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A Specialized Empirical Bayes Single Regression Coefficient Estimation Procedure for Panel Data Linear Regression Models

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Abstract

In this article, a specialized empirical Bayes estimator for the panel data model is applied to real-world data. This estimator is suitable for all the standard classical panel data models. In this study, the simple linear regression model with orthogonal regressors has been used, and the coefficients for the i^{th} unit of the panel have been estimated individually and independently, that is, the intercepts and the slopes coefficients for each unit of the panel are estimated as scalar quantities rather than a vector of both. The estimator used here is, "the precision weighted arithmetic mean of the ordinary least squares coefficient estimates of the k^{th} regressor across all the units of the panel as the estimate of the prior mean and the Zellner's g-prior is used as the estimate of the prior variance and the resultant prior precision". By taking different values by the prior precision parameter, it has the potential to produce all the standard frequentist panel data linear regression model estimators. The estimate of the prior precision parameter is obtained from the data corresponding to all units of the panel therefore, it is considered to be the most reliable.

Disciplinary: Econometrics, Economics.

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1 Introduction

This article is mainly concerned with the procedure related to the application of a specialized empirical Bayes estimation technique to real-world panel data. For this purpose, we take a real-world problem, in order, to show the procedure for the estimation and the analysis of the regression coefficients in a simple panel data linear regression model. In econometrics, there are three main forms of data: cross-sectional data, time-series data, and panel data. The panel data is the combination of cross-sectional data and time-series data. Panel data econometrics has evolved rapidly over the last few decades and researchers often deal in econometrics using panel data in the analysis of the relationship between variables. According to Hsiao (2007), the panel data is preferred and considered better in characteristics over either pure time-series data or purely cross-sectional data, because it considers both sources of variation in the data, i.e., the time dimension as well as the cross-sectional dimension. Chang et al. (2000) prescribe the facts of the econometric model as follows: (i) in economic relationships, the true functional forms of the models are almost unknown; (ii) from every econometric model, at least one unrevealed explanatory variable is excluded; (iii) it is very baseless or futile to assume that the excluded explanatory variables are uncorrelated with the candidate explanatory variables of the model; (iv) economic data are subject to measurement error in many cases, so that is why they are only the approximate values to the underlying exact values. Further, if an econometric model is consistent with all of the above realities of the building of an economic model, then it would be causal and meaningful, in other words, the elucidations attached to the model parameters are reliable with realism. The most commonly used standard panel data models can be found in Arellano (2003), Greene (2003), Gujrati (2003), etc.

In the classical or frequentist panel data framework, before model selection, the pretesting like Hausman (1978), specification test for the model specification is mandatory. After getting some evidence in favor of a particular panel data model, the estimation and other analytical procedures are carried out accordingly. In most instances, these pretesting procedures do not work, and of works but the reliabilities of these tests have also been criticized in the literature (see Clark and Linzer, 2015), and thus the risk of misspecification of models always exists.

On the other hand, in the Bayesian set-up, with the help of this specialized empirical Bayes estimation technique, pretesting for model selection is not required. In this procedure, the prior precision parameter is of great importance. Taking different values by the prior precision parameter yields different models and their estimates. In this set up the prior precision parameter is estimated from the data corresponding to all units of the panel and hence the estimate so obtained is considered to be the most reliable. Because all the data points, rather than data related to any individual unit of the panel, have contributed in the estimation process.

The basic empirical Bayes estimator from (Zaman, 1996) with (Zellner's, 1971) g-priors is

$$EB(\hat{\beta}_k^i) = \left[\left\{(\widehat{DP}_k)^i + \hat{\rho}_k\,(\widehat{DP}_k)^i\right\}^{-1}\ \left\{(\widehat{DP}_k)^i\hat{\beta}_k^i + \hat{\rho}_k\,(\widehat{DP}_k)^i\widehat{\mathrm{B}}_k\right\}\right],$$

For analogous representation see Zaman (1996) or Carrington and Zaman (1994). Here, $(\widehat{DP}_k)^i$ is the estimate of the data precision of the k^{th} regression coefficient of the model corresponding to the i^{th} unit of the panel, $\rho_k \left(\widehat{DP_k}\right)^i$ is the estimate of the prior precision of the k^{th} regression coefficient of the model corresponding to the i^{th} unit of the panel, while $\hat{\rho}_k$ is the estimate of the prior precision parameter of k^{th} regression coefficient of the model corresponding to the i^{th} unit of the panel, $\hat{\beta}_k^i$ is the data mean of the k^{th} regression coefficient of the model corresponding to the i^{th} unit of the panel, $\widehat{\mathrm{B}}_k$ is the prior mean of the k^{th} regression coefficient of the model corresponding to the i^{th} unit of the panel.

