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An Efficient Formulation of Off-line Model Predictive Control for Nonlinear Systems Using Polyhedral Invariant Sets

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ABSTRACT

In this research, an efficient formulation of off-line model predictive control for nonlinear systems is presented. The nonlinear systems are reformulated as linear parameter varying systems so their complexity is reduced without any loss of generality. The on-line computational burdens are decreased by pre-computing off-line the sequences of explicit control laws corresponding to the sequences of polyhedral invariant sets. At each sampling time, the current state and the scheduling parameter are measured. The real-time control law is then calculated by linear interpolation between the pre-computed control laws. The results indicate that the proposed algorithm can achieve better control performance compared to the previously developed off-line robust model predictive control algorithm because the scheduling parameter is incorporated into the controller design.

1. Introduction

Chemical processes are multivariable processes that change one or more chemical compounds to the desired products. Chemical processes are usually involved with many complex chemical reactions which are nonlinear. In order to efficiently control nonlinear chemical processes, a multivariable nonlinear control algorithm needs to be developed (Qin and Badgwell, 2003; Ramesh, *et al.*, 2009; Manenti, 2011).

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Model predictive control (MPC) is an advanced control algorithm for multivariable processes. MPC is widely used in many chemical processes because input and output constraints are considered in a systematic manner (Morari and Lee, 1999; Mayne, *et al.*, 2000; Lee, 2011). A linear model is typically used in MPC formulation because the on-line optimization problem can be easily solved. However, most of the chemical processes are nonlinear. When the operating conditions undergo changes, the performance of linear MPC can significantly deteriorate (Bumroongsri and Kheawhom, 2012a; Yu, *et al.*, 2012; Suzuki and Sugie, 2007).

In order to deal with nonlinear chemical processes, nonlinear MPC was developed by Magni, *et al.* (2001). A full nonlinear model was used in MPC formulation. The complicated nonlinear control problem had to be solved at each sampling instant so the algorithm was computationally prohibitive in practical situations.

The reformulation of nonlinear systems into linear parameter varying (LPV) systems is a promising technique to reduce the complexity of nonlinear systems. LPV systems are linear systems whose dynamics depend on time-varying parameters that can be measured on-line. Therefore, nonlinear systems can be reformulated as LPV systems without any loss of generality (Park and Jeong, 2004; Toth, 2010; Jungers, *et al.*, 2011). Off-line MPC for LPV systems was previously developed by Bumroongsri and Kheawhom (2012b). Although the on-line computational time was significantly reduced, the stabilizable region of the algorithm was small because the ellipsoidal invariant sets were used in problem formulation.

In order to enlarge the size of stabilizable region, an off-line formulation of robust MPC using polyhedral invariant sets was proposed by Bumroongsri and Kheawhom (2012c). The polyhedral invariant sets were used in the problem formulation so a significantly larger stabilizable region was obtained. Although the stabilizable region was enlarged, the conservativeness was obtained because the scheduling parameter was not incorporated into the controller design.

In this research, an efficient formulation of off-line MPC using polyhedral invariant sets is presented. The sequences of explicit control laws corresponding to the sequences of polyhedral invariant sets are pre-computed off-line. At each sampling instant, the current state and the scheduling parameter are measured. The real-time control law is then calculated by linear interpolation between the pre-computed control laws. The proposed algorithm can give a relatively large stabilizable region because the polyhedral invariant sets are computed in the off-line problem formulation. Moreover, the scheduling parameter is used in real-time interpolation between the pre-computed control laws so better control performance compared to an off-line robust MPC algorithm of Bumroongsri and Kheawhom (2012c) can be obtained.

This article is organized as follows. In section 2, the problem description is presented. The proposed algorithm is presented in section 3. In section 4, the proposed algorithm is applied to a case study and the results are discussed. Finally, the paper is concluded in section 5.

