International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies

http://TuEngr.com, http://go.to/Research

Performance Evaluation of Chaotic Mobile Robot Controllers

Ashraf Anwar Fahmy[a*]

[a] *Department of Computer Engineering, Faculty of Computers and Information Technology, Taif University, SAUDIA ARABIA*

ARTICLEINFO

Article history:
Received 12 January 2012
Received in revised form
24 February 2012
Accepted 24 February 2012
Available online
24 February 2012

Keywords:
Chaos;
chaotic motion;
chaotic mobile robot;
Chua's circuit;
Arnold equations;
Lornez equations.

ABSTRACT

The chaotic mobile robot implies a mobile robot with a controller that ensures chaotic motions. A chaotic signal for an autonomous mobile robot is to increase and to take advantage of coverage areas resulting from its travelling paths. Large coverage areas are desirable for many applications such as robots designed for scanning of unknown workspaces with borders and barriers of unknown shape, as in patrol or cleaning purposes. The chaotic behavior of the mobile robot is achieved by adding nonlinear equations into the robot kinematic equations, like Arnold, Lornez, and the Chua's circuit equations, that are well known equations for had a chaotic behavior. The performance of the three controllers is tested in four different scenarios and evaluated in the sense of the wide area coverage, the evenness index, and trajectory length.

1. Introduction

Chaos is a typical behavior of nonlinear dynamical systems, and has been studied deeply in different fields such as mathematics, physics, engineering, economics, and sociology. A robot following a chaotic path, generated by the Arnold's equation, was introduced for the first time in (Y. Nakamura, et al., 2001, A. Sekiguchi, et al., 1999). In many applications,

*Corresponding author (**A. Anwar**). E-mail address: ashrafmanwar90@yahoo.com. ⓒ2012. International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies. Volume 3 No.2 ISSN 2228-9860 eISSN 1906-9642. Online Available at http://TuEngr.com/V03/145-158.pdf.

control of mobile robots designed to perform specific tasks are generated by chaotic signals (Y. Nakamura, et al., 2001, A. Sekiguchi, et al., 1999, A. Jansri, et al., 2004, P. Sooraksa, et al., 2010, L.S. Martins-Filho, 2004, S Martins, et al., 2007, J. Palacin, 2004, M. Islam, et al., 2005, U. Nehmzow, 2003, A. Anwar, et al., 2011).

The main objective in exploiting chaotic signals for an autonomous mobile robot is to increase and to take advantage of coverage areas resulting from its travelling paths. Large coverage areas are desirable for many applications such as robots designed for scanning of unknown workspaces with borders and barriers of unknown shape, as in patrol or cleaning purposes. Two performance indexes are used to evaluate the coverage areas of the chaotic mobile robot, namely a performance index k representing a ratio of areas that the trajectory passes through over the total working area and an evenness index E which refers in general to how close in numbers each species in an environment are. In our case study the robot workspace is split into four quarters which represent the so called species. Not only large coverage areas are desirable in certain applications of mobile robot, but also coverage speed and eventually the shortest path traveled by the robot. The complexity of chaotic motion is increased by the multiple reflections of the robot trajectory on the workspace boundaries and obstacles.

Some experimental and theoretical studies have focused on the chaotic motion of the robot without considering the coverage performance (A. Sekiguchi, et al., 1999, C. Chanvech, 2006, L.S Martins, et al., 2006) where as in other studies, it has been reported that a large coverage area is among the most important performances that may characterize the mobile robot motion (A. Jansri, et al., 2004, P. Sooraksa, et al., 2010, A. Anwar, et al., 2011).

In our previous work (A. Anwar, et al., 2011), we deduced that we can get high performance index *k* of coverage and high evenness index *E* from a non-chaotic behavior of the Chua's circuit as a controller of the mobile robot. Such behavior is an instable focus which is a repeller, obtained by using a particular parameters set of Chua's circuit.

The aim of this paper is to evaluate the performance, of the Lornez, Arnold, and the Chua's circuit equation (by adjusting its parameters to generate the specified instable focus), as a controller of the mobile robot, from point of view of performance index *k*, which reflects how high the coverage area, and the evenness index *E*, which reflects the degree of variation in covering the areas between species, and trajectory length. This performance evaluation is tested in four different scenarios.

The paper structure is as follows: The next section presents the mobile robot model. The chaotic mobile controllers are illustrated in section 3. Section 4 is dedicated to the evaluation criteria to be applied. Section 5 is reserved to the simulation results. Finally section 6 concludes this paper.

2. Mobile Robot Model

The mobile robot used, is shown in Figure 1. Let the linear velocity of the robot *v*[m/s], and the angular velocity *w* [rad/s], be the inputs to the system, and the state equation of the mobile robot is written as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{1}$$

Where x[m], and y[m] is the position of the mobile robot, $\boldsymbol{\theta}[\boldsymbol{rad}]$ is the angle of the robot.

