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Bender's Decomposition Method for a Large Two-stage Linear Programming Model

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ABSTRACT

Linear Programming method (LP) can solve many problems in operations research and can obtain optimal solutions. But, the problems with uncertainties cannot be solved so easily. These uncertainties increase the complexity scale of the problems to become a large-scale LP model. The discussion started with the mathematical models. The objective is to minimize the number of the system variables subjecting to the decision variable coefficients and their slacks and surpluses. Then, the problems are formulated in the form of a Two-stage Stochastic Linear (TSL) model incorporated with the Bender's Decomposition method. In the final step, the matrix systems are set up to support the MATLAB programming development of the primal-dual simplex and the Bender's decomposition method, and applied to solve the example problem with the assumed four numerical sets of the decision variable coefficients simultaneously. The simplex method (primal) failed to determine the results and it was computational time-consuming. The comparison of the ordinary primal, primal-random, and dual method, revealed advantageous of the primal-random. The results yielded by the application of Bender's decomposition method were proven to be the optimal solutions at a high level of confidence.

1. Introduction

Some problems in operations research cannot be solved by traditional calculation methods.

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The classical Linear Programming (LP) method is widely used for modeling feed-mix problems. The general objective in formulating the feed-mix is to minimize cost subjecting to adequate nutrient ingredients (input raw materials) and the required nutrient constraints (output nutrient values) [1]. Among uncertainties, the problem has been extended to become a large-scale LP containing various constraints that need to be conserved. Therefore, LP method is difficult to determine a good balance of the objective function and all constraints in the final solution as formerly. Although, LP has a positive highlight as a deterministic approach because LP can provide the best solution of hundreds of equations simultaneously [2], [12], but the numerous constraints in LP are also rigid leading to an infeasible solution [3]. The LP method, Simplex method, cannot alone overcome all of problem complexities. Recently, there are two appropriate calculation techniques; individual approaches and integrated approaches. The individual approaches or manual formulations such as Pearson's square method, Simultaneous Algebraic Equations, Trial and Error Method, and so on. All of those methods involve with single technique without integration with other method. Whereas the integrated approaches refer to the combination of different methods in one effective aspect such as Integrated LP and Dynamic Programming (DP), LP and Fuzzy, LP and Goal Programming (GP), Genetic Algorithm (GA) and Fuzzy, etc. The comparison of both approaches, the individual approach is more popular than integrated approach. Most of integrated methods were done to introduce new idea in the feed mix problem. [4].

This research aims to find out a new effective calculation method, through a propose of using the Bender's Decomposition Method incorporated with Two-stage Stochastic Linear Programming for survive such a large-scale feed mix problem. Hence, this paper describes the preliminary stage of mathematical formulation. As there is no way to solve such formulated problems manually, therefore the formulated problems have been written in form of the matrix systems before we developed a MATLAB programming for computing the four given numerical sets of a_{ij} and b_i as a trial case. Finally, the optimization results and the calculation performance have been represented.

2. Mathematical Model

As above mentioned the classical LP is widely used for modeling feed-mix problem. The normal objective in formulating the feed mix is to minimize cost Z = $\mathbf{C}^T\mathbf{X}$, subject to $\mathbf{AX} = \mathbf{b}$, and $x \geq 0$. The results, due to the prior limitation of calculation means, were revealed without regarding of some variables with high variance constraint coefficients. Nowadays, because of a higher computational calculation performance, the development of the LP model when the system uncertainties taking in account can be written as [5]:

$$\begin{aligned} \text{Min} \quad & Z = c^T X + g^T U + h^T V \\ \text{subject to} \quad & AX + U - V = b \\ & x, u, v \geq 0 \end{aligned} \tag{a}$$

Where $c^T x$ represents the main cost and $g^T U + h^T V$ the additional corrective costs of materials supplied by subjecting to $AX + U - V = b$, where A is coefficient of the decision variable X (material quantity), U and V stand for the less and the excess mixed output quantities respectively. Awareness of nutrition values containing in U and V amounts play a big role on the right-hand side of equality constraint equations.

2.1 Problem Formulation

The problem has been formulated in form of a Stochastic Linear Programming model, with system uncertainties. Its objective function is to minimize the total cost denoted by z_{min} of various kinds of the input raw materials. Each minimal cost iteratively resolved is the product of the optimal input quantity and the determined cost coefficient of which. The sum of the initial amounts of $x_{j,\, j=1...n}$ and their sum of slack amount of $u_{i,\, i=1...m}$ and excessive amount of $v_{i,\, i=1...m}$. which have been collected from the feasible scenarios through all alternatives subject to the sum product of all those initial amounts of $x_{j,\, j=1...n}$ and their uncertain coefficients a_{ij}. Then, the slacks or surplus of x_j may be added or subtracted at the alternative equations, in order to make the equality to the right-hand side vector b_i. All of those terms have correlated with their individual uncertainties, so that they have generated the numerous stochastic parameters which have been solved with a traditional primal simplex method as the basement of comparison.