Now it can be very easily seen that if the estimate of the prior precision parameter corresponding to each regression coefficient in the model has a very low numerical value, i.e., if $\hat{\rho}_k$ = 0, then the specialized empirical Bayes estimator will have the inclination towards the individual models, one for each unit of the panel. Similarly, if the estimate of the prior precision parameter corresponding to each regression coefficient in the model has a very high numerical value, i.e., if $\hat{\rho}_k = \infty$, then the specialized empirical Bayes estimator will have the disposition towards a pooled model, which will be common to all units of the panel. Further, if the estimate of the prior precision parameter corresponding to each regression coefficient in the model has a very high numerical value except for the intercept, and the intercept has a very low numerical value, then the specialized empirical Bayes estimator will have the temperament toward the fixed effects model. Furthermore, if the estimate of the prior precision parameter corresponding to each coefficient in the model has neither a very low numerical value nor very high, then the specialized empirical Bayes estimator will have the spirit of the random coefficients model. Finally, if the estimate of the prior precision parameter corresponding to each coefficient in the model has a very high numerical value except for the intercept, and the intercept has neither very low numerical value nor very high, then the specialized empirical Bayes estimator will have the temperament towards the random-effects model.

In this piece, we show a single regression coefficient estimation procedure for orthogonal regressors of panel data models. This procedure is based on the specialized empirical Bayes estimation technique. As stated above, this approach itself will display the tendency of the panel data models and the corresponding empirical Bayes estimates. In more simple words, with the help of this specialized empirical Bayes technique, the data itself will lead to the most appropriate model(s) and the corresponding estimates, i.e., whether the data can be the best fit by the individual models corresponding to each unit of the panel, or whether the data can be the best fit by a pooled model common to all units of the panel, or the data can be the best fit by the fixed effects, random effects or random coefficients, etc.

2 Method

The Keynesian consumption function or the simple linear regression model with two regressors, i.e., the column of one's for the intercept and the GDP, is given as

$$c_t^i = y_{1t}^i \, \beta_1^i + y_{2t}^i \, \beta_2^i + \epsilon_t^i \quad (1),$$

where, c_t^i denotes the consumption of the i^{th} country at i^{th} time period, y_{1t}^i denotes, the constant 1 for the intercept term of the i^{th} country at i^{th} time period, y_{2t}^i denotes the GDP of the i^{th} country at i^{th} time period, ϵ_t^i denotes the random errors of the i^{th} country at i^{th} time period, β_1^i denotes the intercept term of the i^{th} country, and β_2^i denotes the slope coefficient of the i^{th} country.

2.1 Matrix Form of the Variables

The matrix structures of the variables in Equation (1) are given as

$$C^i = \begin{bmatrix} C_1^i \\ C_2^i \\ \vdots \\ C_t^i \\ \vdots \\ C_T^i \end{bmatrix}_{T\times 1}, \quad Y_1^i = \begin{bmatrix} y_{11}^i = 1 \\ y_{12}^i = 1 \\ \vdots \\ y_{1t}^i = 1 \\ \vdots \\ y_{1T}^i = 1 \end{bmatrix}_{T\times 1}, \quad Y_2^i = \begin{bmatrix} y_{21}^i \\ y_{22}^i \\ \vdots \\ y_{2t}^i \\ \vdots \\ y_{2T}^i \end{bmatrix}_{T\times 1}, \qquad (1.1a).$$

Let

$$Y^i = [Y_1^i \quad \dot{Y}_2^i] \qquad (1.1b).$$

Then, the expended matrix form of Y^i in Equation (1.1b), get the form

$$Y^i = \begin{bmatrix} 1 & y_{21}^i \\ 1 & y_{22}^i \\ \vdots & \vdots \\ 1 & y_{2t}^i \\ \vdots & \vdots \\ 1 & y_{2T}^i \end{bmatrix}_{T\times 2} \qquad (1.1c).$$