2. Problem Description

The model considered here is the following linear parameter varying (LPV) systems (The techniques to transform nonlinear systems into LPV systems can be found in Toth (2010).)

$$x(k+1) = A(p(k))x(k) + Bu(k) \qquad (1),$$

$$y(k) = Cx(k) \qquad (2),$$

where $x(k)$ is a vector of states, $u(k)$ is a vector of control inputs and $y(k)$ is a vector of outputs. In this research, we assume that the scheduling parameters $p(k) = [p_1(k), p_2(k),..., p_L(k)]$ are measurable on-line at each sampling instant. Moreover, we assume that

$$A(p(k)) \in \Omega, \Omega = Co\{A_1, A_2,..., A_L\} \qquad (3),$$

where Ω is the polytope, Co is the convex hull, A_j are the vertices of the convex hull and L is the number of vertices of Ω. Any $A(p(k))$ within Ω is a linear combination of the vertices such that

$$A(p(k)) = \sum_{j=1}^{L} p_j(k)A_j, \sum_{j=1}^{L} p_j(k) = 1, 0 \le p_j(k) \le 1 \qquad (4),$$

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The objective is to find a state feedback control law that stabilizes LPV systems (1) and (2) subject to the following input and output constraints

$$u_{h,\min} \le u_h(k+i/k) \le u_{h,\max},\ h = 1,2,....,n_u,\ i = 0,1,2,...,\infty \qquad (5),$$

$$y_{r,\min} \le y_r(k+i/k) \le y_{r,\max},\ r = 1,2,....,n_y,\ i = 0,1,2,...,\infty \qquad (6),$$

where n_u is the number of control inputs, n_y is the number of outputs, $u_{\min}$ and $u_{\max}$ are the vectors of input constraints, $y_{\min}$ and $y_{\max}$ are the vectors of output constraints.

3. The Proposed Off-line MPC Algorithm

In this section, an off-line MPC formulation for nonlinear systems is developed. The nonlinear systems are reformulated as LPV systems so their complexity is significantly reduced. Most of the solutions of the control problem are calculated off-line so the on-line computational time is significantly reduced.

3.1 Off-line Procedures

3.1.1 Off-line Step 1: Compute the Sequences of Off-line State Feedback Gains

Choose a sequence of states $x_i, i \in \{1,2,...,N\}$ and solve the optimization problem presented by Bumroongsri and Kheawhom (2012b) off-line to obtain the sequences of state feedback gains $K_{i,j}, \forall i = 1,2,....,N, \forall j = 1,2,....,L$ where N is the number of the chosen states and L is the number of vertices of Ω.

3.1.2 Off-line Step 2: Compute the Sequences of Polyhedral Invariant Sets

Given the state feedback gains $K_{i,j}$ from 3.1.1. For each $K_{i,j}$, the corresponding polyhedral invariant set $S_{i,j} = \{x / M_{i,j} x \le d_{i,j}\}$ is computed by following these steps

(1) Set $M_{i,j} = [C^T, -C^T, K_{i,j}^T, -K_{i,j}^T]^T$, $d_{i,j} = [y_{\max}^T, y_{\min}^T, u_{\max}^T, u_{\min}^T]^T$ and $m = 1$.

(2) Select row m from $(M_{i,j}, d_{i,j})$ and check $\forall l,\ l = 1,...,L$ whether $M_{i,j,m}(A_l + B_l K_{i,j})x \le d_{i,j,m}$ is redundant with respect to $(M_{i,j}, d_{i,j})$ by solving the following problem

$$\max \; W_{i,j,m,l} \quad (7),$$

s.t. $$W_{i,j,m,l} = M_{i,j,m}(A_l + B_l K_{i,j})x - d_{i,j,m} \quad (8),$$

$$M_{i,j} x \le d_{i,j} \quad (9),$$

If $W_{i,j,m,l} > 0$, the constraint $M_{i,j,m}(A_l + B_l K_{i,j})x \le d_{i,j,m}$ is non-redundant with respect to $(M_{i,j}, d_{i,j})$. Then, add non-redundant constraints to $(M_{i,j}, d_{i,j})$ by assigning $M_{i,j} = [M_{i,j}^T, (M_{i,j,m}(A_l + B_l K_{i,j}))^T]^T$ and $d_{i,j} = [d_{i,j}^T, d_{i,j,m}^T]^T$.

