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Application of Bender's Decomposition Solving a Feed–mix Problem among Supply and Demand Uncertainties

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ABSTRACT

The feed-mix problem is primarily transformed into a mixing situation applying a mathematic formulation with uncertainties. These uncertainties generate the numerous expansions of alternative constraint equations. The given problem has been formulated as mathematic models which correspond to a large-scale Stochastic Programming that cannot be solved by the most popular ordinary calculation method, Simplex Method: LINPROG. This research aims to investigate effective methodology to reveal the optimal solution. The authors have examined the method of Bender's decomposition: BENDER and developed both methods into MATLAB® program and calculated comparatively. The results revealed that the nearest optimal solutions can be determined by means of a Two-stage Stochastic Programing incorporated with Bender's decomposition at the most intensive number of uncertainties and take less calculation time than by LINPROG.

1 Introduction

Many animal food mixing industries are confronted with a decision making problem on an appropriate recipe. That is to say that the determination of raw materials which contain various kinds of feed ingredients added to the process are influenced by expectations of

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obtaining a product with lower costs, standardization, or surpass nutritive requirement. Such a feed mix problem is complex and cannot be solved by traditional calculation methods. There are many approaches which have been applied, such as Pearson's square method which is very suit for only two-feed ingredients to be mixed [9] and Trial and Error which is one of the most popular means for feed formulation but it consumes a lot of time for calculation [4],[9]. Classical Linear Programming (LP) is widely used for modeling the animal feed problem. The normal objective in formulating the feed mix is to minimize cost subject to adequate nutrient ingredients (input raw materials) and the required nutrient constraints (output nutrient values) [1]. However, due to the various constraints that need to be conserved, the problem has been extended to become a large-scale problem with uncertainties. Therefore, when using LP method, it is difficult to determine a good balance of nutrients in the final solution. The constraints in LP are also rigid leading to an infeasible solution [1], [2]. However, LP has a positive highlight as a deterministic approach, because it can provide the best solution of hundreds of equations simultaneously [14]. By Stochastic approaches, there are also various methods that have been applied for such a complex problem e. g: Chance Constraint (CCP) and Quadratic Programming (QP), Risk Formulation, and Genetic Algorithm (GA). In addition to these mentioned methods, there are also some methods with other possible algorithms such as Integrated LP and Dynamic Programming (DP), Integrated LP and Fuzzy, Integrated GA and Fuzzy, Integrated GA and Monte Carlo Simulation. All of these methods are arranged as Integrated approaches [13].

This research does not take into consideration all the above mentioned issues, but aims to investigate another new effective calculation method for the feed mix problem and proposes the application of Two-stage Stochastic Linear Programming (TSL) incorporated with the method of Bender's Decomposition (BENDER). Hence, this paper describes the preliminary stage of mathematic formulation, the setup of matrix systems and program development, MATLAB@ program, and represents the optimization results of a case study.

2 Problem Analysis and Methodology

The classical diet problem is considered as a Linear Programming problem with general LP matrix: Min $Z = C^T X$, Subject to $AX = b$, and $X \geq 0$ for all. Because of the limitation of the calculation devices, the prior results were revealed without regard for some variables with high variance constraint coefficients. Nowadays, because of the higher performance of computational calculation, the development of the mentioned LP model when the system uncertainties are taken into account can be written as

$$\begin{aligned} \text{Min} \quad & Z = C^T X + g^T U + h^T V \\ \text{subject to} \quad & AX + U - V = b \\ & x, u, v \geq 0, \end{aligned} \tag{1}$$

where $C^T X$ represents the main cost and $g^T U + h^T V$ the additional corrective costs of materials supplied subject to $AX + U - V = b$, where A is the coefficient of the decision variable X (material quantity), U and V stand for the least and the excess mixed output quantities respectively. Awareness of nutrition values contained in U and V have an effect on the RHS as well. This issue will be discussed in the subtopic 2.3 later. Meanwhile the Two-stages Linear Programming Model [6], [7], [10], [11], [16] can be written as:

$$\text{Maximize} \quad z = \sum_{j=1}^{k} E[c_j] x_j + \sum_{q=1}^{Q} P_q [\sum_{j=k+1}^{n} c_{qj} x_{qj}] \tag{2}$$

Subject to

$$1^{st}.\text{Stage} \quad \sum_{j=1}^{k} a_{ij} x_j = b_i \quad \text{for} \quad i = 1, 2, \ldots, r \tag{3}$$

$$2^{nd}.\text{Stage} \quad \sum_{j=1}^{k} a_{qij} x_j + \sum_{j=1}^{k} a_{qij} x_{qj} = b_{qi} \tag{4}$$

$$i = r+1, \ldots m \quad \text{for} \quad q = 1, 2, \ldots, Q$$

$$x_j \geq 0; \quad x_{qj} \geq 0,$$

where

P_q probability of occurrences of scenerio q (q=1,2,…,Q)

x_{qj} extent variable in the 2^{nd}.stage at constraint j by event q

Notation

1. The value of each random element is independent of the levels of all x_j
2. The levels of x_j for $j = 1, 2, \ldots, k \leq n$ must be fixed at the 1^{st}. stage
3. The constraint $i = 1, 2 \ldots r$ contains only the 1^{st}.stage variables, and the associated a_{ij}

and b_i are known with certainty.

4. The variables of x_{qj} in the 2^{nd}. Stage, where $i = r + 1, 2, \ldots, m$ and $q = 1, 2, \ldots, Q$

5. The values of c_{qj}, a_{qij} and f b_{qi}, for $i = r+1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ are represented by the set of $(c_{qj}, a_{qij}, b_{qi})$ with probability P_q, for $q = 1, 2, \ldots, Q$

2.1 The problem analysis

The animal food mixing as shown below in Table 1 was discussed by a production team. The problem is to determine the optimal quantities of the three main input raw materials to be added to the mixing process.

Table 1: The input ingredient amounts represented as x_1, x_2, and x_3 were unknown.

		Protein (%)	Calcium (%)	cost / unit
x_1	Crushed dried fish	51-53	10-11	70Baht
x_2	Tapioca	2-3	5-7	40Baht
x_3	Sorghum	-	8-9	23Baht
m	Market required	between19-20	Not less than 8	

Note: Baht is the currency used in Thailand (As of January 2013, 30Baht = US$1).