To orthogonalize the intercept and the slopes coefficients, we centralize Y_2^i in Equation (1.1a) and get

$$\dot{Y}_2^i = \begin{bmatrix} \dot{y}_{21}^i \\ \dot{y}_{22}^i \\ \vdots \\ \dot{y}_{2t}^i \\ \vdots \\ \dot{y}_{2T}^i \end{bmatrix}_{T\times 1} \qquad (1.1d),$$

where, $\dot{y}_{2t}^i = (y_{2t}^i - \bar{y}_2^i)$, the top-head dot is to differentiate the centralized $\dot{y}_{2t}^i$ from the simple y_{2t}^i. After centralization of Y_2^i, Equation (1.1c) can now be written as

$$\dot{Y}^i = \begin{bmatrix} 1 & \dot{y}_{21}^i \\ 1 & \dot{y}_{22}^i \\ \vdots & \vdots \\ 1 & \dot{y}_{2t}^i \\ \vdots & \vdots \\ 1 & \dot{y}_{2T}^i \end{bmatrix}_{T\times 2} \qquad (1.1e).$$

After the centralization, we present the modified model in the next section.

2.2 The Modified Model After Centralization

This model is the modified model after the centralization of Y_2^i.

$$C_t^i = y_{1t}^i\ \beta_1^i + \left(y_{2t}^i - \ \bar{y}_2^i\right)\beta_2^i + \epsilon_t^i, \tag{1.2a}$$

$$C_t^i = y_{1t}^i\ \beta_1^i + \dot{y}_{2t}^i\ \beta_2^i + \epsilon_t^i, \tag{1.2b}$$

where

$$\dot{y}_{2t}^i = \left(y_{2t}^i - \ \bar{y}_2^i\right) \tag{1.2c}$$

2.3 Some Basic Quantities

We present some of the important quantities that are needed for this estimation process.

2.3.1 The Model Parameters Estimates

To estimate the model (1.2b) parameters, we have the ordinary least squares coefficients estimate. Let this may be denoted by $\hat{\beta}^i$

$$\hat{\beta}^i = \left(\dot{Y}^{i\prime}\dot{Y}^i\right)^{-1}\dot{Y}^{i\prime}C^i \tag{2.1a}$$

$$\left(\hat{\beta}^i\right) = \begin{bmatrix}\hat{\beta}_1^i \\ \hat{\beta}_2^i\end{bmatrix} \tag{2.1b}$$

These $\hat{\beta}_1^i$ and $\hat{\beta}_2^i$ are now the ordinary least squares coefficients estimates for the intercept term and the slope coefficients of model parameters (1.2b), respectively. We also need the variance of the ordinary least squares coefficients estimates. In the next section, we present the data variances of the ordinary least squares coefficients estimates.

2.3.2 The Data Variance of the Model Parameters Estimates

As mentioned, we need the data variances of the model parameters i.e., the variances of the ordinary least squares coefficients estimate, therefore, by definition,

$$DV\left(\hat{\beta}^i\right) = (\sigma^2)^i\left(\dot{Y}^{i\prime}\dot{Y}^i\right)^{-1} \qquad (2.2a),$$

further, by definition,

$$(\sigma^2)^i = \left(\frac{\epsilon_t^{i\prime}\epsilon_t^i}{T-K}\right) \qquad (2.2b),$$

and in this orthogonal case, we have

$$\left(\dot{Y}^{i\prime}\dot{Y}^i\right) = \begin{bmatrix}T & 0 \\ 0 & \sum_{t=1}^{T}\left(\dot{y}_{2t}^i\right)^2\end{bmatrix} \qquad (2.2c),$$

and

$$\left(\dot{Y}^{i\prime}\dot{Y}^i\right)^{-1} = \begin{bmatrix}(T)^{-1} & 0 \\ 0 & \left(\sum_{t=1}^{T}\left(\dot{y}_{2t}^i\right)^2\right)^{-1}\end{bmatrix} \tag{2.2d}$$