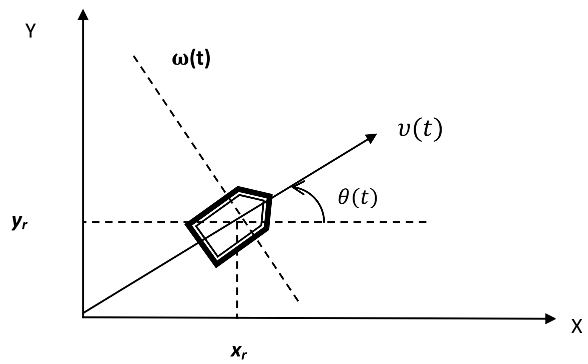


Figure 1: Geometry of the robot motion in Cartesian plane.

3. Chaotic Mobile Robot Controllers

In order to generate chaotic motions of the mobile robot, this is achieved by designing a controller which ensures chaotic motion. The type of chaotic patterns employed to generate the robot trajectory are, the Lornz, the Arnold and the Chua's circuit equations.

*Corresponding author (**A. Anwar**). E-mail address: ashrafmanwar90@yahoo.com. International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies. Volume 3 No.2 ISSN 2228-9860 eISSN 1906-9642. Online Available at http://TuEngr.com/V03/145-158.pdf.

3.1 Lorenz Equation

The Lorenz attractor is generated by the differential equation given by:

$$\begin{aligned} \dot{X} &= -10\,X + 10Y \\ \dot{Y} &= 28\,X - Y - X\,Z \\ \dot{Z} &= -\tfrac{8}{3}\,Z + X\,Y \end{aligned} \qquad (2)$$

The parametric values in the differential equation (2) are needed in order to generate a chaotic behavior. The Lornez attractor is shown in Figure 2.

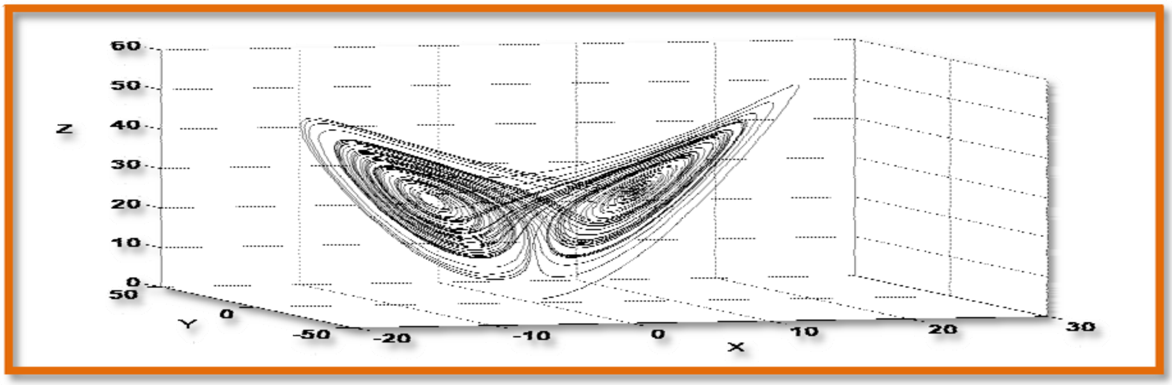


Figure 2: The Lornez attractor in 3-D space.

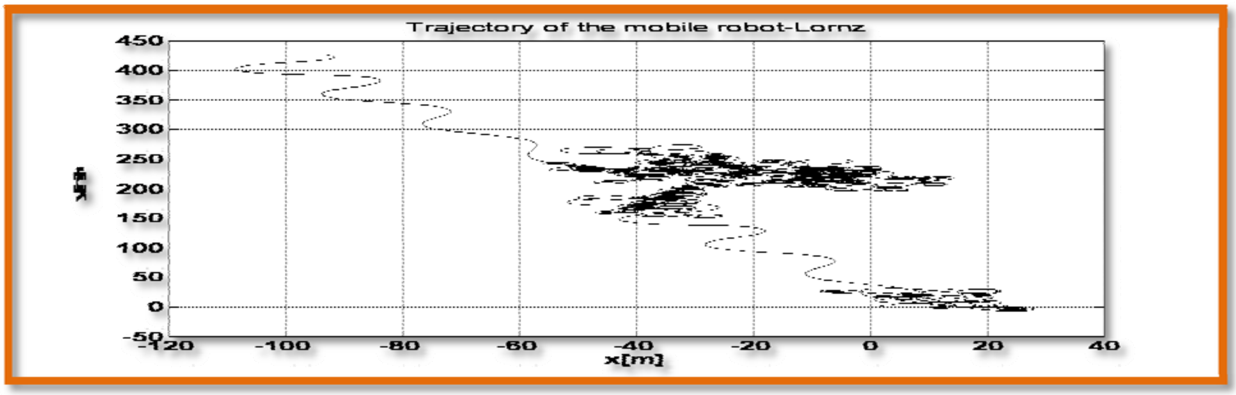


Figure 3: Trajectory of the chaotic mobile robot controlled by Lorenz.