The market demand $d = 1.000 \pm 0.001$ unit weigh t

Let x_1, x_2 and x_3 be the non-negative quantities of the crushed dried fish, tapioca and sorghum respectively. They are mixed to yield 1 unit of the minimum cost diet that satisfies all the specified nutritional requirements m =2 prescribed as following:

protein not in excess of 20 % (Upper Boundary)

not less than 19 % (Lower Boundary)

calcium not less than 8 %

Incremental corrective action cost of nutrient value, respectively

FEXD = fexd = 7 Baht/ Unit of protein

FLES = fles = 5 Baht/ Unit of protein

FEXD = fexd = 7 Baht/ Unit of calcium

FLES = fles = 4 Baht/ Unit of calcium

Incremental corrective action cost of ingredient (raw material), respectively

FEXDD = fexdd = 30 Baht/ kg

FLESD = flesd = 5000 Baht/ kg

2.2 Methodology

Such a problem is a typical large-scale stochastic linear programming with full system uncertainties (tolerances). The decision values of variables x_j are decided by the coefficient of $a_1 \pm \text{tol}$, $a_2 \pm \text{tol}, \ldots, a_n \pm \text{tol}$, right-hand-side (RHS) parameter vector, the nutrition values b_i of $b_1 \pm$tol, $b_2 \pm$tol, ... , $b_m \pm$tol and moreover, the size of the market demand d ±tol as Figure1 below:

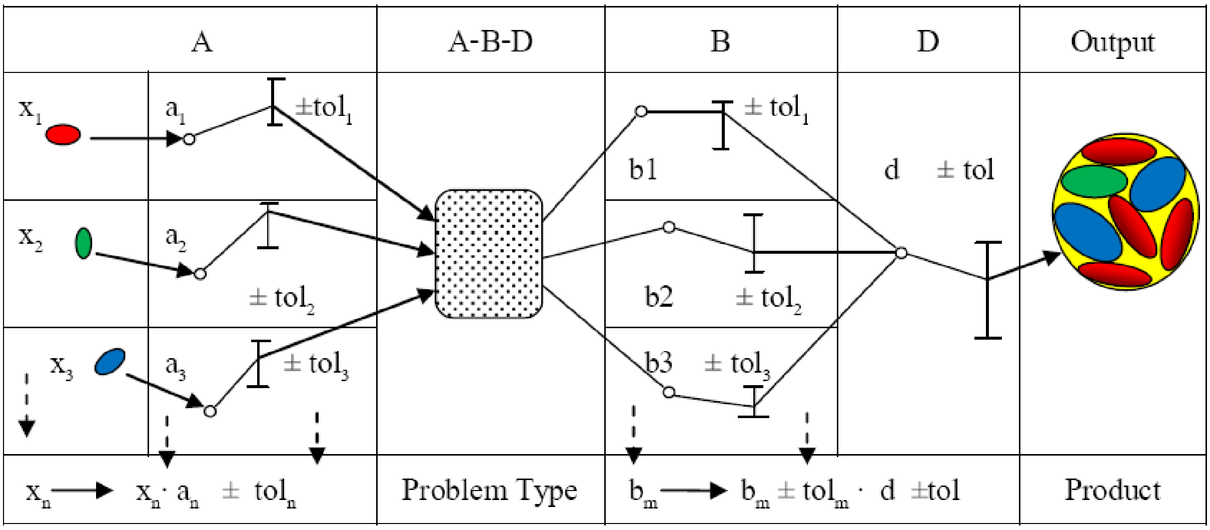

Figure 1: The problem type: A-B-D Uncertainty [15].

2.3 Model Formulation

To formulate the given problem in the form of TSL_ model and to allocate the calculation matrix system, some occurrence possibilities were assumed as follows:

Assumptions A):

P_N the occurrence possibility for each nutrient constraint

P_D the occurrence possibility for each demand constraint

$P_N = PD = P$ (Point) for this case study, the distributions of the probability of P_N and P_D are assumed to be uniform distributions. Thus, the possibilities P_N and P_D will be equal and also equal to P (Point) where the P (Point) is the initial input number for allocating the division number of all system uncertainty intervals.

E incremental event step, for this study, $E = P_N \times P_D$

Constr Constraint, $C = m \times \text{Event} + P_D$

Var Variable, Var = n + 2Constr

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Assumptions B):

The lower and upper boundaries of all constraint variables are allocated from the middle point of their tolerances:

$$\begin{aligned} & a_{ij} \in \left[a_{ij}^{(mid)} - a_{ij}^{(tol)} \; ; \; a_{ij}^{(mid)} + a_{ij}^{(tol)}\right] \\ & b_{i} \in \left[b_{i}^{(mid)} - b_{i}^{(tol)} \; ; \; b_{i}^{(mid)} + b_{i}^{(tol)}\right] \\ & d_{k} \in \left[d^{(mid)} - d^{(tol)} \; ; \; d^{(mid)} + d^{(tol)}\right] \\ & a_{ij}^{(mid)} \; ; \; a_{ij}^{(tol)} \; ; \; b_{i}^{(mid)} \; ; \; b_{i}^{(tol)} \; ; \; d^{(mid)} \text{ and } d^{(tol)} \in R \end{aligned} \tag{5}$$

Referring to the previous notations (1), (2), (3), (4) and (5) including analyzing the above given diet problem in Table 1, the calculation models were developed to be (6), (7) and (8). The objective function of the given problem was to minimize the total cost z min. which is the sum of the raw material unit cost c_j multiplied by the amount of x_j, plus the sum of all necessary incremental corrective action costs for both qualitative and quantitative values not meeting the specification, Equation (6). The sum of the product of the quantity xj and its uncertain coefficients $a_{ij\ell}$ including the sum of nutritive slack $u_{i\ell k}$ and surplus $v_{i\ell k}$ of each calculation scenario have to be balanced to the right-hand-side RHS , i.e. the vectors of required nutritive value of $b_{i\ell}$ multiplied by the demand d , in Equation (7). Simultaneously, the LHS, the sum of x_j and addition of the slacks u_k and surplus v_k has to be associated with the quantity requirement of d. In practice, the demand d cannot be exactly equal to 1 unit, but rather it comprises an allowance of $\pm$ 0.001 in unit weight (0.1% error).