Let the coefficients a_{ij} be a denoted set which consist of elements $a_{\min_{ij}} + \Delta_{ij}$, $\Delta = \varepsilon_{ij}, 2\varepsilon_{ij}, ..., N(ij)\varepsilon_{ij}$ and b_i is a right-hand-side denoted set which consists of elements $b_{\min_i} + \delta_i$, $\delta_i = \varepsilon_i', 2\varepsilon_i', ..., M(i)\varepsilon_i'$. As a_{ijk}, b_{ik} a union set of a_{ij} and b_{ik} for all i and j.

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Hence:

$$\text{Minimize } z = \sum_{j=1}^{3} x_j + \sum_{i=1}^{m} \left[u_i + v_i \right] \tag{1}$$

$$\text{Subject to } \sum_{j=1}^{3} a_{ij} x_j + u_i - v_i = b_i \text{ , for all } i$$

$$x_j \text{ , } u_i \text{ , } v_i \geq 0 \tag{2}$$

a_{ijk}, b_{ik} is union set of a_{ij} *and* b_{ik} , $\forall i, j, k$

Where

$$a_{ij} = \left\{ a_{\min_{ij}} + \Delta_{ij} \text{ , } \Delta = \varepsilon_{ij}, 2\varepsilon_{ij}, \ldots, N(ij)\varepsilon_{ij} \right\} \tag{3}$$

$$b_i = \left\{ b_{\min_i} + \delta_i \text{ , } \delta_i = \varepsilon_i', 2\varepsilon_i', \ldots, M(i)\varepsilon_i' \right\} \tag{4}$$

Assumed four numerical sets:

a_{i1} = {1.0, 1.1, 1.2, 1.3… 2.0}
a_{i2} = {2.00, 2.01, 2.02, 2.03… 3.00}
a_{i3} = {3.000, 3.001, 3.002, 3.003… 4.000} and
b_i = {100.00,100.25,100.50,100.75, 101.00…120.00}

To verify these four models, the four numerical sets of real numbers and their resolution steps in decimal figures are assumed. The coefficient a_{ij} and the vector b_i are stepwise varied in tiny divisions. The finer the coefficient interval divided are, the more constraint alternatives and calculation scenario numbers are yielded.

2.2 Mathematical Transformation

Referred to (1), (2), (3), and (4) the models have been transformed as follows:

$$\sum_{j=1}^{3} x_j \rightarrow x_1 + x_2 + x_3 \tag{5}$$

$$\sum_{i=1}^{m} \left[u_i + v_i \right] \rightarrow \left[u_1 + v_1 \right] + \left[u_2 + v_2 \right] + \ldots + \left[u_m + v_m \right] \tag{6}$$

Subject to

$$\sum_{j=1}^{3} a_{ij} x_j + u_i - v_i \rightarrow \left[a_{i1} x_1 + a_{i2} x_2 + a_{i3} x_3 \right] + u_i - v_i = b_i \tag{7}$$

Where,

m = 11x101x1001x81 = 90,080,991

Hence;

$$\text{Min } z = x_1 + x_2 + x_3 + \left[u_1 + v_1\right] + \left[u_2 + v_2\right] + \ldots + \left[u_m + v_m\right] \tag{8}$$

$$\text{Subject to } \left[a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3\right] + u_i - v_i = b_i$$
$$x_j, \quad u_i, \quad v_i \quad \geq \quad 0 \tag{9}$$

Alternative consideration

$a_{i1} = 11, \quad a_{i2} = 101, \quad a_{i3} = 1001, \quad b_i = 81$

m = 11x101x1001x81 = 90,080,991 alternatives

2.3 Distribution over all constraints

Overall constraints are obtained from coefficients of x_j and slacks or surplus. The formulation will have size increase to be as follows

$$\begin{aligned}
&\left[a_{11}x_1 + a_{12}x_2 + a_{13}x_3\right] + u_1 - v_1 = b_1 \\
&\left[a_{21}x_1 + a_{22}x_2 + a_{23}x_3\right] + u_2 - v_2 = b_2 \\
&\left[a_{31}x_1 + a_{32}x_2 + a_{33}x_3\right] + u_3 - v_3 = b_3 \\
&. \qquad . \qquad . \qquad . \qquad . \\
&. \qquad . \qquad . \qquad . \qquad . \\
&\left[a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3\right] + u_m - v_m = b_m
\end{aligned} \tag{10}$$

2.4 Formulation of Objective Function

Added Slacks or subtracted surplus affects the objective function:

$$Min \quad z = x_1 + x_2 + x_3 + \left[u_1 + v_1\right] + \left[u_2 + v_2\right] + \ldots + \left[u_m + v_m\right] \tag{11}$$