(3) Let $m = m+1$ and return to step (2). If m is strictly larger than the number of rows in $(M_{i,j}, d_{i,j})$ then terminate.

3.2 On-line Procedures

At each sampling time, measure the current state $x(k)$ and the scheduling parameter $p(k)$. When $x(k) \in S_{i,j}$, $x(k) \notin S_{i+1,j}$, $\forall j = 1,2,\ldots,L$, $i \ne N$, the real-time state feedback gain $K(\alpha(k)) = \alpha(k)(\sum_{j=1}^{L} p_j(k) K_{i,j}) + (1-\alpha(k))(\sum_{j=1}^{L} p_j(k) K_{i+1,j})$ can be calculated from $\alpha(k)$ obtained by solving the following optimization problem

$$\min \; \alpha(k) \quad (10),$$

s.t. $$u_{\min} \le K(\alpha(k))x(k) \le u_{\max} \quad (11),$$

$$A(p(k))x(k) + BK(\alpha(k))x(k) \in S_{i,j}, \forall j = 1,2,\ldots,L \quad (12),$$

$$0 < \alpha(k) \le 1 \quad (13),$$

It is seen that the on-line optimization problem is only linear programming so it can be efficiently solved (Boyd and Vandenberghe, 2004). (11) is for guaranteeing input constraint satisfaction and (12) is for guaranteeing that the next predicted state still lies in the polyhedral

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invariant sets computed off-line.

4. Results and Discussion

Consider the nonlinear two-tank system (Angeli, *et al.*, 2000) which is described by the following equation

$$\rho S_1 \dot{h}_1 = -\rho A_1 \sqrt{2gh_1} + u \quad (14),$$

$$\rho S_2 \dot{h}_2 = \rho A_1 \sqrt{2gh_1} - \rho A_2 \sqrt{2gh_2} \quad (15),$$

where h_1 is the water level in tank 1, h_2 is the water level in tank 2 and u is the inlet water flow. The schematic diagram of the nonlinear two-tank system is shown in Figure 1.

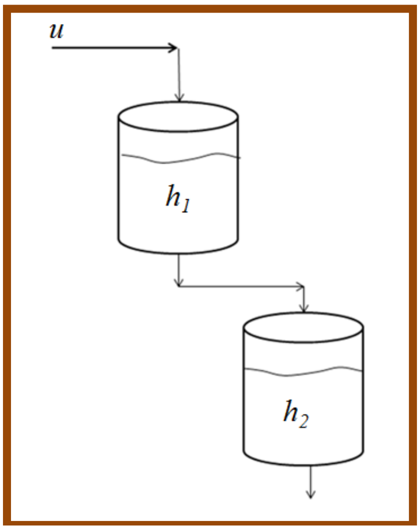


Figure 1: The schematic diagram of the nonlinear two-tank system.

The operating parameters of the nonlinear two-tank system are shown in Table 1.

Table 1: The operating parameters of the nonlinear two-tank system.

Parameter	Value	Unit
S_1	2,500	cm^2
S_2	1,600	cm^2
A_1	9	cm^2
A_2	4	cm^2
g	980	cm/s^2
ρ	0.001	kg/cm^3
$h_{1,eq}$	14	cm
$h_{2,eq}$	70	cm

Let $\overline{h_1} = h_1 - h_{1,eq}$, $\overline{h_2} = h_2 - h_{2,eq}$ and $\overline{u} = u - u_{eq}$ where subscript eq is used to denote the corresponding variable at equilibrium condition, the objective is to regulate $\overline{h_2}$ to the origin by manipulating $\overline{u}$. The input and output constraints are given as follows

$$\left|\overline{u}\right| \le 1.5 \text{ kg/s}, \ \left|\overline{h_1}\right| \le 13 \text{ cm}, \ \left|\overline{h_2}\right| \le 50 \text{ cm} \qquad (16),$$