Minimize z

$$\sum_{j=1}^{n} c_j x_j + \sum_{i=1}^{m}\sum_{k=1}^{P}\sum_{\ell=1}^{P} (g_{i\ell k} u_{i\ell k} + h_{i\ell k} v_{i\ell k}) + \sum_{k=1}^{P} (g_k u'_k + h_k v'_k) \tag{6}$$

Subject to

$$\sum_{j=1}^{n} a_{ij\ell} \times x_j + u_{i\ell k} - v_{i\ell k} = b_{i\ell} \times d_k \quad ; \quad \forall_{i\ell k} \tag{7}$$

$$\sum_{j=1}^{n} x_j + u'_k - v'_k = d_k \quad ; \quad \forall_k \tag{8}$$

$$x_j ; \quad u_{i\ell k} ; \quad v_{i\ell k} \; ; \; u'_k \; ; \; v'_k \geq 0 \; ; \; \forall_{i,j,k}$$

Where

k	denotes the constraint alternatives
j	denotes the type of ingredient to be input to mixing process ($j=1,2,\ldots,n$).
i	denotes the type of nutrient composition which the market needs ($i=1,2,\ldots,m$)
c_j	denotes the cost factor of raw material j- type (cost/ unit weight = fx)
x_j	denotes the quantity of raw material j- type (weight unit = kg).
a_{ij}	denotes the nutrient value type i in material type j
b_i	denotes the nutrient value (of product) type i /unit weight.
$u_{i\ell k}$	denotes the nutrient value (of product) i , at the event ℓ which misses (slack) the target in the alternative k
$v_{i\ell k}$	denotes the nutrient value (of product) i , at the event ℓ which exceeds (surplus) the target in the alternative k
$g_{i\ell k}$	denotes the expected cost of nutrient value i / unit which misses the target alternative k . FLES = fles
$h_{i\ell k}$	denotes the expected cost of nutrient value i / unit which exceeds the target alternative k . FEXD = fexd
g_k	denotes the expected cost of u'_k (by lack of demand) = FLESD
h_k	denotes the expected cost of v'_k (by exceeding demand) = FEXDD
u'_k	denotes the lower quantity of raw material in the alternative k
v'_k	denotes the excess quantity of raw material in the alternative k
d_k	denotes the market demand 1.000 weight unit with the standardized allowance of ± 0.001 in unit weight (0.1 % error) for animal food production.

2.4 Minimum Cost Calculation Model

Minimize z =

$$(fx_1 \cdot x_1 + fx_2 \cdot x_2 + fx_3 \cdot x_3) + \sum_{i=1}^{m} \sum_{k=1}^{P} \sum_{\ell=1}^{P} (FLES \cdot u_{i\ell k} + FEXD \cdot v_{i\ell k}) + \sum_{k=1}^{P} (FLESD \cdot u'_k + FEXDD \cdot v'_k) \quad \textbf{(9)}$$

Subject to

$$\sum_{j=1}^{n} a_{ij\ell} \times x_j + u_{i\ell k} - v_{i\ell k} = b_{i\ell} \times d_k \qquad ; \forall_{i\ell k}$$

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$$\sum_{j=1}^{n} x_j + u'_k - v'_k = d_k \; ; \forall_{i,k}$$

$$x_j; \quad u_{i\ell k}; \quad v_{i\ell k} \; ; \; u'_k \; ; \; v'_k \geq 0 \; ; \forall_{i,j,k}$$

Substitute the given data from Table 1 into the minimal cost calculation model above

$$\text{Minimize } Z = (70x_1 + 40x_2 + 23x_3) + \sum_{i=1}^{2} \sum_{k=1}^{P} \sum_{\ell=1}^{P} ((5,4) \cdot u_{i\ell k} + (7,7) \cdot v_{i\ell k}) + \sum_{k=1}^{P} \left(5000 \cdot u'_k + 30 \cdot v'_k\right)$$

Subject to

$$\begin{aligned}
[51\text{--}53]x_1 + [2\text{--}3]x_2 + [0]x_3 + u_{1\ell k} - v_{1\ell k} &= [19\text{--}20] && \forall \ell, k \\
[10\text{--}11]x_1 + [5\text{--}7]x_2 + [8\text{--}9]x_3 + u_{2\ell k} - v_{2\ell k} &= 8 && \forall \ell, k \\
x_1 + x_2 + x_3 + u'_k - v'_k &= 1 \pm 0.001 && \forall k \\
x_1, x_2, x_3 \geq 0, \quad u'_{i\ell k}, v'_{i\ell k} &\geq 0 && \forall i, \ell, k
\end{aligned}$$

1^{st}.Event; for $i = 1, \ell = 1, k = 1$ (at lower boundary)

$$\begin{aligned}
51x_1 + 2x_2 + 0x_3 + u_{111} - v_{111} &= 19 \cdot (0.999) \\
10x_1 + 5x_2 + 8x_3 + u_{211} - v_{211} &= 8 \cdot (0.999) \\
x_1 + x_2 + x_3 + u'_1 - v'_1 &= 0.999
\end{aligned}$$

2^{nd}.Event; for $i = 2, \ell = 2, k = 2$ (at middle value)

$$\begin{aligned}
52x_1 + 2.5x_2 + 0x_3 + u_{122} - v_{122} &= 19.5 \cdot (1.000) \\
10.5x_1 + 6x_2 + 8.5x_3 + u_{222} - v_{222} &= 8 \cdot (1.000) \\
x_1 + x_2 + x_3 + u'_2 - v'_2 &= 1.000
\end{aligned}$$

3^{rd}.Event; for $i = 3, \ell = 3, k = 3$ (at upper boundary)

$$\begin{aligned}
53x_1 + 3x_2 + 0x_3 + u_{133} - v_{133} &= 20 \times (1.001) \\
11x_1 + 7x_2 + 9x_3 + u_{233} - v_{233} &= 8 \times (1.001) \\
x_1 + x_2 + x_3 + u'_3 - v'_3 &= 1.001
\end{aligned}$$

The above formulations just demonstrate how to set up only three events (l): lower boundary (LB), middle value, and upper boundary (UB). But, in this research, the calculation models were planned to be set up throughout the tolerance intervals of all relevant variables and divide those with the same number of P, by which the resolution (res), through all uncertainty intervals can be definable. One must beware of the LB and UB of each event which correspond with the given tolerances.