After integration the Lornz equation (2) into the controller of the mobile robot equation (1), the state equation of the mobile robot becomes:

$$\begin{aligned} \dot{X} &= -10\,X + 10Y \\ \dot{Y} &= 28\,X - Y - X\,Z \\ \dot{Z} &= -\tfrac{8}{3}\,Z + X\,Y \end{aligned}$$

$$\dot{x} = v \cos Z$$
$$\dot{y} = v \sin Z \quad (3)$$

The resultant trajectory of the mobile controlled by Lornz equation, at initial conditions: X_0=1, Y_0=0, Z_0=1, x_0=1, y_0=0, and 5000 time unit, is shown in Figure 3.

3.2 Arnold Equation

The equation of the Arnold is written as follows:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} A\, sinx_3 + C\, cosx_2 \\ B\, sin\, x_1 + A\, cosx_3 \\ C\, sinx_2 + B\, cosx_1 \end{bmatrix} \quad (4)$$

Where A, B, and C are constants. It is known that the Arnold equation shows periodic motion when one of the constants, for example C, is 0 or small and shows chaotic motion when C is large. The chaotic pattern of the Arnold equation, for the following parameters: A=0.27, B=0.135, C=0.135 and initial conditions: x_{10}= 4, x_{20}=3.5, x_{30}=0, is shown in Figure 4

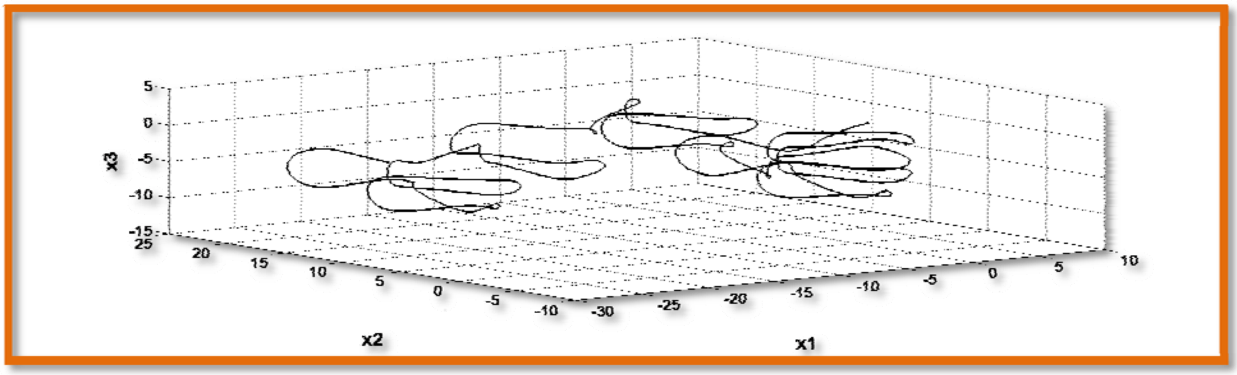


Figure 4: Arnold chaotic pattern in 3-D space.

After integration the Arnold equation (4) into the controller of the mobile robot equation (1), the state equation of the mobile robot becomes:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} A\, sinx_3 + C\, cosx_2 \\ B\, sin\, x_1 + A\, cosx_3 \\ C\, sinx_2 + B\, cosx_1 \\ v\, cos\, x_3 \\ v\, sin\, x_3 \end{bmatrix} \quad (5)$$

*Corresponding author (A. Anwar). E-mail address: ashrafmanwar90@yahoo.com. International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies. Volume 3 No.2 ISSN 2228-9860 eISSN 1906-9642. Online Available at http://TuEngr.com/V03/145-158.pdf.

The integrated system of the Arnold equation with the mobile robot equation with appropriate adjusting parameters and initial conditions guaranteed that a chaotic orbit of the Arnold's equation behaves chaotically. The resultant trajectory of the mobile robot is shown in Figure 5.

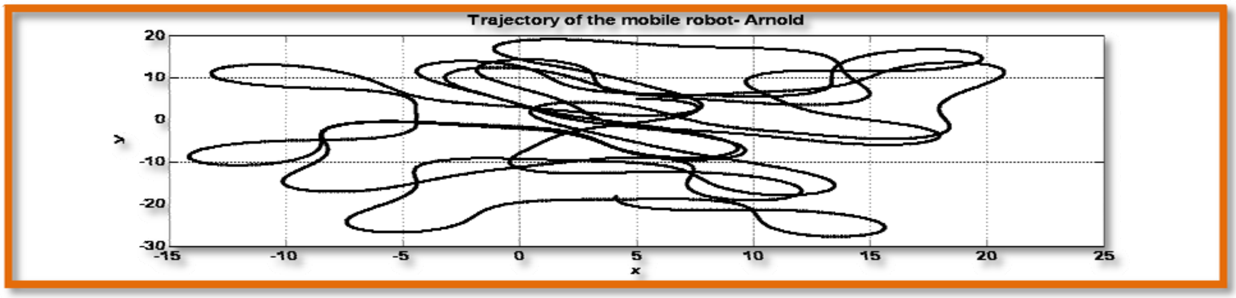


Figure 5: Trajectory of the chaotic mobile robot controlled by Arnold.