By evaluating the Jacobian matrix of (14) and (15) along the vertices of the constraints set (16), we have that all the solutions of (14) and (15) are also the solutions of the following differential inclusion

$$\begin{bmatrix} \rho S_1 \dot{\overline{h}}_1 \\ \rho S_2 \dot{\overline{h}}_2 \end{bmatrix} \in \left(\sum_{j=1}^{4} p_j A_j \right) \begin{bmatrix} \overline{h}_1 \\ \overline{h}_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \overline{u} \qquad (17),$$

where $A_j, j = 1,\ldots,4$ are given by

$$A_1 = \begin{bmatrix} -\rho A_1 \sqrt{\dfrac{2g}{h_{1,\min}}} & 0 \\ \rho A_1 \sqrt{\dfrac{2g}{h_{1,\min}}} & -\rho A_2 \sqrt{\dfrac{2g}{h_{2,\min}}} \end{bmatrix}, \ A_2 = \begin{bmatrix} -\rho A_1 \sqrt{\dfrac{2g}{h_{1,\max}}} & 0 \\ \rho A_1 \sqrt{\dfrac{2g}{h_{1,\max}}} & -\rho A_2 \sqrt{\dfrac{2g}{h_{2,\min}}} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -\rho A_1 \sqrt{\dfrac{2g}{h_{1,\min}}} & 0 \\ \rho A_1 \sqrt{\dfrac{2g}{h_{1,\min}}} & -\rho A_2 \sqrt{\dfrac{2g}{h_{2,\max}}} \end{bmatrix}, \ A_4 = \begin{bmatrix} -\rho A_1 \sqrt{\dfrac{2g}{h_{1,\max}}} & 0 \\ \rho A_1 \sqrt{\dfrac{2g}{h_{1,\max}}} & -\rho A_2 \sqrt{\dfrac{2g}{h_{2,\max}}} \end{bmatrix} \qquad (18),$$

and $p_j, j = 1,\ldots,4$ are given by

$$p_1 = \left(\frac{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_1})}{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_{1,\min}})} \right) \cdot \left(\frac{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_2})}{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_{2,\min}})} \right)$$

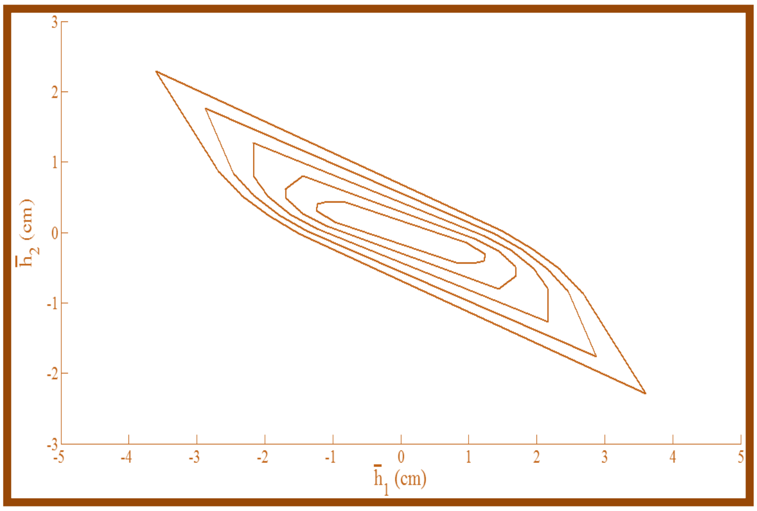
$$p_2 = \left(\frac{(1/\sqrt{h_1}) - (1/\sqrt{h_{1,\min}})}{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_{1,\min}})}\right) \cdot \left(\frac{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_2})}{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_{2,\min}})}\right)$$

$$p_3 = \left(\frac{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_1})}{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_{1,\min}})}\right) \cdot \left(\frac{(1/\sqrt{h_2}) - (1/\sqrt{h_{2,\min}})}{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_{2,\min}})}\right)$$