2.5 The TSL incorporated with Bender's Decomposition

The concept of the Bender's decomposition is to predict the second-stage costs by a scalar θ and replace the second-stage constraints by cuts, which are necessary conditions expressed in terms of the first-stage variables x and θ [6], [10], [12]. The initial model can be modified, and written as shown below:

The 1st.stage: Primal problem

$$\begin{aligned} &\text{Min} \quad Z = C^T X + g^T U + h^T V \\ &\text{subject to} \quad AX + U - V = b \\ &\qquad x, u, v \geq 0, \end{aligned} \tag{10}$$

The 2nd.stage: The Dual Bender's Decomposition

$$\begin{aligned} &U - V = b - AX \\ &b - AX \geq 0 \\ &\text{Max} \quad \omega = (b - AX)^T Y + C^T X \\ &\qquad \theta = \sum (b - AX)^T \\ &\quad \text{cond} = \left[\text{cond}; \ \theta + \left(Y^T A - C^T\right) X \geq b^T Y\right] \end{aligned} \tag{11}$$

$$\begin{aligned} &\text{Min } \theta + X; \\ &\text{Subject to} \quad \theta + \left(Y^T A - C^T\right) X \geq b^T Y \end{aligned} \tag{12}$$

Stop condition

$$\begin{aligned} &\text{Check} \quad \theta = \omega \ ? \quad \text{No} \rightarrow \text{Get } Xnew \rightarrow 1X, \ 2X, \ldots.. \\ &\qquad\qquad\qquad \text{Yes} \rightarrow \text{Get } \theta \text{ and } X \Rightarrow \text{Min } Z = \text{Max } \omega \Rightarrow \text{Stop} \end{aligned}$$

The selected algorithm to attain a satisfactory solution was the integration between TSL and Bender's decomposition. As shown in Figure 2, the main concept of Bender's decomposition is to split the original problem into a master problem and a sub-problem, which in turn decomposes into a series of independent sub problems, one for each. The latter are used to generate cuts. The $X_{initial}$ is to be randomly selected to substitute in terms of

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the constraints inequality equation. If the result, by substitution of X $_{initial}$, is equal to or greater than 0, then $y_i = g_i$. If the result (substitution of X $_{initial}$) is less than 0, then $y_i = -h$. The $x_{initial}$ and the selected y_i are substituted in the Dual equation and in the inequality (to generate the optimal cut) to attain the maximum value of ω and θ respectively. By minimization of the $\theta + 0X_{new}$; subject to $\theta + (y^T A - c^T) \cdot x_{new} \geq b^T y$, Equation (12) renders θ and x_{new}. If the obtained ω and θ are equal at the step of convergence test, ω and θ are to be approximately equivalent to z_{Dual} which can be obtained as an optimal solution.

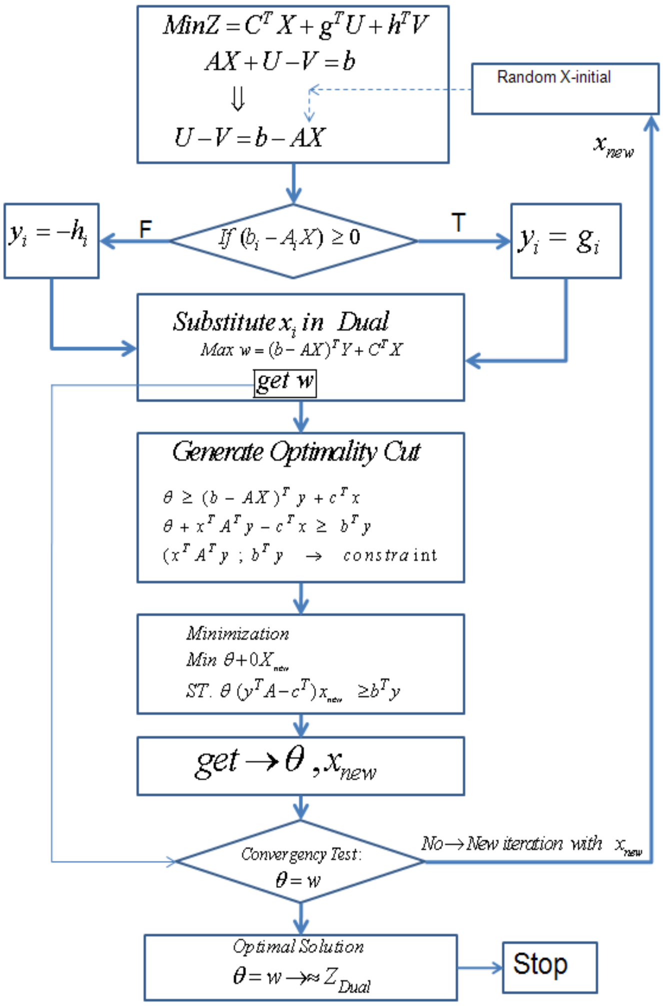

Figure 2: TSL incorporated with Bender's decomposition [6], [10], [11], [15].

2.6 Calculation Tools

The mathematical calculation tool, MATLAB® Program was selected to solve this problem. The MATLAB®_Software / Verion.2006a and a HP_Pavillion_IntelCore_2Qurd Inside, No.: 016-120610000, personal computer, at the Department of Industrial Engineering, Thammasat University, Pathumtani were used. The calculation steps were as follows

Step1. In accordance with the mathematic formulation, previous topic 2.4, the matrices systems were established according to the ordinary simplex method. The initial input variable matrix was expressed in two possible substitutions: randomization and dual-simplex algorithm.

Step2. Model building according to the method of Bender's decomposition followed the programming flowchart as shown in Figure 2.

Step3. Preparation of MATLAB® programming named: lstart, start, setupmodel, solvemodel; linMAT, bender, dataABD_Uncertainty.mat, result, pline, selfcat, speye, and vspace

Step4. Creation of data file: a, a_tol, b, b_tol, d, d_tol, fx, fles, fexd, flesd and fexdd

Step5. Program execution by the program named: lstart (input parameter is P-Number).