Subject to;

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$$
\begin{array}{l}
[1.0x_1 + 2.00x_2 + 3.000x_3] + u_i - v_i = 100.00 \\
[1.0x_1 + 2.00x_2 + 3.001x_3] + u_i - v_i = 100.00 \\
[1.0x_1 + 2.00x_2 + 3.002x_3] + u_i - v_i = 100.00 \\
\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \\
\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \\
[1.0x_1 + 2.00x_2 + 4.000x_3] + u_{81} - v_{81} = 100.00 \\
\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
[1.0x_1 + 2.01x_2 + 3.000x_3] + u_{82} - v_{82} = 100.00 \\
[1.0x_1 + 2.01x_2 + 3.001x_3] + u_{83} - v_{83} = 100.00 \\
[1.0x_1 + 2.01x_2 + 3.002x_3] + u_{84} - v_{84} = 100.00 \\
\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow
\end{array}
\tag{12}
$$

Therefore, all Constraints can be written as

$$
\begin{array}{lll}
a_{i1} = \{1.0, 1.1, 1.2, ..., 2.0\} & 11 & \textit{Alternatives} \\
a_{i2} = \{2.00, 2.01, 2.02, ..., 3.00\} & 101 & \textit{Alternatives} \\
a_{i3} = \{3.000, 3.001, 3.003, ...4.000\} & 1001 & \textit{Alternatives} \\
b_i = \{100.00, 100.25, 100.50, ..., 120.00\} & 81 & \textit{Alternatives} \\
m = \{11 * 101 * 1001 * 81\} & 90,080,991 & \textit{Alternatives}
\end{array}
\tag{13}
$$

2.7 Matrix Systems A and b

The system is given as

A	Co-efficient matrix of x	Dimension:	(11x101x1001) x 3
b	RH-side constraint matrix	Dimension:	(81x11x101x1001) x 1

Matrix A

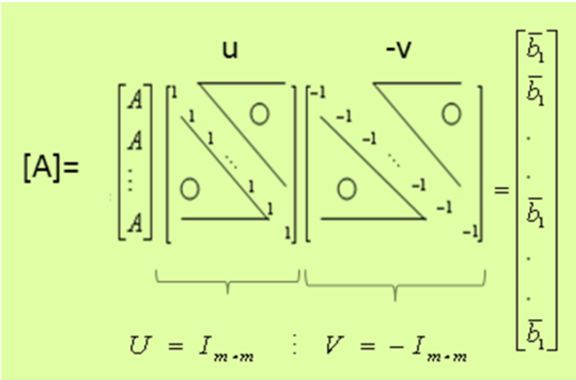

Figure 1: Set up of matrix system, matrix A

Matrix b and b_1

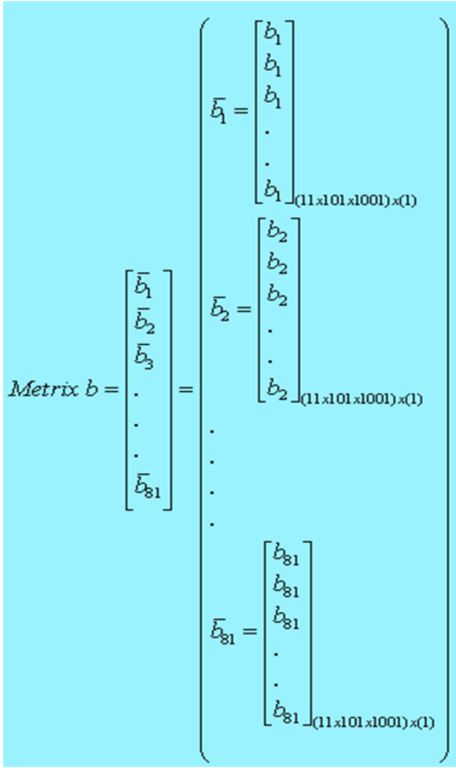

Figure 2: Set up of matrix b and b_1

Matrix b and b1 (cont.)

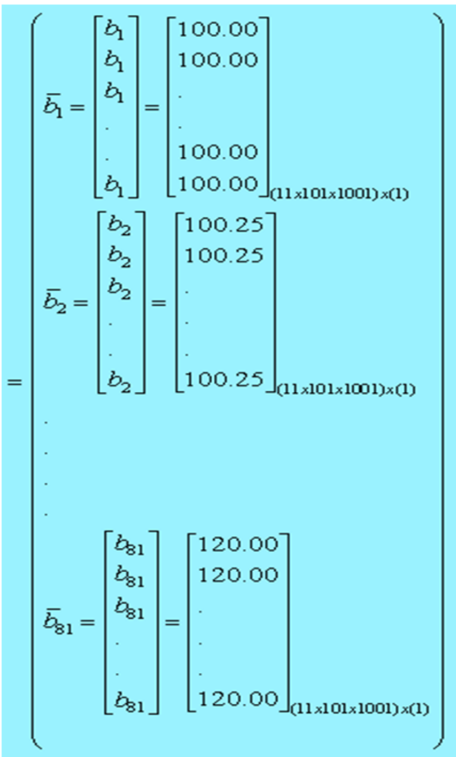

Figure 3: Set up of matrix b and b_1 (cont.)