3.3 Chua's circuit

The chaotic controller used herein as a trajectory generator is Chua's circuit which is low cost and easy to construct for trajectory generators. The general equations of Chua's circuit are:

$$\dot{X}_1 = \alpha \left(X_2 - X_1 - f(X_1)\right)$$
$$\dot{X}_2 = X_1 - X_2 - X_3$$
$$\dot{X}_3 = -\beta\, X_2 \qquad (6)$$

Where: $f(X_1) = bX_1 + \frac{1}{2}(a-b)[|X_1+1| - |X_1-1|]$, α= 9, β = 100/7, a = -8/7, b = -5/7. These parameters generate instable focus pattern shown in Figure 6.

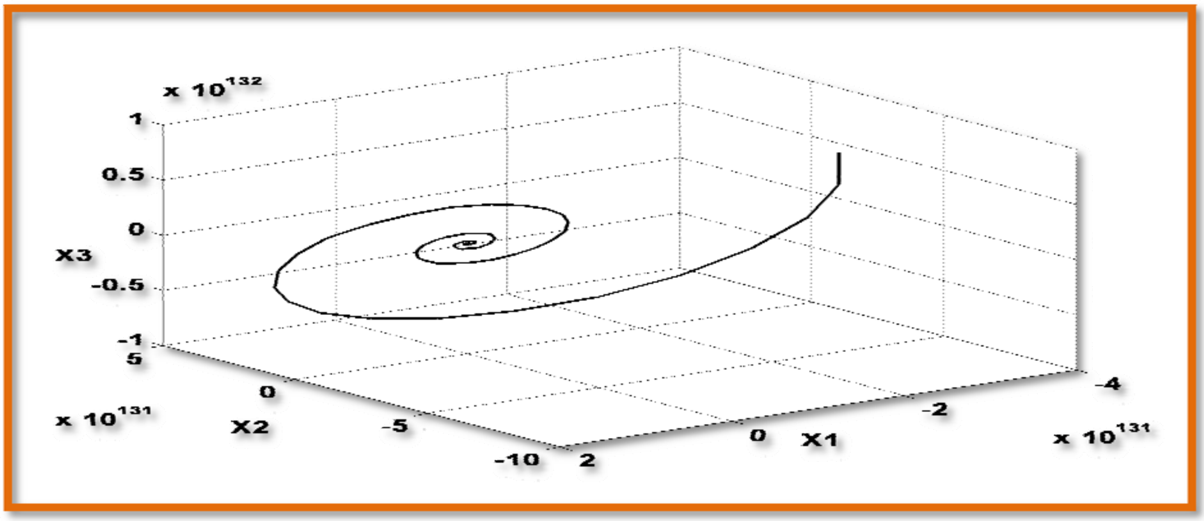


Figure 6: Chua's pattern in the 3-D space.

The integrated system of the Chua's circuit equation as a controller of the mobile robot will be as follows:

$$\dot{X}_1 = \alpha\left(X_2 - X_1 - f(X_1)\right)$$
$$\dot{X}_2 = X_1 - X_2 - X_3$$
$$\dot{X}_3 = -\beta\, X_2$$
$$\dot{\boldsymbol{x}} = v\cos X_2$$
$$\dot{y} = v\sin X_2 \qquad (7)$$

The trajectory of the mobile robot with Chua's circuit as a controller to generate chaotic behavior**,** is shown in Figure 7**.**

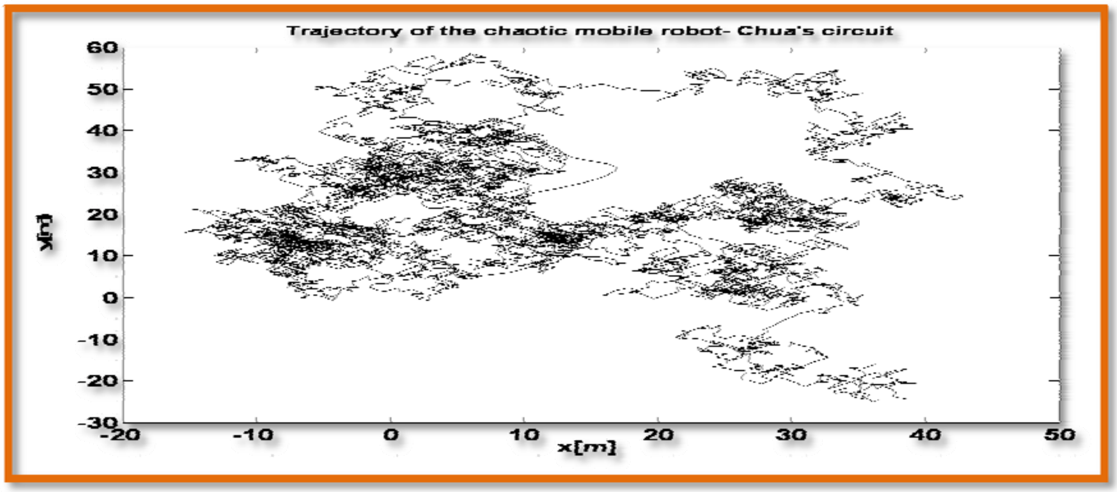


Figure 7: Trajectory of the chaotic mobile robot controlled by Chua's circuit.