2.7 Computational Calculation

To solve the above formulated problem, the primal-dual Simplex method /LINPROG, Equations (1) was applied to compare with two-stage stochastic linear programming incorporated with the method of Bender's Decomposition, Equations (9), (10), (11), (12) referred to Equations (2), (3), (4), (6), (7), and (8).

Assumptions:

1. The quantity of each nutrient value in each type of raw material is a continuous event. The values, which are independent of each other, are uncertain. However, the values intervals are recognized to be uniform distributions.
2. All scenarios are also uniformly distributed and independent on each other.

As the solving tools for comparative calculation, the results of both selected method can be collected after the calculation iteration has terminated. The programs for primal-dual simplex and Bender's decomposition were developed. The starting program referred to as 'Start.m' constitutes the main application to perform the execution of all relevant functions. The matrix systems corresponding with Equations (9), (10), (11), (12) are established. The program referred to as 'linMAT.m' is a matrix to receive all loaded input data whereas the

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program name: ‘Speye.m’ serves to construct a large identity matrix system with the ‘Selfcat.m’ application to await the parameter patterns modification prior to the continuation of the Bender’s decomposition program ‘Bender.m’ [15].

Table 2: Lower section cut off at the uncertainty number of **P=2:2:50**.

P	Event	Constr	Var	x1_BEN	x2_BEN	x3_BEN	Sx_BEN	Z_BEN	Ti_BEN	x1_LIN	x2_LIN	x3_LIN	Sx_LIN	Z_LIN	Ti_LIN
2	4	10	23	0.3631	0.2420	0.3949	1.0000	48.61	0.4924	0.3631	0.2420	0.3949	1.0000	48.61	2.3399
4	16	36	75	0.3633	0.2422	0.3946	1.0000	48.60	0.1767	0.3633	0.2422	0.3946	1.0000	48.60	0.0558
6	36	78	159	0.3632	0.2422	0.3947	1.0000	48.59	0.1995	0.3632	0.2422	0.3947	1.0000	48.59	0.0868
8	64	136	275	0.3617	0.2761	0.3622	1.0000	48.57	0.1284	0.3617	0.2762	0.3621	1.0000	48.57	0.2207
10	100	210	423	0.3617	0.2768	0.3615	1.0000	48.54	0.1818	0.3617	0.2766	0.3617	1.0000	48.54	0.3620
12	144	300	603	0.3617	0.2769	0.3614	1.0000	48.52	0.2010	0.3617	0.2769	0.3615	1.0000	48.52	0.8774
14	196	406	815	0.3618	0.2739	0.3643	1.0000	48.52	0.1472	0.3618	0.2739	0.3643	1.0000	48.52	1.4670
16	256	528	1059	0.3617	0.2762	0.3621	1.0000	48.51	0.1660	0.3617	0.2762	0.3621	1.0000	48.51	1.1920
18	324	666	1335	0.3617	0.2761	0.3621	1.0000	48.51	0.1456	0.3617	0.2765	0.3618	1.0000	48.51	0.9501
20	400	820	1643	0.3617	0.2766	0.3617	1.0000	48.51	0.1393	0.3617	0.2766	0.3617	1.0000	48.51	0.2607
22	484	990	1983	0.3617	0.2764	0.3619	1.0000	48.50	0.1514	0.3617	0.2768	0.3616	1.0000	48.50	0.3611
24	576	1176	2355	0.3617	0.2768	0.3615	1.0000	48.50	0.1820	0.3617	0.2769	0.3614	1.0000	48.50	0.7045
26	676	1378	2759	0.3617	0.2763	0.362	1.0000	48.50	0.1695	0.3617	0.2763	0.362	1.0000	48.50	1.5480
28	784	1596	3195	0.3617	0.2763	0.362	1.0000	48.50	0.1490	0.3617	0.2765	0.3618	1.0000	48.50	0.6279
30	900	1830	3663	0.3617	0.2763	0.362	1.0000	48.50	0.1589	0.3617	0.2766	0.3617	1.0000	48.50	0.7172
32	1024	2080	4163	0.3617	0.2769	0.3614	1.0000	48.49	0.1537	0.3617	0.2767	0.3616	1.0000	48.49	0.5023
34	1156	2346	4695	0.3617	0.2767	0.3616	1.0000	48.49	0.2377	0.3617	0.2768	0.3615	1.0000	48.49	0.6613
36	1296	2628	5259	0.3617	0.2769	0.3614	1.0000	48.49	0.1867	0.3617	0.2769	0.3614	1.0000	48.49	2.2707
38	1444	2926	5855	0.3617	0.2765	0.3618	1.0000	48.49	0.1819	0.3617	0.2765	0.3618	1.0000	48.49	1.2727
40	1600	3240	6483	0.3617	0.2770	0.3614	1.0000	48.49	0.1650	0.3617	0.2766	0.3617	1.0000	48.49	0.8648
42	1764	3570	7143	0.3617	0.2764	0.3619	1.0000	48.49	0.1649	0.3617	0.2767	0.3616	1.0000	48.49	1.1724
44	1936	3916	7835	0.3617	0.2767	0.3616	1.0000	48.49	0.1892	0.3617	0.2768	0.3615	1.0000	48.49	0.8411
46	2116	4278	8559	0.3617	0.2768	0.3615	1.0000	48.49	0.1923	0.3617	0.2768	0.3615	1.0000	48.49	3.1807
48	2304	4656	9315	0.3617	0.2768	0.3615	1.0000	48.49	0.1462	0.3617	0.2769	0.3614	1.0000	48.49	3.4919
50	2500	5050	10103	0.3617	0.2767	0.3616	1.0000	48.49	0.1504	0.3617	0.2766	0.3617	1.0000	48.49	2.8000