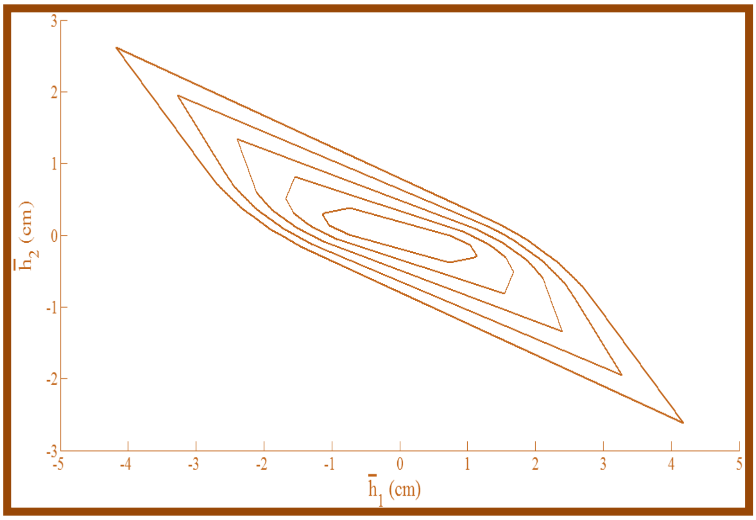
$$p_4 = \left(\frac{(1/\sqrt{h_1}) - (1/\sqrt{h_{1,\min}})}{(1/\sqrt{h_{1,\max}}) - (1/\sqrt{h_{1,\min}})}\right) \cdot \left(\frac{(1/\sqrt{h_2}) - (1/\sqrt{h_{2,\min}})}{(1/\sqrt{h_{2,\max}}) - (1/\sqrt{h_{2,\min}})}\right) \qquad (19),$$

The discrete-time model is obtained by discretization of (17) using Euler first-order approximation (Seborg, *et al.*, 2004) with a sampling period of 0.5 s and it is omitted here for brevity.

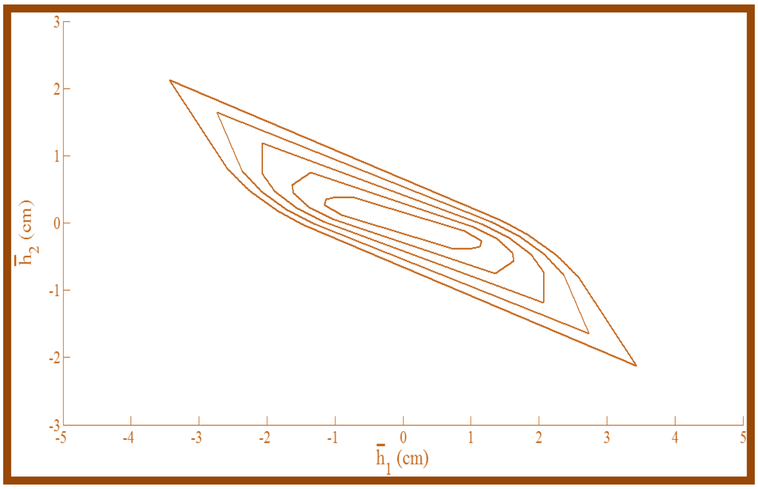
Figure 2 shows the polyhedral invariant sets computed off-line by the proposed algorithm. Figure 3 shows the polyhedral invariant sets computed off-line by an off-line robust MPC algorithm of Bumroongsri and Kheawhom (2012c). For both algorithms, the polyhedral invariant sets are computed by choosing the same sequence of states $x_i, i \in \{1,2,\ldots,5\}$. Note that with the same number of chosen states, the proposed algorithm requires larger number of polyhedral invariant sets than an off-line robust MPC algorithm of Bumroongsri and Kheawhom (2012c). This is due to the fact that for the proposed algorithm, the number of sequences of polyhedral invariant sets computed is equal to the number of the vertices of the polytope Ω. In comparison, only a sequence of polyhedral invariant sets needs to be computed off-line for an off-line robust MPC algorithm of Bumroongsri and Kheawhom (2012c).