Matrix X and f

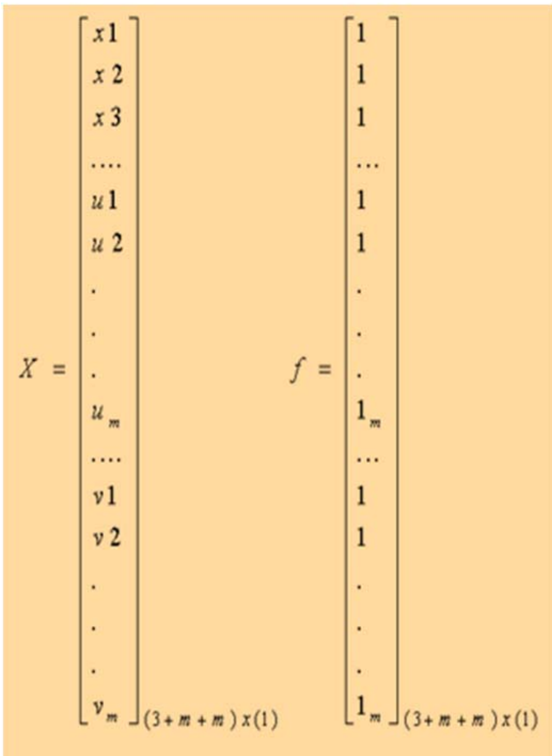

Figure 4: Matrix X and f system

2.8 Set up of Matrix X and f

After the assumed data are transformed into mathematical symbols, the problem is to setup in the matrix X and f systems, as shown in Figure 4:, available for program development with MATLAB programming.

Remarks:

f	= Matrix of raw materials costs of x_1, x_2, x_3	dimension: (3) x 1
f_{les}	= Matrix of raw materials costs of u_i (slack)	dimension: (m) x1
f_{exd}	= Matrix of raw materials costs of v_i (surplus)	dimension: (m) x1

Notation:

Herein, the cost factors of f = f_{les} = f_{exd} = 1 for temporary use at this developing phase of MATLAB Programming in general syntax.

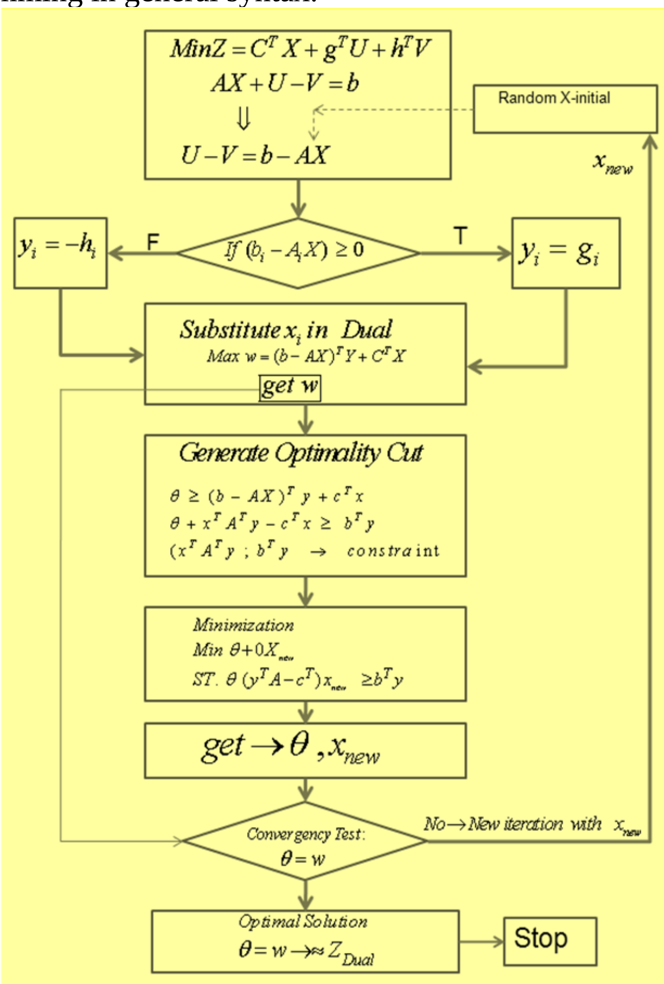

Figure 5: Flow chart of Algorithm according to the Bender's decomposition method.