4. Evaluation Criteria

The evaluation criteria are set according to the application purpose. Since we would like to use the robot in wandering around area in the area of no maps, the chaotic trajectory should cover the entire areas of patrolling as much as possible. The following two performances criteria are to be considered to evaluate the coverage rate of the chaotic mobile robot, namely the performance index *k* and the evenness index *E*.

a) A performance index K representing a ratio of areas that the trajectory passes through or used space (A_u), over the total working area (A_t)

$$K = \frac{A_u}{A_t} \tag{8}$$

Similarly, let us consider a rectangular shape area, Figure 8. The total area can be partitioned into four quarter, denoted Q=1, 2, 3, 4. The quantitative measurement of the trajectory can be evaluated by using the following equation:

$$K_Q = \frac{A_{uQ}}{A_{tQ}} \tag{9}$$

Where K_Q is the performance index of the Q^{th} quadrant, A_{uQ} is the area used by the trajectory in the Q^{th} quadrant. In our case, we have

$$A_{tQ} = \frac{A_t}{4} \tag{10}$$

Equations (8)-(10) will be used as performance indices in section 4.

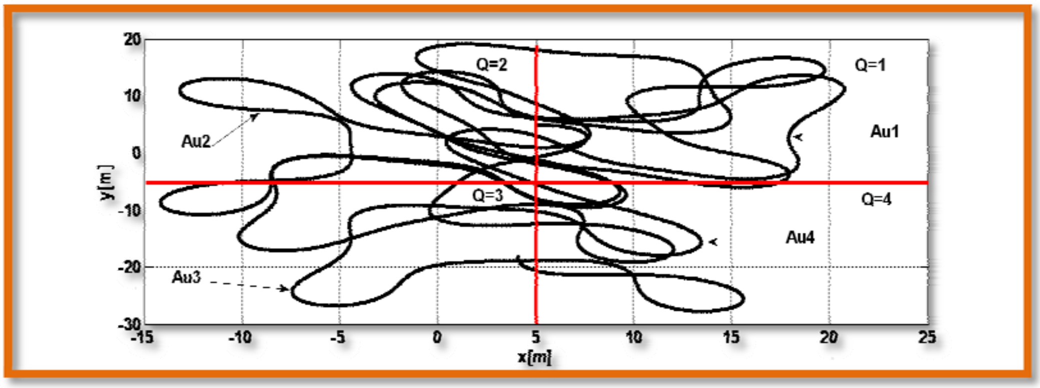


Figure 8: Partition of the specified area.

b) An evenness index E refers to how close in numbers each species in an environment are. The evenness index can be represented in our situation by (J. Nicolas et., 2003):

$$E = 1 - \frac{\sum_{Q=1}^{s} K_Q \ln(k_Q)}{\ln(s)} \tag{11}$$

Where s: No of species = 4 Quarters in our case.

E is constrained between 0 and 1. The less variation in covering the areas between the species, the higher E is.

c) The total length of the trajectory L – The total distance of the generated trajectory of each controller should be taken in the account to measure the performance of the controller in coverage a certain area, and it can be calculated by the following formula:

$$\boldsymbol{L} = \sqrt{(\boldsymbol{x}_{i+1} - \boldsymbol{x}_i)^2 + (\boldsymbol{y}_{i+1} - \boldsymbol{y}_i)^2} \qquad (12)$$

Where: $\boldsymbol{x}_{i+1}$ and $\boldsymbol{x}_i$ are the x- coordinates at successive instants & $\boldsymbol{y}_{i+1}$ and $\boldsymbol{y}_i$ are the y- coordinates at successive instants.

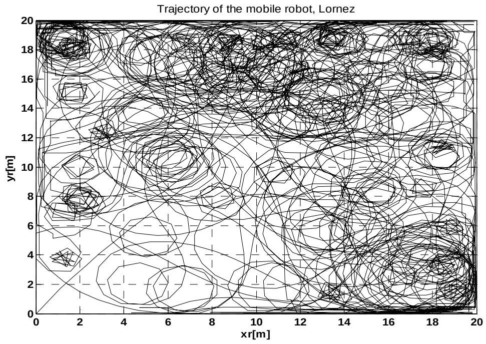
5. Simulation Results

In order to evaluate the performance of the three controllers, Lornez, Arnold, and Chua's equations, used to generate the chaotic motion of the mobile robot, we simulate each of the three systems of equations (3), (5), and (7) given in section 3, in four different scenario tests.

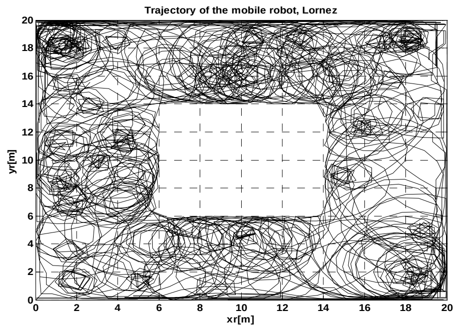
We simulate the system of equations using the parameters given in section 3 for each controller and the velocity v of the robot is 1m/s. We use the performance index K, the evenness index E and the total distance of the chaotic mobile robot D, as the evaluation criteria to distinguish the performance between the three controllers.