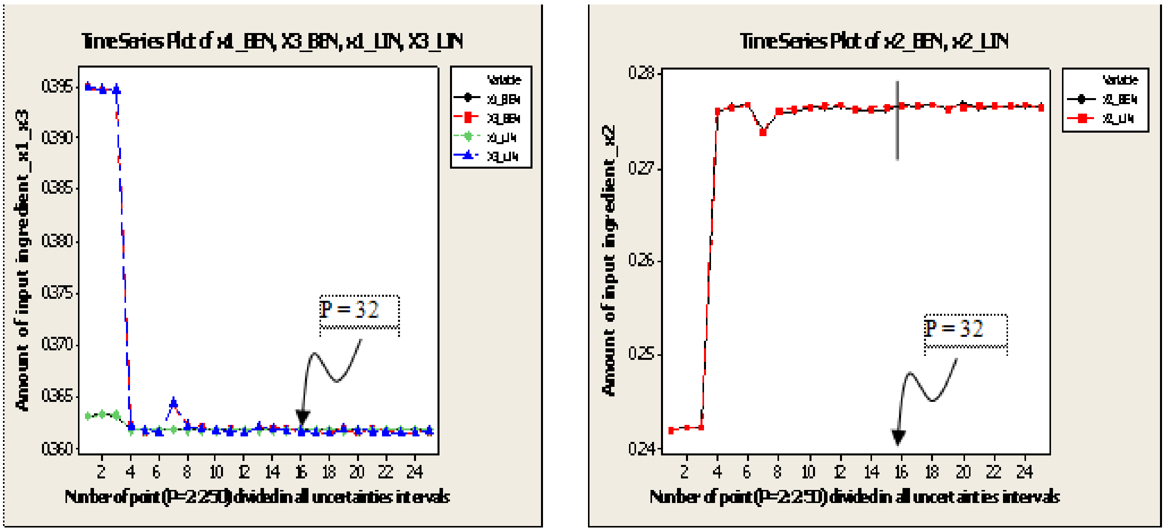

Figure 3: The congruence of x1_BEN vs. x1_LIN; x2_BEN vs. x2_LIN and x3_BEN vs. x3_LIN, x3_LI

3 Results and Discussion

The calculation results were enumerated to check the calculation efficiencies of both the applied methodologies and the programming development. To be discussed in this research paper are the expected values of all response factors and their calculation times on a limited set of personal computers. Hence, there are three sections discussed as follow:

3.1 The result at the lower section (P = 2:2:50)

P = 2:2:50 denotes for the value of P starting at 2: increasing step 2: and ending at 50. According to assumption a) on page 6, the incremental event step E = P^2 for uniform distribution and constraint number C = m × Event + $P_{D.}$ The values of x1_BEN, x1_LIN; x2_BEN, x2_LIN; x3_BEN, x3_LIN are congruent and consistent variants as represented in Table 2 and in Figure 3. The results of Zmin_BEN and Zmin_LIN represent their respective congruencies and at P= 32 (in Figure 4, at 16 on the axis) . However, consideration of the series plot of Ti_BEN and Ti_LIN in Figure 5 shows the calculation time fluctuations of Ti_LIN, but not for Ti_BEN.

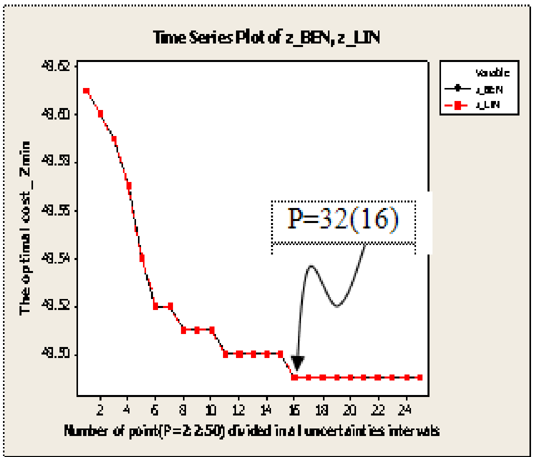

Figure 4: Zmin_BEN and Zmin_LIN

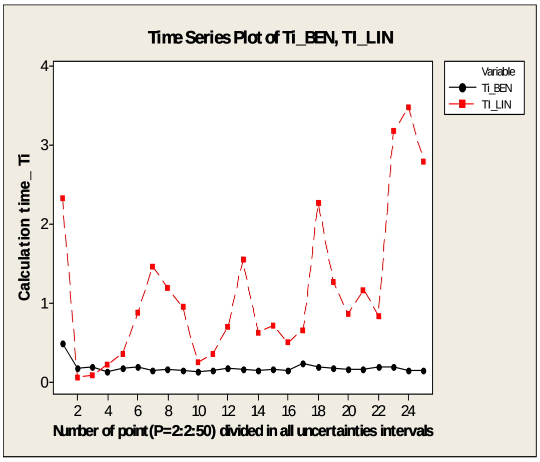

Figure 5: Ti_BEN and Ti_LIN

3.2 The results at the middle section (P = 50:2:106)

At the intensive computational events, the values of x1_BEN, x3 _BEN, z_BEN and x1_LIN, x3 _LIN, z_LIN are stable congruent with a very small consistent variance as reported in Table 3 below:

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Table 3: Upper section cut off at the uncertainty number P = 50:2:106.