2.9 Calculation Tool

The mathematical calculation tool has been applied: MATLAB Software / Version 2006a on the Hardware HP_Pavillion_IntelCore _2Qurd_Inside: Number 016-120610000 hardware.

At the Department of Industrial Engineering, Thammasat University, Pathumtani, Thailand.

2.10 Bender's Decomposition

The main concept of Bender's decomposition is to split the original problem into a master problem and a sub problem, which in turn decomposes into a series of independent sub problems, one for each $\omega \in \Omega$. The latter are used to generate cuts [5], [6], [8], [10], [14]. The master problem, the sub problems, and the cuts are on below diagram in Figure 5.

Explanation

1. Randomly select the X $_{\text{initial}}$ (in matrix form) to substitute in term of the inequality constraint equations
2. If the result (by substitution of X $_{\text{initial}}$) is equal to or greater than 0, then y_i = g
3. If the result (substitution of X $_{\text{initial}}$) is less than 0, then y_i = - h
4. From this point onwards, the values of *X* and *y* is known. Next, to find ω in term of *f (X, y)* and receive the result of ω to compare with θ
5. Optimality cut and builds up of constraints
6. by minimization θ + 0X $_{\text{new}}$ will get θ and X $_{\text{new}}$
7. Compare the value of θ and ω. If $\theta \neq \omega$, then take X $_{\text{new}}$ instead of X $_{\text{initial}}$ to substitute in the next iteration until:
8. $\theta = \omega$, Denotes the consequent of the matrix X $_{\text{new}}$ and the single value of θ and ω can be comparable with *Z* $_{\text{Dual}}$

The authors applied the method of Bender's decomposition to address this problem. The values of $X_{(Bender)}$, $Z_{(Bender)}$ and $T_{(Bender)}$, are represented in graphical diagrams for comparison with the results from the Primal, Prima$_{\text{(random)}}$, and Dual methods.

3. Results and Discussion

Prior to the trial of this large-scale problem with m = 90,080,991 alternatives was solved with the application of the ordinary primal simplex method, any such calculation was time intensive. It is noteworthy that the preparation of the matrix system, it consumed approximately seven hours with an actual calculation time of about 40 hours (144000 seconds) on 30 GB of RAM. However, no solution was defined. The x $_{\text{initial}}$ inputs were subsequently randomized for iteration and only the solutions within particular feasible intervals were gathered. As for the elapsed time, such an approach required interval of [0.09344, 7.15680] seconds. With the subsequent solution with the dual simplex to attain the maximal dual solution instead of the

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primal was attempted. The results were revealed to be almost identical with the exception of the z_{min} to be more exact than the randomized means. However, both results were failed to approximate the optimal solution expected as shown in Table 1.

Table 1: The summarized calculation results of 3 solution methodologies.

Methodology	x1 [weight]	x2 [weight]	x3 [weight]	z_min [currency]	Cal.Time [sec.]
Primal	N/A	N/A	N/A	N/A	144,000
Primal(random)	0	0	$\in$ [29.35457, 29.9253]	$\in$ [37.39135,39.45131]	$\in$ [0.09344, 7.15680]
Dual	0	0	$\in$ [29.3545, 29.9253]	$z_{max\,(Dual)} = 39.5608$	5,640
Bender's Decom.	x1 = 0	x2 = 6.6883	x3 = 1.0187	$z_{min} = 36.5152$	46.511729

At last, the problem hard to be solved by means of the method of Bender's decomposition with computation with the application of the MATLAB program (Appendix) with the subsequent inclusion of the optimal results. Those results were found to be satisfactory, especially since they can be obtained with a small sample size [11].

As the conclusion of the research, the upper bound of the feasible region was gradually modified. By increases of the values of the upper bound (UB) between 100,000 to 100,000,000, while the lower bound (LB) = zero.

The optimal solutions are represented for each calculating scenario in Table 2. The final results for the selected amount of x1 = 0.000000, x2 = 6.688300, x3 = 1.018700 in unit weight, Z min = 36.515200 unit currency with the errors (aerr) of -0.179900 and a calculation time in seconds of [44.645367,46.511729]. The calculation time was found to be dependent upon the number of the upper bound, the higher upper bound, the more calculation time, however without any effects on the optimal xs, x1, x2, x3 and Z_min values.

Thus, the calculation with the application of the Bender's decomposition method can solve the problem more precisely and effectively, and is thus suited to address the feed-mix problem. Furthermore, such an approach does not require the availability of a very high performance computer.