The chaotic trajectory of the mobile robot for the three specified controllers: Lornz, Arnold, and Chua's equations, in (20mx20m) workspace at iteration n=5000, number of pixels to cover area= NxN, N=2000, integration step h=0.1 and the parameters given in section 3. The robot moves as if is reflected by the boundary "mirror mapping".

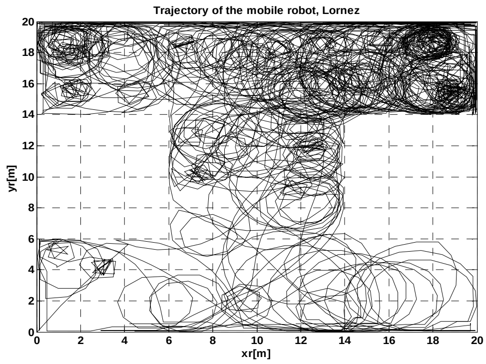
The resultant chaotic mobile robot trajectory of the system controlled by Lornz, Arnold, and Chua's equations in four different scenario tests are given in Figures 9-11 respectively.



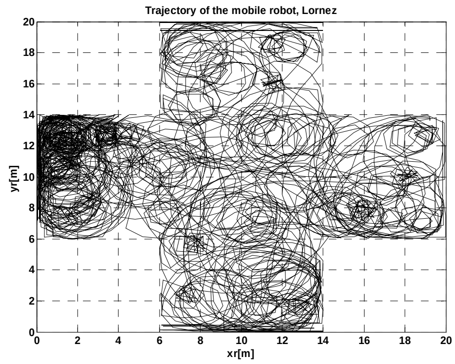
case (a)



case (b)

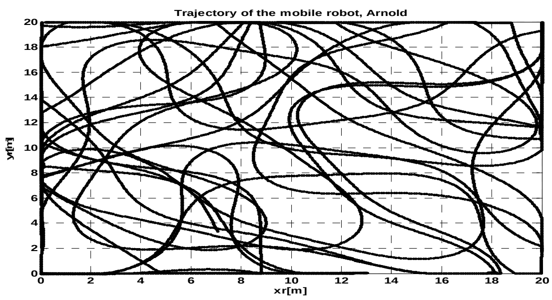


case (c)

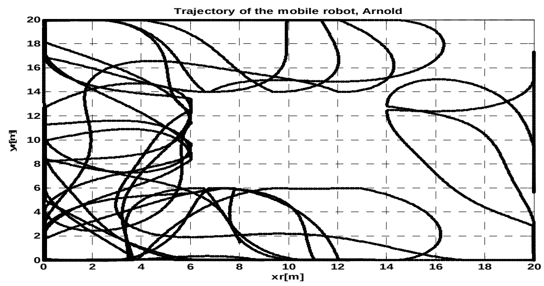


case (d)

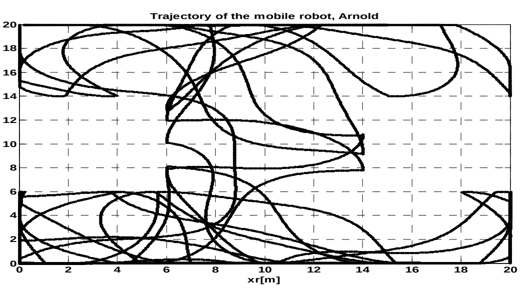
Figure 9: Trajectory of the chaotic mobile robot controlled by Lornez equation in the four testing scenarios.



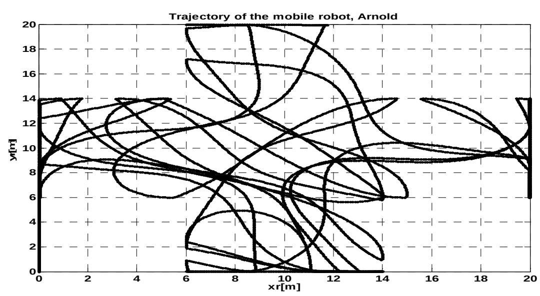
case (a)



case (b)

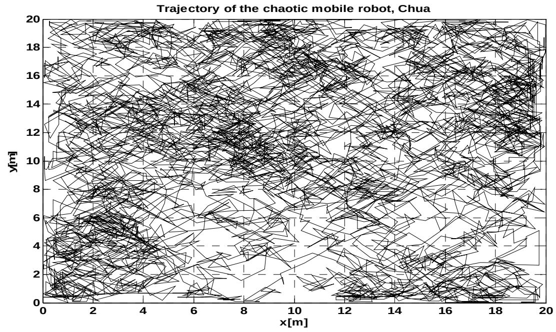


case (c)

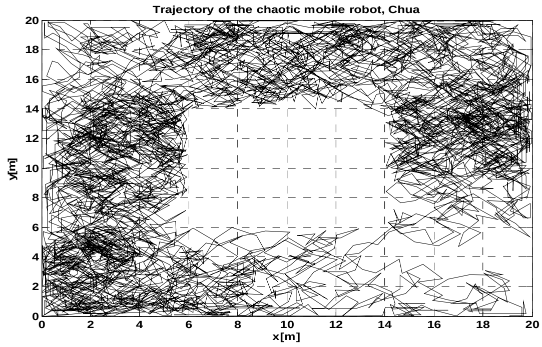


case (d)