thus finally,

$$(\sigma^2)^i(\dot{Y}^{i\prime}\dot{Y}^i)^{-1} = (\sigma^2)^i \begin{bmatrix} (T)^{-1} & 0 \\ 0 & \left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)^{-1} \end{bmatrix} \quad (2.2e)$$

or

$$(\sigma^2)^i(\dot{Y}^{i\prime}\dot{Y}^i)^{-1} = \begin{bmatrix} (\sigma^2)^i(T)^{-1} & 0 \\ 0 & (\sigma^2)^i\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)^{-1} \end{bmatrix} \quad (2.2f),$$

and thus

$$DV(\hat{\beta}^i) = \begin{bmatrix} (\sigma^2)^i(T)^{-1} & 0 \\ 0 & (\sigma^2)^i\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)^{-1} \end{bmatrix} \quad (2.2g).$$

In Equation (2.2g), the left-hand top diagonal or the first diagonal, quantity is the variance of the first regression coefficient estimate, i.e., the intercept while the right-hand bottom diagonal or the second diagonal, quantity is the variance of the second regression coefficient estimate, i.e., the slope.

Let, $DV(\hat{\beta}_1^i)$ denotes the data variance of the first regression coefficient and $DV(\hat{\beta}_2^i)$ denotes the data variance of the second regression coefficient. Then,

$$DV(\hat{\beta}_1^i) = (\sigma^2)^i(T)^{-1} \quad (2.2h),$$

and

$$DV(\hat{\beta}_2^i) = (\sigma^2)^i\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)^{-1} \quad (2.2i).$$

On the basis of these quantities, we show the corresponding data precisions in next section.

2.3.3 The Data Precision of the Model Parameters Estimates

On the basis of the data variances, the corresponding data precisions, which are denoted by $DP(\hat{\beta}_1^i)$ and $DP(\hat{\beta}_2^i)$ respectively for both of the regression coefficient estimates become

$$DP(\hat{\beta}_1^i) = \left[(\sigma^2)^i\right]^{-1}T \quad (2.3a),$$

and

$$DP(\hat{\beta}_2^i) = \left[(\sigma^2)^i\right]^{-1}\left[\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right] \quad (2.3b).$$

Further, we describe the data densities for both coefficients estimates.

2.3.4 The Data Densities for Both of the Coefficients Estimates

The data density in terms of means and variances of $\hat{\beta}_1^i$ is

$$\hat{\beta}_1^i \overset{IID}{\sim} N\left(\beta_1^i, DV(\hat{\beta}_1^i)\right) \quad (2.4a).$$

Thus, $\hat{\beta}_1^i$ is normally distributed with mean β_1^i and variance $DV(\hat{\beta}_1^i)$, and similarly, the data density in terms of means and variances of $\hat{\beta}_2^i$ is also given as

$$\hat{\beta}_2^i \overset{IID}{\sim} N\left(\beta_2^i, DV\left(\hat{\beta}_2^i\right)\right) \quad (2.4b).$$

Thus, $\hat{\beta}_2^i$ is normally distributed with mean β_2^i and variance $DV\left(\hat{\beta}_2^i\right)$. Now in the following section, we describe the prior densities too.

2.3.5 The Prior Densities of Both Regression Coefficients

The prior density in terms of mean and variance of β_1^i is

$$\beta_1^i \overset{IID}{\sim} N\left(\mathrm{B}_1, \Lambda_1^i\right) \quad (2.5a),$$

and similarly, the prior density in terms of mean and variance of β_2^i is

$$\beta_2^i \overset{IID}{\sim} N\left(\mathrm{B}_2, \Lambda_2^i\right) \quad (2.5b).$$

Here, B_1 and B_2 denotes the prior means of β_1^i and β_2^i, i.e., the 1^{st} and 2^{nd} regression coefficient in the model respectively, and Λ_1^i and Λ_2^i denotes the prior variances of β_1^i and β_2^i respectively.

Let

$$(PV_1)^i = \Lambda_1^i, \quad (2.5c),$$

and

$$(PV_2)^i = \Lambda_2^i, \quad (2.5d).$$

2.4 Computation of the Prior Variance

To use Zellner's (1971) g-priors in general and the g-prior precision in specific, let us define the g-prior precision for both of the regression coefficients.