Table 2: The calculation results from Bender's decomposition method.
Note: LB = zeros (n, 1)

UB=100,000

xs	x1	x2	x3	z_min	ub	aerr	Cal_time
0	100.7852	100.2912	-8.485	0	302.5913	302.5913	6.397752
0	4.6168	0.4907	2.2638	0		302.5913	12.775251
20.1997	0	11.8475	0.3599	20.1997	70.8543	50.6546	19.148775
26.9801	0	5.2224	0.994	26.9801	37.8536	10.8735	25.51115
34.7749	0	8.4171	1.0075	34.7749		3.0787	31.88439
35.4315	0	8.5688	0.7372	35.4315		2.4221	38.260048
36.5152	**0**	**6.6883**	**1.0187**	**36.5152**	**36.3354**	**-0.1799**	**44.645367**

UB=500,000

xs	x1	x2	x3	z_min	ub	aerr	Cal_time
0	100.7852	100.2912	-8.485	0	302.5913	302.5913	6.428753
0	4.6168	0.4907	2.2638	0		302.5913	12.903343
20.1997	0	11.8475	0.3599	20.1997	70.8543	50.6546	19.355393
26.9801	0	5.2224	0.994	26.9801	37.8536	10.8735	25.786211
34.7749	0	8.4171	1.0075	34.7749		3.0787	32.234832
35.4315	0	8.5688	0.7372	35.4315		2.4221	38.682433
36.5152	**0**	**6.6883**	**1.0187**	**36.5152**	**36.3354**	**-0.1799**	**45.134651**

UB=1,000,000

xs	x1	x2	x3	z_min	ub	aerr	Cal_time
0	100.7852	100.2912	-8.485	0	302.5913	302.5913	6.391282
0	4.6168	0.4907	2.2638	0		302.5913	12.762113
20.1997	0	11.8475	0.3599	20.1997	70.8543	50.6546	19.122427
26.9801	0	5.2224	0.994	26.9801	37.8536	10.8735	25.47826
34.7749	0	8.4171	1.0075	34.7749		3.0787	31.86641
35.4315	0	8.5688	0.7372	35.4315		2.4221	38.27182
36.5152	**0**	**6.6883**	**1.0187**	**36.5152**	**36.3354**	**-0.1799**	**44.671446**

UB=5,000,000

xs	x1	x2	x3	z_min	ub	aerr	Cal_time
0	100.7852	100.2912	-8.485	0	302.5913	302.5913	6.43265
0	4.6168	0.4907	2.2638	0		302.5913	12.891945
20.1997	0	11.8475	0.3599	20.1997	70.8543	50.6546	19.319871
26.9801	0	5.2224	0.994	26.9801	37.8536	10.8735	25.758732
34.7749	0	8.4171	1.0075	34.7749		3.0787	32.197715
35.4315	0	8.5688	0.7372	35.4315		2.4221	38.663319
36.5152	**0**	**6.6883**	**1.0187**	**36.5152**	**36.3354**	**-0.1799**	**45.102412**

UB=10,000,000

xs	x1	x2	x3	z_min	ub	aerr	Cal_time
0	100.7852	100.2912	-8.485	0	302.5913	302.5913	6.380424
0	4.6168	0.4907	2.2638	0		302.5913	12.75361
20.1997	0	11.8475	0.3599	20.1997	70.8543	50.6546	19.12451
26.9801	0	5.2224	0.994	26.9801	37.8536	10.8735	25.496045
34.7749	0	8.4171	1.0075	34.7749		3.0787	31.869051
35.4315	0	8.5688	0.7372	35.4315		2.4221	38.239459
36.5152	**0**	**6.6883**	**1.0187**	**36.5152**	**36.3354**	**-0.1799**	**44.627897**

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Table2: (Continued)

UB=50,000,000

xs	x1	x2	x3	z_min	ub	aerr	Cal_time
0	100.7852	100.2912	-8.485	0	302.5913	302.5913	6.440356
0	4.6168	0.4907	2.2638	0		302.5913	12.911864
20.1997	0	11.8475	0.3599	20.1997	70.8543	50.6546	19.358927
26.9801	0	5.2224	0.994	26.9801	37.8536	10.8735	25.798323
34.7749	0	8.4171	1.0075	34.7749		3.0787	32.251175
35.4315	0	8.5688	0.7372	35.4315		2.4221	38.699808
36.5152	**0**	**6.6883**	**1.0187**	**36.5152**	**36.3354**	**-0.1799**	**45.134729**

UB=100,000,000

xs	x1	x2	x3	z_min	ub	aerr	Cal_time
0	100.7852	100.2912	-8.485	0	302.5913	302.5913	6.627243
0	4.6168	0.4907	2.2638	0		302.5913	13.259088
20.1997	0	11.8475	0.3599	20.1997	70.8543	50.6546	19.928218
26.9801	0	5.2224	0.994	26.9801	37.8536	10.8735	26.570579
34.7749	0	8.4171	1.0075	34.7749		3.0787	33.220471
35.4315	0	8.5688	0.7372	35.4315		2.4221	39.872046
36.5152	**0**	**6.6883**	**1.0187**	**36.5152**	**36.3354**	**-0.1799**	**46.511729**

These following diagrams are illustrated with the identical values. As shown in Figure 6 the results comparison between errors (set 1 to set 7), whereas in Figure 7 between the calculation times, (Cal.Tim sets 1-7). In Figure 8 and Figure 9 are represented the optimal values of x2 and x3 also for 7 sets. In addition, Figure 10 the optimal value of z, 7 set. The optimal solution can be ensured by convergence of the result data. It is noteworthy that the errors of all seven sets are represented error figures of-0.179900 as shown in Figure 6 with a probability plot of all seven sets of errors, as shown in Figure 11.

