in the newly obtained series. As to indicators of the Brent oil price of, it was equal to 0.092, and the dollar to ruble exchange rate was 0.078. The regression equations also turned out to be significant, since the F-statistics turned out to be 163 and 295, respectively. That is, our hypothesis that these series have long-term memory, was confirmed.

Next, we built a graph of V-statistics and a graph $E((R/S)_n)$. The graph of theoretically calculated indicator $E((R/S)_n)$ corresponds to the null hypothesis and shows the behaviour of a system that is a completely independent process. For comparison, Figures 4 and 5 show the V-statistics graphs for the first two series under consideration.

These graphs also confirm the presence of persistence for the financial series of Brent crude oil price quotes and the dollar/ruble exchange rates.

All the above financial series were examined for volatility. The result is shown in Table 2.

Table 2: The result of the analysis of the volatility of financial time series

No	Financial time series	Hurst's indicator	Correlation	Determination	F-statistics
1	Quotations of Brent oil	0.41	0.978	0.957	110
2	USD / RUB quotations	0.45	0.993	0.985	330
3	Quotations of ViaSat IT-company	0.27	0.984	0.969	154

Interpreting the obtained result, we find that the volatility of each of the above considered financial series is antipersistent in nature, which is characterized by more frequent changes in directions than it happens in a random sequence. This means that the process implies the absence of a stable average value, the size of the change itself is random, and it is returnable. Comparing the values of F-statistics with the Table 2, we can conclude about the significance of the constructed regression models for each of the situations considered above.

Rows that turned out to be persistent were examined for the presence of cycles in them. To do this, each of the financial series with long-term memory was divided into subperiods and for each of them, the Hurst index was calculated. Tables 3 and 4 present the study result. Comparing the F-statistics and tabular values of the Fisher index in Tables 3 and 4 for each sub-period, we can conclude about the significance of the regression equation for each of the sections of the partition.

Table 3: The regression analysis result for the subperiods on the Brent Quotations

Subperiods (days)	[12; 18]	(18; 36]	(36; 64]	(64; 448]	(448; 2016]
Hurst's indicator	0.557	0.63	0.54	0.65	0.71
Correlation	0.9996	0.9988	0.9986	0.998	0.9804
F-statistics	2690	1260	1049	2680	98.86
F table for α = 0.05	18.51	10.13	10.13	4.84	7.71

Table 4: The regression analysis result for the subperiods on the USD/RUB quotations

Subperiods (days)	[12; 24]	(24; 36]	(36; 126]	(126; 2016]
Hurst's indicator	0.61	0.52	0.56	0.61
Correlation	0.9987	0.9985	0.9997	0.9944
F-statistics	1172	341	12333	1148
F table for α = 0.05	10.13	161.45	5.32	4.67

4. DISCUSSION

Due to the fact that the Hirst index for the price of Brent crude oil and dollar to ruble exchange rate is more than 0.5, then these financial series are persistent. In addition, their volatility is anti-persistent, since the Hirst index for volatility is below 0.5.

For persistent series, non-periodic cycles have been defined.
Interpreting the result in Tables 3 and 4, it can be concluded that for a financial series, the prices for Brent oil cycles have periods of length 36, 448, and 2016 days; for the financial range of the dollar against the ruble - periods of 24 and 2016 days.

Note that there may be non-periodic cycles longer than 2016 days. But since the available data on the time series are presented over a period of 10 years, it is not possible to draw conclusions about this.

An R/S analysis conducted for the financial series characterizing the stock quotes of the IT-company ViaSat showed that the series under investigation is random. The volatility of this series is also antipersistent.

5. CONCLUSION

The financial time series were investigated by the method of fractal analysis. Using R/S analysis, persistent and antipersistent series were identified. Rows with long-term memory are of the greatest interest to investors. The higher are the values of the Hurst index, the higher is the predictability of quotes over time we can expect. In conjunction with the presence of identified financial cycles, this makes it possible to miscalculate a good time to invest in the product being studied and the approximate time when these funds need to be withdrawn in order to get maximum profit. Investments, in this case, can give effect for long periods of investment.

Low values, less than 0.5, of the Hurst index will indicate frequent fluctuations of rates, which means such a tool can be used to implement speculative tactics. Quotes will constantly increase, and then subside, which (with constant tracking of the rates) will allow us to receive income on a short time interval. For traders, these series are of interest. Taking advantage of short-term memory and short-term volatility of the financial series, they are able to make a profit in the short term.

Fractal analysis is one of the effective methods for studying financial series. For an investor, the most interesting is the persistent financial series, which can be considered for long-term investments. At this stage of the study, the persistence hypothesis of the studied series was tested using various methods of fractal analysis. And cycles are also found there.

Since in the third case we obtained the Hirst index of 0.501, this indicates the randomness of the process under study. In other words, this indicator shows the absence of any dependence on the subsequent values on the previous ones. This means that an investor who decides to invest in securities of this company will find it difficult to calculate the behaviour of quotes in the future. Thus, it makes working with such an asset to be quite risky and not very suitable for both long-term investments and the use of speculative tactics.

6. AVAILABILITY OF DATA AND MATERIAL

All relevant information in this study is available by requesting to the corresponding author.

7. ACKNOWLEDGEMENT

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