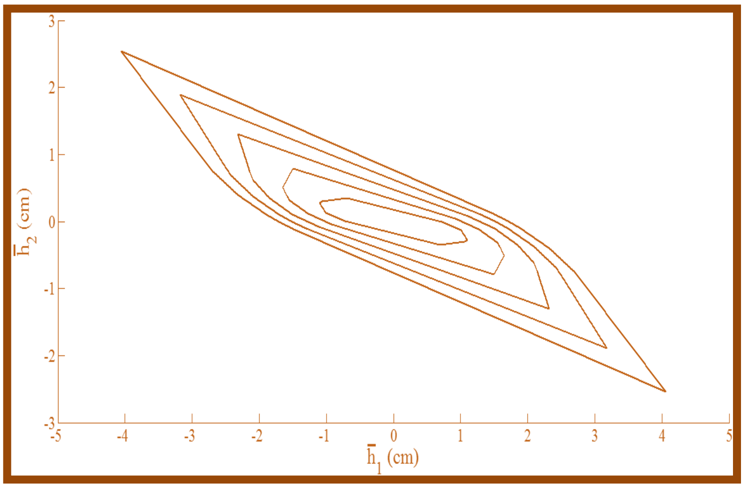
(2.1) $S_{i,1}, i \in \{1,2,\ldots,5\}$



(2.2) $S_{i,2}, i \in \{1,2,\ldots,5\}$



(2.3) $S_{i,3}, i \in \{1,2,\ldots,5\}$



(2.4) $S_{i,4}, i \in \{1,2,\ldots,5\}$

Figure 2: The polyhedral invariant sets computed off-line by the proposed algorithm.

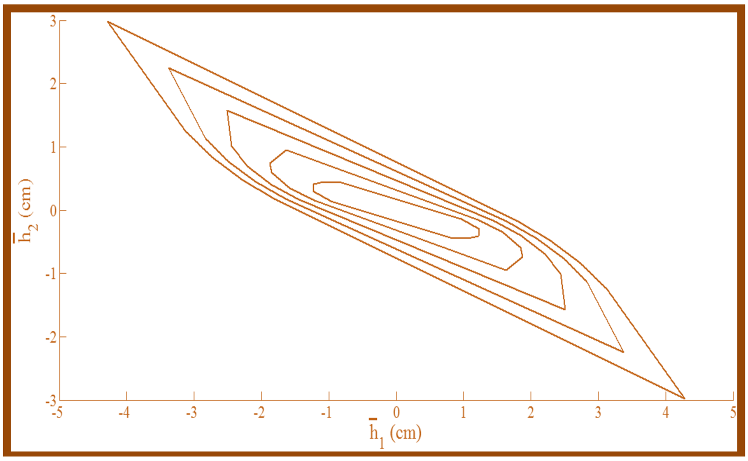


Figure 3: The polyhedral invariant sets computed off-line by an off-line robust MPC algorithm of Bumroongsri and Kheawhom (2012c).

Figure 4 shows the regulated output. For the proposed algorithm, the scheduling parameter is measured on-line at each sampling time so less conservativeness compared to an off-line robust MPC algorithm of Bumroongsri and Kheawhom (2012c) can be obtained. It can be observed that the proposed algorithm requires less time to enter and remain within the settling band ($\pm 0.1\%$ of $h_{2,\text{equilibrium}}$) compared to an off-line robust MPC algorithm of Bumroongsri and Kheawhom (2012c).