P	Event	Constr	Var	x1_BEN	x2_BEN	x3_BEN	Sx_BEN	Zmin_BEN	Ti_BEN	x1_LIN	x2_LIN	x3_LIN	Sx_LIN	Zmin_LIN	Ti_LIN
50	2500	5050	10103	0.3617	0.2767	0.3616	1.0000	48.49	0.2884	0.3617	0.2766	0.3617	1.0000	48.49	3.3737
52	2704	5460	10923	0.3617	0.2766	0.3617	1.0000	48.49	0.1726	0.3617	0.2767	0.3616	1.0000	48.49	1.2569
54	2916	5886	11775	0.3617	0.2772	0.3611	1.0000	48.49	0.2258	0.3617	0.2767	0.3616	1.0000	48.49	1.7699
56	3136	6328	12659	0.3617	0.2771	0.3612	1.0000	48.49	0.1420	0.3617	0.2768	0.3615	1.0000	48.49	1.3229
58	3364	6786	13575	0.3617	0.2768	0.3615	1.0000	48.48	0.1641	0.3617	0.2768	0.3615	1.0000	48.48	2.8755
60	3600	7260	14523	0.3617	0.2771	0.3612	1.0000	48.48	0.2000	0.3617	0.2769	0.3614	1.0000	48.48	2.3254
62	3844	7750	15503	0.3617	0.2768	0.3615	1.0000	48.48	0.2491	0.3617	0.2767	0.3616	1.0000	48.48	5.5466
64	4096	8256	16515	0.3617	0.2767	0.3616	1.0000	48.48	0.2022	0.3617	0.2767	0.3616	1.0000	48.48	1.5803
66	4356	8778	17559	0.3617	0.2766	0.3617	1.0000	48.48	0.2676	0.3617	0.2768	0.3615	1.0000	48.48	2.5353
68	4624	9316	18635	0.3617	0.2768	0.3604	0.9989	48.48	0.1525	0.3617	0.2766	0.3607	0.9990	48.48	3.0060
70	4900	9870	19743	0.3617	0.2764	0.3567	0.9947	48.47	0.2050	0.3617	0.2762	0.3568	0.9947	48.47	6.6884
72	5184	10440	20883	0.3742	0.0000	0.5166	0.8908	48.39	0.1865	0.3741	0.0000	0.5166	0.8908	48.39	3.0824
74	5476	11026	22055	0.3742	0.0000	0.5141	0.8882	48.18	0.1798	0.3742	0.0000	0.5140	0.8882	48.18	2.0440
76	5776	11628	23259	0.3742	0.0000	0.5116	0.8858	47.98	0.1804	0.3742	0.0000	0.5116	0.8858	47.98	1.4806
78	6084	12246	24495	0.3742	0.0000	0.5094	0.8835	47.78	0.2126	0.3742	0.0000	0.5094	0.8835	47.78	2.0009
80	6400	12880	25763	0.3742	0.0000	0.5085	0.8827	47.60	0.1892	0.3742	0.0000	0.5085	0.8827	47.60	1.5094
82	6724	13530	27063	0.3742	0.0000	0.5062	0.8804	47.42	0.1766	0.3742	0.0000	0.5064	0.8806	47.42	1.4550
84	7056	14196	28395	0.3742	0.0000	0.5045	0.8787	47.24	0.1828	0.3742	0.0000	0.5045	0.8787	47.24	3.0142
86	7396	14878	29759	0.3742	0.0000	0.5026	0.8768	47.07	0.1794	0.3742	0.0000	0.5026	0.8768	47.07	6.0361
88	7744	15576	31155	0.3742	0.0000	0.5009	0.8750	46.91	0.1692	0.3742	0.0000	0.5008	0.8750	46.91	5.9829
90	8100	16290	32583	0.3742	0.0000	0.4991	0.8733	46.75	0.1697	0.3742	0.0000	0.4991	0.8733	46.75	4.6769
92	8464	17020	34043	0.3742	0.0000	0.4987	0.8729	46.59	0.1991	0.3742	0.0000	0.4987	0.8729	46.59	1.9615
94	8836	17766	35535	0.3742	0.0000	0.4971	0.8713	46.45	0.1917	0.3742	0.0000	0.4971	0.8713	46.45	4.8717
96	9216	18528	37059	0.3742	0.0000	0.4956	0.8698	46.30	0.1981	0.3742	0.0000	0.4956	0.8698	46.30	10.8197
98	9604	19306	38615	0.3742	0.0000	0.4942	0.8684	46.16	0.2047	0.3742	0.0000	0.4942	0.8684	46.16	4.3436
100	10000	20100	40203	0.3742	0.0000	0.4929	0.8671	46.03	0.2016	0.3742	0.0000	0.4929	0.8670	46.03	3.0834
102	10404	20910	41823	0.3742	0.0000	0.4925	0.8668	45.90	0.2233	0.3742	0.0000	0.4925	0.8667	45.90	3.4999
104	10816	21736	43475	0.3742	0.0000	0.4913	0.8655	45.77	0.1791	0.3742	0.0000	0.4913	0.8655	45.77	7.3899
106	11236	22578	45159	0.3742	0.0000	0.4901	0.8643	45.65	0.1778	0.3742	0.0000	0.4901	0.8643	45.65	3.6450

According to the numerical consideration shown in Table 3, the values of x1_BEN , x1_LIN; x2_BEN, x2_LIN ; x3_BEN, x3_LIN are congruent and consistent variants. The optimal values of Zmin_BEN and Zmin_LIN are also congruent and have tendencies to converge to the most optimal solution. By contrast with this, the calculation times Ti_LIN and Ti_BEN are very different. The ratio of them is 2.5353:0.2676 = 9.474:1 at P = 66. The sums of Sx_BEN and Sx_LIN start to decrease after P = 66 as do the Zmin values also. Figure 6, 7 and 8 show that the components of x3_BEN and x3_LIN were selected , but not x2_LIN and x2_BEN at P=72.

In an actual factory, the producers can make a decision at this stage with Sx = 0.8908 unit weight and Zmin = 48.39. If they want to obtain the sum Sx = 1.000 unit weight to meet demand, they can select the status of P = 66 with the optimal Zmin = 48.48 (higher cost). A further perspective, Figure 9 represents the series plot of Ti_BEN and Ti_LIN with major fluctuations. the calculation times of Ti_LIN are extremely variable, whereas the calculation time Ti_BEN tends to be constant.