By the definition of the g-prior, the g-prior variance is proportional to the data variance. In our case,

$$(PV_1)^i \propto DV\left(\hat{\beta}_1^i\right) \quad (2.6a),$$

or

$$(PV_1)^i = \rho_1^{-1} DV\left(\hat{\beta}_1^i\right) \quad (2.6b)$$

and

$$(PV_2)^i \propto DV\left(\hat{\beta}_2^i\right) \quad (2.6c)$$

or

$$(PV_2)^i = \rho_2^{-1} DV\left(\hat{\beta}_2^i\right) \quad (2.6d).$$

Now plugging the value of $DV\left(\hat{\beta}_1^i\right)$ and $DV\left(\hat{\beta}_2^i\right)$ from (2.3a) and (2.3b) in (2.6c) and (2.6d) respectively, we have

$$(PV_1)^i = \rho_1^{-1}\left[(\sigma^2)^i (T)^{-1}\right] \quad (2.6e),$$

$$(PV_2)^i = \rho_2^{-1}\left[(\sigma^2)^i \left\{\sum_{t=1}^{T}\left(\dot{y}_{2t}^i\right)^2\right\}^{-1}\right] \quad (2.6f).$$

2.4.1 The Prior Precisions

Let, $(PP_1)^i$ and $(PP_2)^i$ denote the two prior precisions respectively, then, in terms of precision (2.6e) and (2.6f) can also be expressed as

$$(PP_1)^i = \rho_1 \left[\left\{ (\sigma^2)^i \right\}^{-1} T \right] \quad (2.7a),$$

$$(PP_2)^i = \rho_2 \left[(\sigma^2)^{i^{-1}} \left\{ \sum_{t=1}^{T} \left(\dot{y}_{2t}^i \right)^2 \right\} \right] \quad (2.7b),$$

where

$$(PP_1)^i = \left(\Lambda_1^i \right)^{-1} \quad (2.7c),$$

and

$$(PP)_2^i = \left(\Lambda_2^i \right)^{-1} \quad (2.7d).$$

Now, with the help of Equations (2.6e) and (2.6f), the prior densities of both of the coefficients are

$$\hat{\beta}_1^i \overset{IID}{\sim} N\left(\mathrm{B}_1, \{ \rho_1^{-1} [(\sigma^2)^i (T)^{-1}] \} \right) \quad (2.7e)$$

and

$$\beta_2^i \overset{IID}{\sim} N\left(\mathrm{B}_2, \left\{ (\rho_2)^{-1} (\sigma^2)^i \left(\sum_{t=1}^{T} \left(\dot{y}_{2t}^i \right)^2 \right)^{-1} \right\} \right) \quad (2.7f).$$

2.5 The Marginal Densities of the Estimates

Here, the marginal density for the 1^{st} regression coefficient estimate becomes

$$m(\hat{\beta}_1^i) \overset{IID}{\sim} N\left(\mathrm{B}_1, \{ [(\sigma^2)^i (T)^{-1}] + \rho_1^{-1} [(\sigma^2)^i (T)^{-1}] \} \right) \quad (3a).$$

Now, we describe the marginal density for the 2^{nd} regression coefficient estimate, which becomes

$$m(\hat{\beta}_2^i) \overset{IID}{\sim} N\left(\mathrm{B}_2, \left\{ \left[(\sigma^2)^i \left(\sum_{t=1}^{T} \left(\dot{y}_{2t}^i \right)^2 \right)^{-1} \right] + \rho_2^{-1} \left[(\sigma^2)^i \left(\sum_{t=1}^{T} \left(\dot{y}_{2t}^i \right)^2 \right)^{-1} \right] \right\} \right) \quad (3b).$$

2.6 The Estimates of the Prior Means

We show the estimates of the prior means separately. First, we show the estimates of the prior mean for the intercept and then the estimates of the prior mean for the slope.