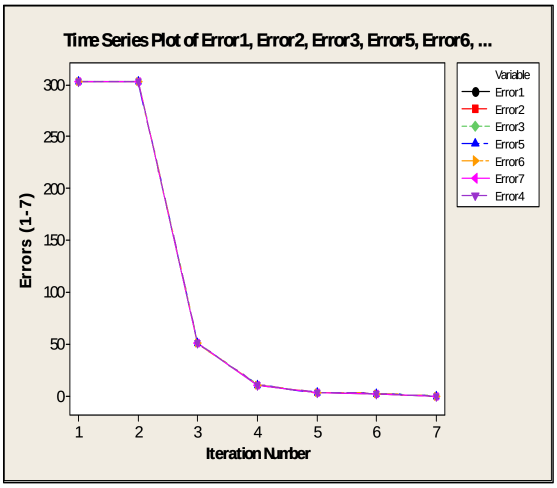

Figure 6: Error Plot (Set1-7)

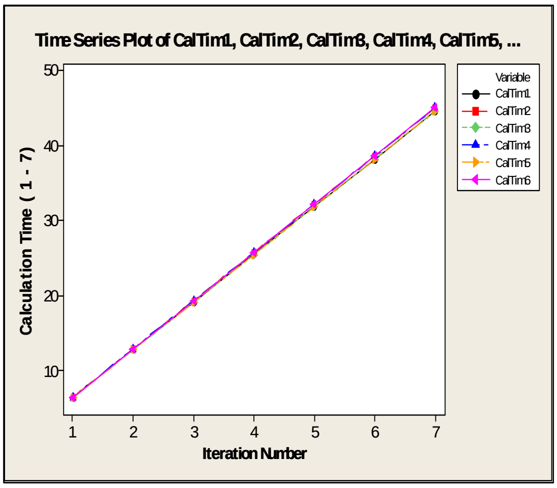

Figure 7: CalTime (Set 1-7)

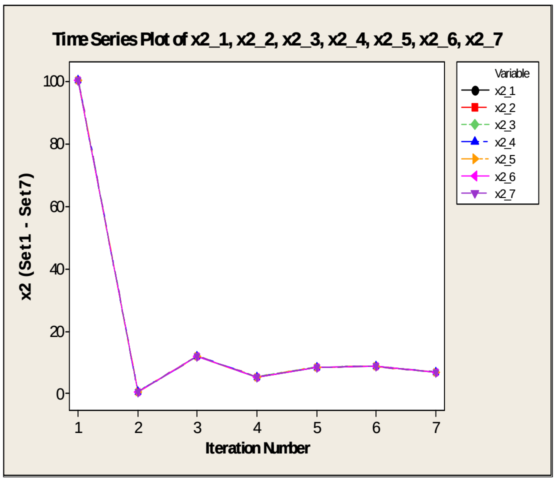

Figure 8: Value x2 (Set1-7)

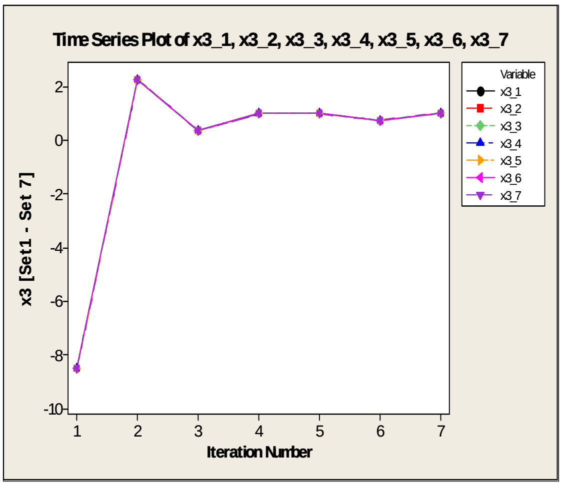

Figure 9: Value x3 (Set1-7)