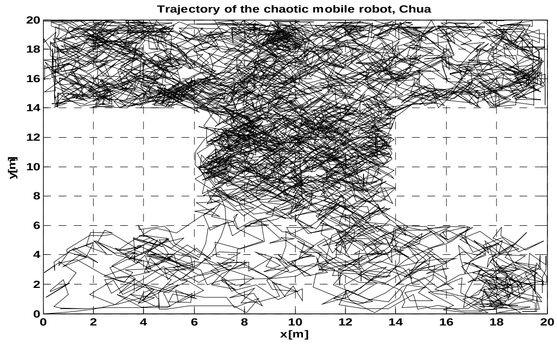
Figure 10: Trajectory of the chaotic mobile robot controlled by Arnold equation in the four testing scenarios.



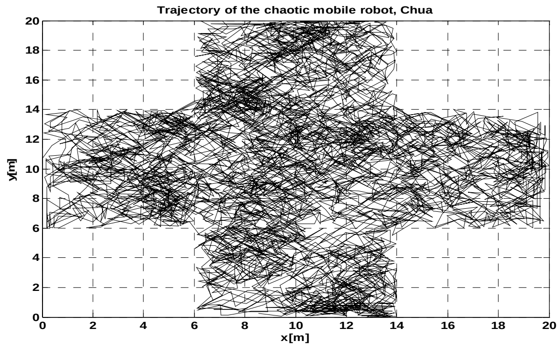
case (a)



case (b)



case (c)



case (d)

Figure 11: Trajectory of the chaotic mobile robot controlled by Chua's equation in the four testing scenarios.

The results of the investigation present in Tables 1-12, where various run time: 3000, 5000, and 10000 second have been set for the four test scenarios.

Table 1: case (a), n=3000

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	54.11	49.45	63.49	58.20	45.34	5.48	5682
Lornez	48.03	62.25	51.99	35.45	42.48	1.43	2646
Arnold	45.51	40.88	46.39	50.39	44.42	2.99	248.6

Table 2: case (a), n=5000

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	81.03	84.15	85.46	77.85	76.67	51.10	9449
Lornez	74.68	84.17	77.44	65.69	71.43	38.01	4528
Arnold	75.41	74.64	76.27	76.18	74.57	38.61	431.8

Table 3: case (a), n=10000.

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	98.15	98.20	98.40	98.10	97.91	94.73	10891
Lornez	96.32	98.40	97.21	94.26	95.42	89.64	8905
Arnold	94.81	94.85	97.61	94.73	92.06	85.50	849.8

*Corresponding author (**A. Anwar**). E-mail address: ashrafmanwar90@yahoo.com.
International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies. Volume 3 No.2 ISSN 2228-9860 eISSN 1906-9642. Online Available at http://TuEngr.com/V03/145-158.pdf.

Table 4: case (b), n=3000.

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	48.67	41.31	62.64	54.66	36.06	2.17	5545.5
Lornez	43.21	47.52	42.51	39.11	43.75	4.3	2399
Arnold	26.83	13.99	43.33	37.80	12.19	8.9	186.6

Table 5: case (b), n=5000.

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	76.29	78.69	83.95	73.56	68.98	41.04	9229.4
Lornez	72.07	72.74	75.60	71.31	68.63	32.02	4179.3
Arnold	41.01	24.64	61.45	55.63	22.28	5.8	288.9

Table 6: case (b), n=10000.

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	95.35	96.22	97.11	94.47	93.60	86.94	18490
Lornez	91.93	92.34	94.06	91.50	89.83	77.73	8207.9
Arnold	79.76	67.77	85.72	92.44	73.09	49.70	600.5

Table 7: case (c), n=3000.

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	47.59	48.65	59.68	45.19	36.86	0.1	5542.1
Lornez	39.04	55.56	41.09	25.31	34.26	1.47	2369.2
Arnold	29.20	30.30	32.53	27.94	26.06	3.43	187.4

Table 8: case (c), n=5000.

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	69.71	77.90	79.30	61.37	60.31	29.09	9216.6
Lornez	55.98	77.92	64.37	36.93	44.75	13.03	4048.9
Arnold	45.13	40.29	40.02	49.93	50.30	2.8	290.42

Table 9: case (c), n=10000.

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	93.11	95.84	95.64	90.38	90.57	80.93	18423
Lornez	83.16	92.36	89.56	74.20	76.54	56.86	7914
Arnold	76.06	71.75	77.49	80.51	74.56	40.18	576.1

Table 10: case (d), n=3000.

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	43.17	35.02	46.29	52.02	39.38	3.21	5518.6
Lornez	33.18	26.05	31.90	41.16	33.58	4.3	2131.1
Arnold	19.73	26.46	21.74	13.86	16.90	9.2	144.3

Table 11: case (d), n=5000.