2.6.1 The Estimates of the Prior Mean for the Intercept

Now, the estimate of the prior mean $\widehat{\mathrm{B}}_1$ for the first regression coefficient may be given as

$$\widehat{\mathrm{B}}_1 = \frac{\sum_{i=1}^{N} \left\{ \left[\left[\left\{ (\hat{\sigma}_1^2)^i \right\}^{-1} T \right] + \hat{\rho}_1 \left[\left\{ (\hat{\sigma}_1^2)^i \right\}^{-1} T \right] \right\} \hat{\beta}_1^i}{\sum_{i=1}^{N} \left[\left\{ (\hat{\sigma}_1^2)^i \right\}^{-1} T \right] + \hat{\rho}_1 \left[\left\{ (\hat{\sigma}_1^2)^i \right\}^{-1} T \right]} \quad (4.1a)$$

$$\widehat{\mathrm{B}}_1 = \frac{\sum_{i=1}^{N} \left\{ (1 + \hat{\rho}_1) \left[\left\{ (\hat{\sigma}_1^2)^i \right\}^{-1} T \right] \right\} \hat{\beta}_1^i}{\sum_{i=1}^{N} \left\{ (1 + \hat{\rho}_1) \left[\left\{ (\hat{\sigma}_1^2)^i \right\}^{-1} T \right] \right\}^{-1}} \quad (4.1b),$$

$$\widehat{B}_1 = \frac{(1+\hat{\rho}_1)\sum_{i=1}^{N}\left[\left\{(\hat{\sigma}_1^2)^i\right\}^{-1} T\right]\hat{\beta}_1^i}{(1+\hat{\rho}_1)\sum_{i=1}^{N}\left[\left\{(\hat{\sigma}_1^2)^i\right\}^{-1} T\right]} \quad (4.1c),$$

$$\widehat{B}_1 = \frac{\sum_{i=1}^{N}\left[\left\{(\hat{\sigma}_1^2)^i\right\}^{-1} T\right]\hat{\beta}_1^i}{\sum_{i=1}^{N}\left[\left\{(\hat{\sigma}_1^2)^i\right\}^{-1} T\right]} \quad (4.1d).$$

Here it is to be noted that, $\widehat{B}_1$ becomes independent of $\hat{\rho}_1$.

2.6.2 The Estimates of the Prior Mean for the Slope

Also, the estimate of the prior mean B_2 for the second regressor may be given as follow,

$$\widehat{B}_2 = \frac{\sum_{i=1}^{N}\left\{\left[\left\{(\hat{\sigma}_2^2)^i\right\}^{-1}\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)\right] + \hat{\rho}_2\left[\left\{(\hat{\sigma}_2^2)^i\right\}^{-1}\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)\right]\right\}\hat{\beta}_2^i}{\sum_{i=1}^{N}\left[\left\{(\hat{\sigma}_2^2)^i\right\}^{-1}\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)\right] + \hat{\rho}_2\left[\left\{(\hat{\sigma}_2^2)^i\right\}^{-1}\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)\right]} \quad (4.2a),$$

$$\hat{B}_2 = \frac{\sum_{i=1}^{N}\left\{(1+\hat{\rho}_2)\left[\left\{(\hat{\sigma}_2^2)^i\right\}^{-1}\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)\right]\right\}\hat{\beta}_2^i}{\sum_{i=1}^{N}\left\{(1+\hat{\rho}_2)\left[\left\{(\hat{\sigma}_2^2)^i\right\}^{-1}\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)\right]\right\}} \quad (4.2b),$$

$$\widehat{B}_2 = \frac{(1+\hat{\rho}_2)\sum_{i=1}^{N}\left[\left\{(\hat{\sigma}_2^2)^i\right\}^{-1}\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)\right]\hat{\beta}_2^i}{(1+\hat{\rho}_2)\sum_{i=1}^{N}\left[\left\{(\hat{\sigma}_2^2)^i\right\}^{-1}\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)\right]} \quad (4.2c),$$

$$\widehat{B}_2 = \frac{\sum_{i=1}^{N}\left[\left\{(\hat{\sigma}_2^2)^i\right\}^{-1}\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)\right]\hat{\beta}_2^i}{\sum_{i=1}^{N}\left[\left\{(\hat{\sigma}_2^2)^i\right\}^{-1}\left(\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right)\right]} \quad (4.2d).$$

Here again, it is to be noted that, $\widehat{B}_2$ becomes independent of $\hat{\rho}_2$.