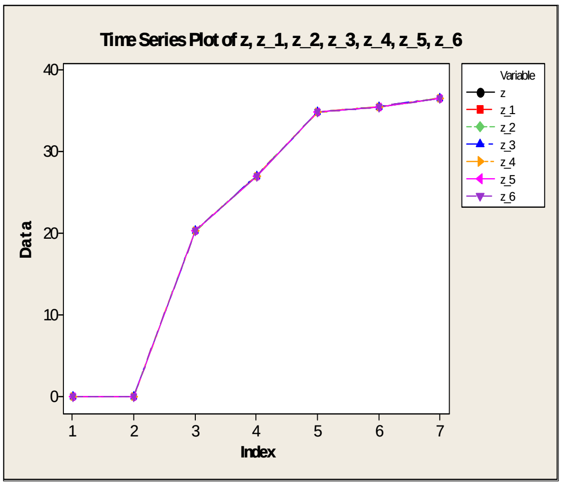

Figure 10: Value z (Set1-7)

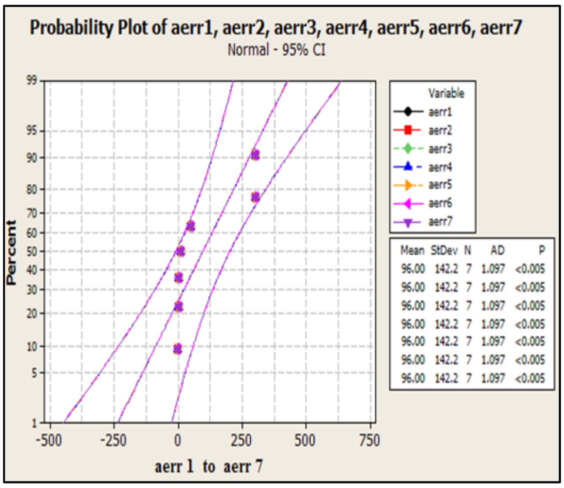

Figure 11: Probability plot of all a_{err} sets

4. Conclusion

There are two noticeable criterions for the summary. First, the Bender's decomposition method incorporation with a large-scale stochastic linear programming developed in a MATLAB program for computational calculation can produce great solution for numerous extending constraints variables. Second, through the Method of Bender's decomposition, the problem can be solved very efficiently and the nearest optimal solution can be obtained in a short period of time in comparison to the primal simplex method as the summary in Table 1**.** The plotted diagrams clearly indicate that all errors located are within an acceptable probability interval. This assures that the results can be converged to the optimum solution.

Therefore, this proposed technique, the Method of Bender's Decomposition incorporated with TSL will be well-suited to such a large-scale problem, especially for the feed mix problems. We plan to apply this methodology to solve the mixing problems in some other related fields.

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7. Appendix: MATLAB Programming

```
clear;
tic
n1 = 11;
a1 = zeros(n1,1);
for i=1:n1
    a1(i) = 1+0.1*(i-1);
end
n2 = 101;
a2 = zeros(n2,1);
for i=1:n2
    a2(i) = 2+0.1*(i-1);
end
n3 = 1001;
a3 = zeros(n3,1);
for i=1:n3
    a3(i) = 3+0.1*(i-1);
end
nb = 81;
bi = zeros(nb,1);
for i=1:nb
    bi(i) = 100+0.25*(i-1);
end
nalt = n1*n2*n3*nb;
n = 3;
x = [0 0 0]';
c = [1 0 0 0]';
icons = 0;
aerr = 1;
ub =100000;
% increase the UB=0 to 100,000,000 for observing the results.
while aerr>0.001
cut_coeff = zeros(n,1);
cut_rhs = 0;
uvcost = 0;
for i1=1:n1
    for i2=1:n2
```

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```
            for i3=1:n3
                for i4=1:nb
                    coeff_obj = bi(i4)-(a1(i1)*x(1)+a2(i2)*x(2)+a3(i3)*x(3));
                    if coeff_obj > 0
                        y = 1/nalt;
                    else
                        y = -1/nalt;
                    end
                    cut_rhs = cut_rhs + y*bi(i4);
                    uvcost = uvcost + y*coeff_obj;
                    cut_coeff(1) = cut_coeff(1) + y*a1(i1);
                    cut_coeff(2) = cut_coeff(2) + y*a2(i2);
                    cut_coeff(3) = cut_coeff(3) + y*a3(i3);
                end
            end
        end
    end
    icons = icons+1;
    cutcons(icons,1) = -1;
    for j=1:n
        cutcons(icons,j+1) = 1-cut_coeff(j);
    end
    lb = zeros(n,1);
    % Occurrence of aerr =-3.1048
    % and   Elapsed time is 63.054255 sec.
    rhs(icons) = -cut_rhs;
    [xs,z]= linprog(c,cutcons,rhs',[],[],lb,[])
    for i=1:n
        x(i) = xs(i+1);
    end
    if x(1)+x(2)+x(3)+ uvcost < ub
        ub = x(1)+x(2)+x(3)+ uvcost
    end
    aerr = ub-z
```

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