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	67.89	70.38	68.51	65.44	67.24	24.21	9185
Lornez	56.55	48.35	57.90	65.37	54.60	7.9	3758
Arnold	33.53	39.75	26.13	27.07	41.21	3.6	225.9

Table 12: case (d), n=10000.

System	K%	Q=1%	Q=2%	Q=3%	Q=4%	E%	L[m]
Chua	89.81	91.79	82.77	87.57	97.12	72.61	18393
Lornez	83.03	77.60	84.73	88.63	81.18	55.75	7417
Arnold	65.86	63.21	62.45	68.47	69.31	20.85	496.4

6. Conclusion

In this paper, we evaluate the performance of three controllers: the Arnold, the Lorenz, and the Chua's equations, which were adopted as the chaotic dynamics to be integrated into the mobile robot model and the behaviors of these equations were evaluated from point of view of performance index *k*, which reflects how high the coverage area, the evenness index *E*, which reflects the degree of variation in covering the areas between species, and the total length L of the generated trajectory.

The effects of the shape of the workspace were studied in four different test scenarios. The results show that the performance of the Chua's trajectory outperforms that of the Arnold and Lornez for all cases and all run time conditions. The performance of the Arnold equation as a controller is the worst among the other controllers. The total length of the trajectory is increased semi-linear with the time depending on the linear velocity of the mobile robot and the assumed obstacles and boundary area, and the Chua's controller generates the longest trajectory in all cases.

It is implied that the Chua's circuits are appropriated to be used for generating trajectories of mobile robots to take advantage of coverage areas resulting from its travelling paths. Large coverage areas are desirable for many applications such as robots designed for scanning of unknown workspaces with borders and barriers of unknown shape, as in patrol or cleaning purposes.

7. References

A. Anwar, H. Khammari. (2011). An Investigation on patrol Robot Coverage Performance Based on Chaotic and non Chaotic Guiding Signals. *In International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies,* 2(4), 405-421.

C. Chanvech, K. Klomkarn, and P. Sooraksa. (2006). Combined Chaotic Attractors Mobile Robots. *In Proc. SICE-ICASE Int. Joint Conf.*, 3079-3082.

M. Islam and K.Murase. (2005). Chaotic dynamics of a behavior-based miniature mobile robot: effects of environment and control structure. *Neural Networks*, 18 (2), 123–

144.

A. Jansri, K. Klomkarn, and P. Sooraksa. (2004). Further Investigation on Trajectory of Chaotic Guiding Signals For Robotics system. *In Proc. Int. Symp. Communication & Information Technology,* 1166-1170.

A. Jansri, K. Klomkarn, and P. Sooraksa. (2004). On Comparison of Attractors For Chaotic Mobile Robots *in Proc. 30th Annu. Conf. IEEE Industrial Electronics Society, IECON*, 3, 2536-2541.

L.S. Martins-Filho and Elbert E. N. Macau (2007). Patrol Mobile Robots and Chaotic Trajectories. *In mathematical problems in engineering*, 2007, Article ID61543, 13 pages.

L.S. Martins-Filho, R. F. Machado, R. Rocha, and V. S. Vale. (2004) Commanding mobile robots with chaos. In ABCM Symposium Series in Mechatronics,J.C. Adamowski, E. H. Tamai, E. Villani, and P. E. Miyagi, EdsABCM, Rio de Janeiro, Brazil, 1, 40–46.

L.S. Martins-Filho, EEN Macau, R Rocha. (2006). Kinematic Control of Mobile Robots to Produce Chaotic Trajectories. *ABCM Symposium Series in Mechatronics*, 2, 258-264.

Y. Nakamura and A. Sekiguchi. (2001). Chaotic Mobile Robot. *IEEE Transaction on Robotics and Automation*, 17(6), 898–904.

U. Nehmzow. (2003). Quantitative Analysis of Robot-Environment Interaction-Towards Scientific Mobile Robotics. *Robotics and Autonomous Systems*, 44 (1), 55–68.

J. Nicolas et al. (2003). A comparative Analysis of Evenness Index Sensitivity. *Int. Review Hydrobiology*, 88 (1), 3-15.

J. Palacin, J. A. Salse, I. Valganon, and X. Clua. (2004). Building A Mobile Robot For A Floor-cleaning Operation In Domestic Environments. *IEEE Transactions on Instrumentation and Measurement*, 53 (5), 1418–1424.

A. Sekiguchi and Y. Nakamura. (1999). The Chaotic Mobile Robot. *In Proc. IEEE/RSJ. Int. Conf. Intelligent Robots and Systems,* (1), 172-178.

P. Sooraksa and K. Klomkarn. (2010). No-CPU Chaotic Robots From Classroom To Commerce. *In IEEE circuits and systems magazine,* 10(1109), 46-53.

Dr. Ashraf Anwar received his PH.D degree in Electronics and Electrical Communication from Faculty of Engineering, Cairo University in 2005. Currently, he is an Assistant Professor at college of computers and Information Technology, Taif University. His research interests are mainly in the area of tracking system, control, and robotics.

Peer Review: This article has been internationally peer-reviewed and accepted for publication according to the guidelines given at the journal's website.