2.7 The Specialized Empirical Bayes Estimates

By applying the specialized empirical Bayes estimation technique, we obtain the empirical Bayes estimates without estimating the prior precision parameter, thus, the empirical Bayes estimates for the intercept term and the slope coefficients respectively, are described in the following sections.

2.7.1 The Specialized Empirical Bayes Estimates for the Intercept

To get rid of the estimation of the prior precision parameter, after some algebraic manipulation, the specialized empirical Bayes estimator can be obtained as

$$EB(\hat{\beta}_k^i) = \left(\left\{1 - \left[\sum_{i=1}^{N}\left\{\frac{(N-2)\left[DV(\hat{\beta}_k^i)\right]}{(\hat{\beta}_k^i - \hat{B}_k)^2}\right\}\right]\right\}\hat{\beta}_k^i + \left\{\left[\sum_{i=1}^{N}\left\{\frac{(N-2)\left[DV(\hat{\beta}_k^i)\right]}{(\hat{\beta}_k^i - \hat{B}_k)^2}\right\}\right]\right\}\hat{B}_k\right) \quad (5.1a).$$

Modifying Equation (5.1a) for the intercept term only, we replace the subscript 'k' by '1' to get,

$$EB(\hat{\beta}_1^i) = \left(\left\{1 - \left[\sum_{i=1}^{N}\left\{\frac{(N-2)\left[DV(\hat{\beta}_1^i)\right]}{(\hat{\beta}_1^i - \hat{B}_1)^2}\right\}\right]\right\}\hat{\beta}_1^i + \left\{\left[\sum_{i=1}^{N}\left\{\frac{(N-2)\left[DV(\hat{\beta}_1^i)\right]}{(\hat{\beta}_1^i - \hat{B}_1)^2}\right\}\right]\right\}\hat{B}_1\right) \quad (5.1b),$$

or

$$EB(\hat{\beta}_1^i) = \left(\left\{1 - \left[\sum_{i=1}^{N}\left\{\frac{(N-2)\{(\hat{\sigma}_1^2)^i (T)^{-1}\}}{(\hat{\beta}_1^i - \hat{B}_1)^2}\right\}\right]\right\}\hat{\beta}_1^i + \left\{\left[\sum_{i=1}^{N}\left\{\frac{(N-2)\{(\hat{\sigma}_1^2)^i (T)^{-1}\}}{(\hat{\beta}_1^i - \hat{B}_1)^2}\right\}\right]\right\}\hat{B}_1\right) \quad (5.1c).$$

Equation (5.1c) is the empirical Bayes estimate of the first regression coefficient i.e., the intercept term of the model independent of the prior precision parameter.

2.7.2 The Specialized Empirical Bayes Estimate for the Slope

Similarly, modifying Equation (5.1a) for the slope term only, we get,

$$EB(\hat{\beta}_2^i) = \left(\left\{1 - \left[\sum_{i=1}^{N}\left\{\frac{(N-2)[DV(\hat{\beta}_2^i)]}{(\hat{\beta}_2^i - \hat{B}_2)^2}\right\}\right]\right\}\hat{\beta}_2^i + \left\{\left[\sum_{i=1}^{N}\left\{\frac{(N-2)[DV(\hat{\beta}_2^i)]}{(\hat{\beta}_2^i - \hat{B}_2)^2}\right\}\right]\right\}\hat{B}_2\right) \quad (5.2a)$$

or

$$EB(\hat{\beta}_2^i) = \left\{1 - \left[\sum_{i=1}^{N}\left\{\frac{(N-2)\left\{(\hat{\sigma}_2^2)^i\left[\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right]^{-1}\right\}}{(\hat{\beta}_{22}^i - \hat{B}_2)^2}\right\}\right]\right\}\hat{\beta}_2^i$$
$$+ \left\{\left[\sum_{i=1}^{N}\left\{\frac{(N-2)\left\{(\hat{\sigma}_2^2)^i\left[\sum_{t=1}^{T}(\dot{y}_{2t}^i)^2\right]^{-1}\right\}}{(\hat{\beta}_2^i - \hat{B}_2)^2}\right\}\right]\right\}\hat{B}_2 \quad (5.2b).$$

Equation (5.2b) is the empirical Bayes estimate of the second regression coefficient i.e., the slope coefficient of the model. Now, it is very simple to find the empirical Bayes estimates from Equations (5.1c) and (5.2b).