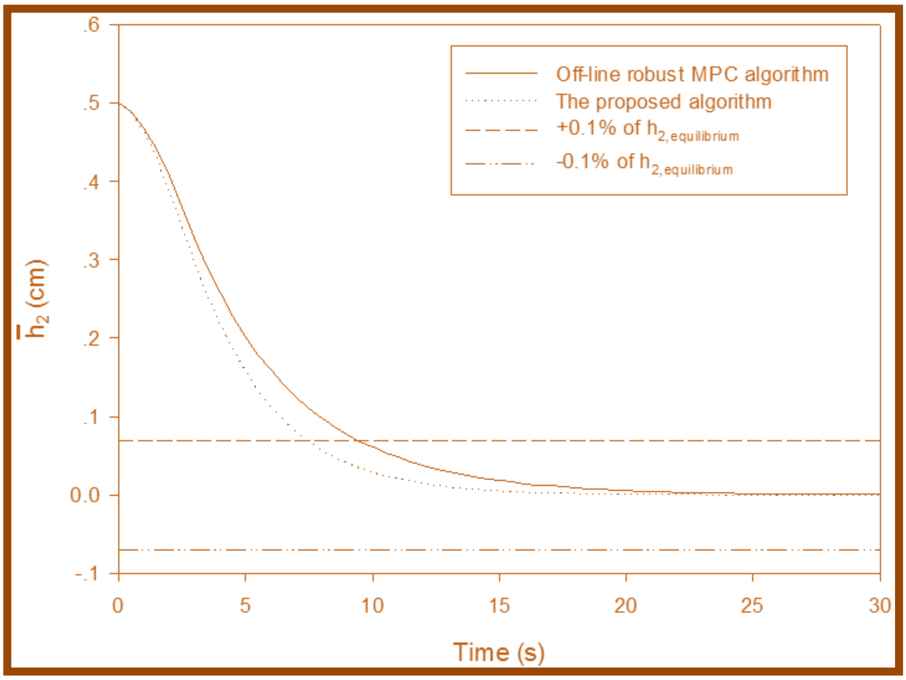


Figure 4: The regulated output.

The control input is shown in Figure 5. For the proposed algorithm, the pre-computed state feedback gains are interpolated on-line so a smoother input response is obtained.

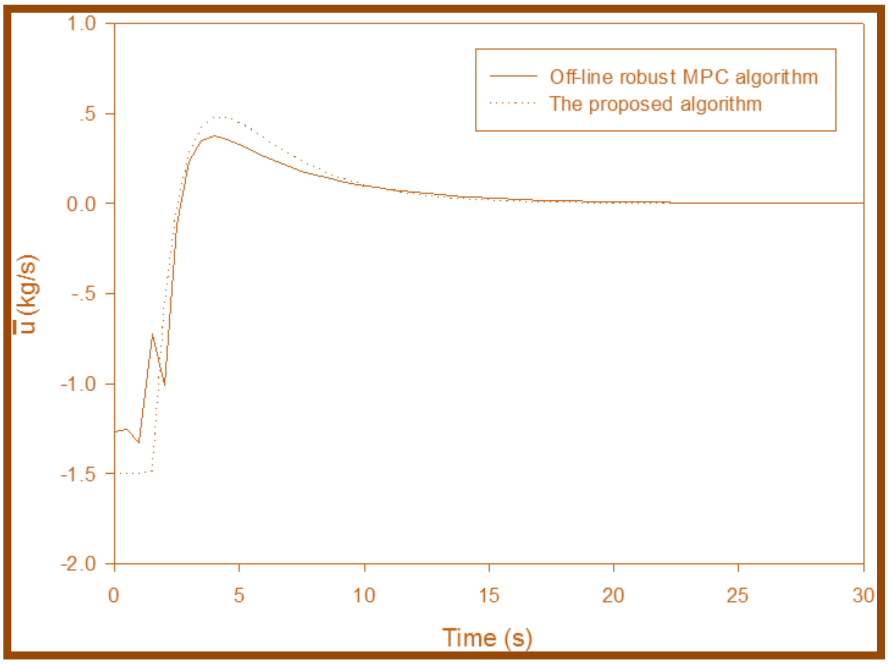


Figure 5: The control input.

The overall computational burdens are shown in Table 2. Although the proposed algorithm requires larger off-line computational time than an off-line robust MPC algorithm of Bumroongsri and Kheawhom (2012c), the on-line computation is tractable because only linear programming needs to be solved on-line. All of the simulations have been performed in Intel Core i-5 (2.4GHz), 2 GB RAM, using SeDuMi (Sturm, 1999) and Yalmip (Löfberg, 2012) within Matlab 2008a environment.

Table 2: The overall computational burdens.

Algorithm	Off-line CPU time (s)	On-line CPU time (s)
An off-line robust MPC algorithm	3.612	-
The proposed algorithm	6.738	0.001

5. Conclusion

In this research, an efficient formulation of off-line MPC for nonlinear systems using polyhedral invariant sets has been developed. The results show that the proposed algorithm can give better control performance than the previously developed off-line robust MPC algorithm. This is due to the fact the scheduling parameter is incorporated into the controller design. The controller design is illustrated with an example of nonlinear two-tank system.

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