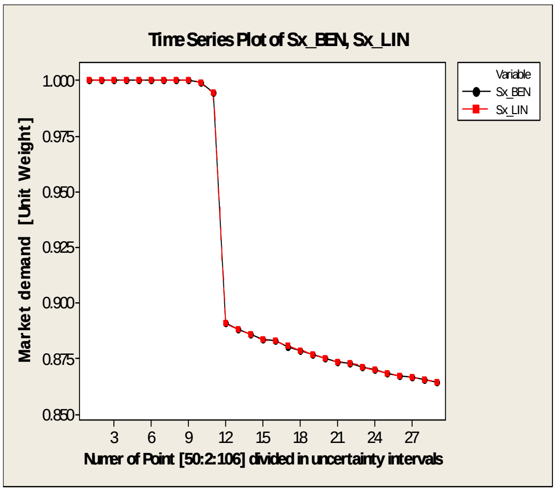

Figure 6: Sx_BEN and Sx_LIN

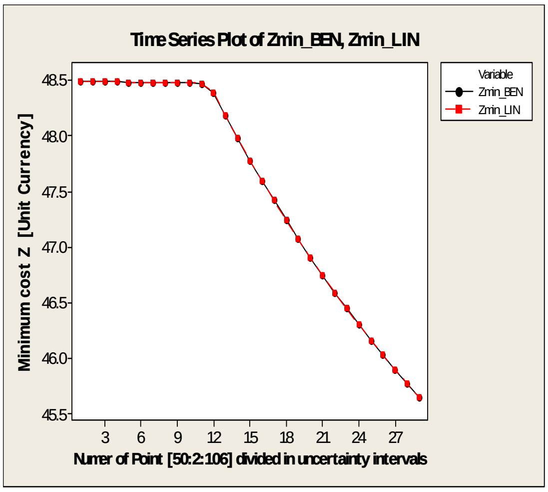

Figure 7: Min.cost Z_{min}_BEN and Z_{min}_LIN

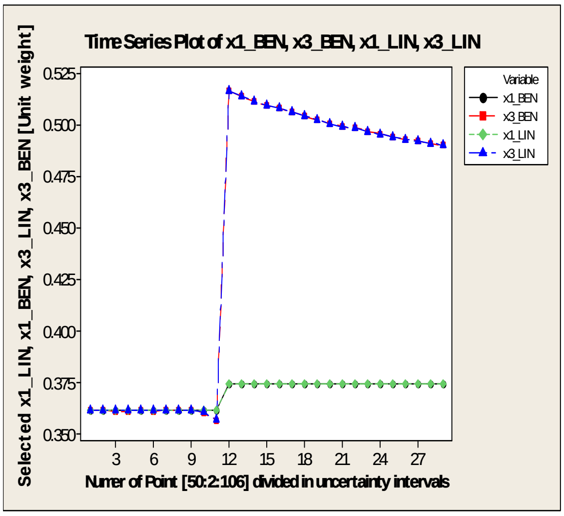

Figure 8: The congruence of x1_BEN, x3_BEN,x1_LIN, x1_BEN

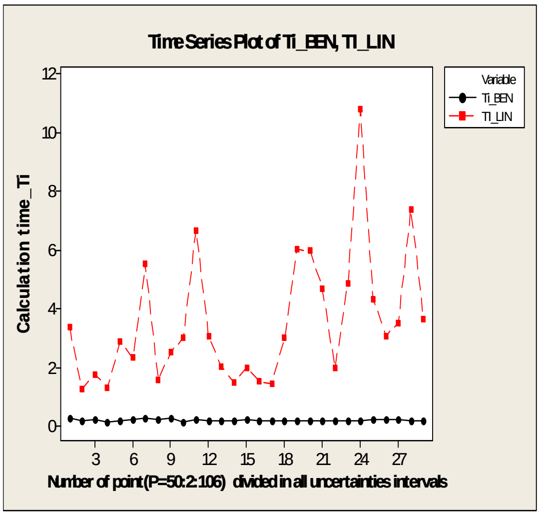

Figure 9: Ti_LIN and Ti_BEN.

3.3 The results at the upper section

Continuing the computational calculation by the Bender's decomposition method with P = (106:2:1584) until the calculation terminated (out of memory on the HP_Pavillion_ IntelCore_2Quard Inside, No.: 016-120610000, personal computer), the nearest optimal solution can be accepted at P=1584 corresponding to Zmin of 38.63Baht and Sx_BEN of 0.8185 unit weight. Sx_BEN is not equal to 1 as the market demand. It depends upon the cost factors of *flesd* and *fexdd* . If flesd is less than fexdd, it will be reasonable to produce the mixed product with lower amount from the demand. But, the optimization can reveal the lowest value of the Zmin as shown in Table 4 below.

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Table 4: The upper section cut off at the uncertainty number **P = (1560:2:1584)**

P	Events	Constr	Var	x1_BEN	x2_BEN	x3_BEN	Sx_BEN	Z_BEN	Ti_BEN
1560	2433600	4868760	9737523	0.3743	0	0.4445	0.8187	38.64	10.9534
1562	2439844	4881250	9762503	0.3743	0.0002	0.4442	0.8186	38.64	10.2998
1564	2446096	4893756	9787515	0.3743	0	0.4442	0.8185	38.64	10.3893
1566	2452356	4906278	9812559	0.3743	0	0.4442	0.8185	38.64	10.5075
1568	2458624	4918816	9837635	0.3743	0	0.4453	0.8197	38.63	10.4842
1570	2464900	4931370	9862743	0.3743	0.0001	0.4444	0.8187	38.63	11.056
1572	2471184	4943940	9887883	0.3743	0	0.4442	0.8185	38.63	10.0722
1574	2477476	4956526	9913055	0.3743	0	0.4445	0.8188	38.63	11.0531
1576	2483776	4969128	9938259	0.3743	0.0002	0.4445	0.8188	38.63	11.0438
1578	2490084	4981746	9963495	0.3743	0.0011	0.4446	0.8189	38.63	11.373
1580	2496400	4994380	9988763	0.3743	0.0055	0.4441	0.8185	38.63	10.543
1582	2502724	5007030	1E+07	0.3743	0	0.4445	0.8187	38.63	11.7056
1584	2509056	5019696	1E+07	0.3743	0	0.4442	0.8185	38.63	10.5942
				OUT OF MEMORY					

4 Conclusions

At the first start with a lower division number of points P, the results obtained from the simplex method-LINPROG and Bender's decomposition were consistently equivalent. The calculation results were almost identical. These two algorithms are very suitable for small-scale problems but when increasing the division numbers (point P) there is, a rise of uncertainties numbers, the consequence of the enlargement of the constraint numbers. Some of response factors were found to deviate from the target and thus failed in condition. Nevertheless, the calculation by both methods can be performed. The LINPROG method is extensive in calculation time and thus requires a large memory storage. On the personal computer, the calculation failed to determine the results at the earlier stage.

Conversely, the Bender's decomposition method can quickly and consistently obtain the nearest optimal solution up to the calculation termination due to being out of memory. The same problem and the same calculation tool, MATLAB® program, were also performed on a high performance computer. The results showed the enormous effect of the system uncertainties mainly influencing the calculation times. It is noteworthy that the ratio of the mean time consumption of the LINPROG : BENDER is approximately equal to 232.77 :1 at P=2:2:500 on a general PC, whereas the results of the other response factors can be congruent.

The authors hope that this calculation method, the integration of Two-stage stochastic linear programming incorporated with the Bender's decomposition method, compacted in general form of a MATLAB® programming, can contribute to supporting decision making in other operations research areas as a low cost effective calculation tool. For future research, this program is to be developed in form of a Graphic User Interface for convenience of use.

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