3 Result and Discussion

In this study, we took the simple panel data linear regression model, for five European Union countries namely, Austria, France, Italy, Sweden, and Britain. The data was on two variables, i.e., gross domestic product 'GDP' and consumption for the period of 1970 to 2016. Each variable for each county consisted of 47 observations. The data had been taken from International Financial Statistics (IFS) data. The log transforms of the variables had been made in order, to condense the data. Further, the GDP as the regressor had been centralized, in order, to make the regressors orthogonal and bring about independence in the intercepts and slope coefficients.

The most important aspect of this article is that here we are not interested in the economic characteristics of the variables related to the above countries, i.e., how and how much does a GDP of a country affect the consumption of the country, rather, we are interested the procedure of applying the specialized empirical Bayes technique of estimation to panel data linear regression models. Because this technique has the capability to be fitted to all standard panel data models. Therefore, the main focus of this article was around the procedure of application of this specialized empirical Bayes estimation for panel data models and not the other way around.

Further, the procedure of the specialized empirical Bayes technique of estimation to multiple panel data linear regression models is the simple extension of the specialized empirical Bayes technique of estimation to simple panel data linear regression models.

Table 1: The Empirical Bayes Estimates for the Intercepts

	Austria	France	Italy	Sweden	Britain
Intercept Coefficient Estimates	0.2694	0.3012	0.2861	0.2767	0.2921
Standard Error of Intercepts	0.0097	0.0183	0.0234	0.0065	0.0203

Table 2: Empirical Bayes Estimates for the Slopes

	Austria	France	Italy	Sweden	Britain
Slope Coefficient Estimates	0.9804	0.9436	0.9302	0.9338	0.9989
Standard Error of Slopes	0.0084	0.0078	0.0064	0.0092	0.0074

Table 3: The Empirical Bayes Coefficients Estimates for All Countries

Country	Intercept		Slope	
	Coefficients Estimates	Standard Error of the Coefficients Estimates	Coefficients Estimates	Standard Error of the Coefficients Estimates
Austria	0.2694	0.0097	0.9804	0.0084
France	0.3012	0.0183	0.9436	0.0078
Italy	0.2861	0.0234	0.9302	0.0064
Sweden	0.2767	0.0065	0.9338	0.0092
Britain	0.2921	0.0203	0.9989	0.0074

Table 1 contains the estimates of the intercept with their corresponding standard errors for all the five countries of the analysis. Similarly, Table 2 contains the slopes with their corresponding standard errors for all the five countries of the analysis. Further, Table 3 contains the intercepts as well as the slopes with their corresponding standard errors simultaneously, for all the five countries of the analysis. Here as per the theoretical relationship between consumption and GDP the sign and magnitude of both the intercepts and the slopes seem very much consistent with the theory. Also, the coefficients estimates among different countries seem randomly fluctuate.

4 Conclusion

This study applied the empirical Bayes estimation techniques to the real-world data of five European Union countries, Austria, France, Italy, Sweden, and Britain. Further, the GDP as the regressor has been centralized, in view, to make the regressors orthogonal and attain independency in the intercepts and slope coefficients. Thereafter, the whole procedure was described analytically.

The salient features of the estimates are that we have not done any pretesting procedure for model selection and neither have decided in advance whether we fit the subject-specific coefficient model or the subject common coefficients model or any other model of the frequentist setup. Rather we estimated the coefficients for each unit by the specialized empirical Bayes estimator.

The symbols and magnitudes of the coefficient estimates are in accordance with theory, the standard errors are also of acceptable size. Also, the coefficient estimates for different units are not too much different. If the common subject was appropriate then the two coefficients, i.e., the intercept and the slope would be identical for all units of the panel.

The quality of the empirical Bayes estimate is that one does not need pretesting for model selection, the technique itself first designs the vectors of the coefficients estimates, and then resultantly the corresponding models can be framed.

5 Availability of Data and Material

Data can be made available by contacting the corresponding author.

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