

Coding and Classification Based Heuristic Technique for Workpiece Grouping Problems in Cellular Manufacturing System

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ABSTRACT

Part family detection problem is an NP-complete problem in the vicinity of Cellular Manufacturing System (CMS). In the past literature several part family detection techniques have been proposed by researchers which are ordinarily grounded on Production Flow Analysis (PFA). Coding and Classification (CC) techniques are merely attempted in CMS which is believed to be the highly effective method to identify the part families. This article portrays a novel heuristic approach namely Heuristic for Part Family using Opitz Coding System (HPFOCS), to materialize the efficient part families by incorporating similarity metric which utilizes part coding attributes, adopted from Offodile (1992) and the proposed technique is verified on six generic datasets of size (5×9) to (30×9) and results are compared with Average linkage Clustering (ALC) algorithm. The computational results report that the HPFOCS method is extremely effective and has outperformed ALC techniques in all instances.

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1 Introduction

Group technology (GT) portrays a significant role in improving productivity for the cellular manufacturing systems (CMS) which classifies homogeneous parts and clusters them into part families based on their manufacturing designs, attributes and geometric shapes

(Burbidge, 1963). It scrutinizes products, parts and assemblies and then assembles homogeneous items to simplify design, manufacturing, purchasing and other business processes. Group Technology reduces the time required for practicing engineering drawings for homogeneous parts, and the cost and time required for designing supplementary machining apparatus such as typically designed cutting tools, jigs and fixtures etc. A successful implementation of GT can eventually minimize the engineering and tooling costs, quicken product development, enhance costing accuracy, simplify process planning and the overall purchasing process (Galan et al., 2007). A major prerequisite in implementing GT is the recognition of part families (Wemmerlov and Hyer, 1987), a group of parts sharing homogeneous design and manufacturing attributes. Early research in this domain has been dedicated primarily on the formation of production-oriented part families. However these methodologies are inadequate in achieving the needs of other extents of manufacturing. For example, parts with homogeneous shape, size, dimension or other design characteristics are believed to be clustered in a single family for design justification and elimination of part varieties.

Therefore the scope of this domain of investigation is believed to be expanded and examined to a wider span of part similarities, which are assumed to be identified sooner than the formation of part families based on shape, length/diameter ratio, material type, part function, dimensions, tolerances, surface finishing, process, operations, machine tool, operation sequence, annual production quantity, fixtures needed, lot sizes (Groover and Zimmers 1984). This paper proposes a state-of-the-art part family identification technique called HPFOCS, to investigate the nature of similarities and to describe the effectiveness of the technique in solving the problem in hand.

2 Literature Review

Two different approaches are traced in past literature in order to form part families, first is production flow analysis (PFA) which deals with processing requirements, operational sequences and operational time of the parts on the machines (Burbidge, 1996). Second approach is the classification and coding system which utilizes predefined coding schemes to facilitate the process using several attributes of parts such as geometrical shapes, materials, design features and functional requirements etc (Mitrofanov, 1966).

Classification and coding (CC) is practiced in this study as an essential and effective tool for successful implementation of GT concept. A code may be numbers (numerical) or alphabets (alphabetical) or a hybridization of numbers and alphabets (alphanumeric) which are allotted to the parts to process the information (Ham et al. 1985). Parts are categorized based on significant attributes such as dimensions, type of material, tolerance, operations required, basic shapes, surface finishing etc. and assigned a code which is a string of numerical digits that store information about the part. Generally coding systems depict either hierarchical structure (monocode), or chain structure (polycode) or hybrid mode structure mixed with monocode and polycode (Singh and Rajamani, 1996). Several CC systems have been developed, e.g. Opitz (Opitz, 1970), MICLASS (TNO, 1975), DCLASS (Gallagher and Knight, 1985) and FORCOD (Jung and Ahluwalia, 1992). Han and Ham (1986) have claimed that part families could be established more realistically by practicing the CC due to the advantage of using the manufacturing and design attributes concurrently. Offodile reported a similarity metric based on the numeric codes for any pair of parts which could be utilized to form efficient part families (Offodile, 1992).

Various techniques are developed to solve manufacturing cell formation problems since last few decades, some researchers have proposed to measure dissimilarity or distance instead of similarity metric (Prabhakaran et al., 2002) for generalized cell formation problems. Few of the agglomerative clustering methods adopted earlier are Single linkage clustering algorithm with similarity metric (McAuley, 1972), Average linkage clustering algorithm (Seifoddini and Wolfe, 1986). Other clustering methods are practiced by Carrie (1973), Chandrasekharan and Rajagopalan (1986a, 1986b), King (1980), King and Nakornchai (1982). The rank order clustering (ROC) algorithm is the most familiar array-based technique for cell formation (King, 1980). Substantial alterations and enhancements over rank order clustering algorithm have been described by King and Nakornchai (1982) and Chandrasekharan and Rajagopalan (1986a). Rajagopalan and Batra (1975) proposed the use of graph theory to form machine cells. Chandrasekharan and Rajagopalan (1986a) proposed an ideal seed non-hierarchical clustering algorithm and Srinivasan (1994) implemented a method using minimum spanning tree (MST) for the machine-part cell formation problem.

In this paper a novel heuristic approach based on classifications and coding systems is adopted. A brief study of past CMS literature based on heuristic approaches is discussed next.

Since past two decades application of heuristics is evolving in CMS, exclusively which are nature inspired and mimic the biological phenomena to find ‘fittest’ solution by incorporating ‘survival of the fittest’ theory (Darwin, 1929). A detail study based on such NP-complete problem solving methods in CMS could be obtained from recent survey (Ghosh et al., 2011). Lee-Post (2000) proposed that GT coding system (DCLASS) could be efficiently used with simple Genetic Algorithm (GA) method to cluster part families which is well suited for part design and process planning in production. Lei and Wu (2006) worked with multi-objective cell formation (CF) problem and proposed a Pareto-optimality based on multi-objective tabu search (TS) with different objectives. Arkat et al. (2007) developed a sequential model based on Simulated Annealing (SA) for large-scale problems and compared with GA. Ateme-Nguema and Dao (2007) investigated an Ant algorithm based TS heuristic for cellular system design problem. Safaei et al. (2008) proposed a model of dynamic cellular manufacturing system and solved their model using mean field annealing and SA. Defersha and Chen (2008) studied a mathematical programming model to form manufacturing cells and developed a parallel SA incorporating several problem specific perturbation operators. Tavakkoli-Moghaddam et al. (2008) introduced an integer programming model for dynamic CF and implemented SA to obtain the optimal solutions. Ateme-Nguema and Dao (2009) further proposed quantized Hopfield network for CF to find near-optimal solution and TS was employed to improve the performance and the quality of solution of the network. Tavakkoli-Moghaddam et al. (2009) further presented improved SA to CF and compared with LINGO 6 software package. A hybrid methodology based on Boltzmann function from SA and mutation operator from GA was proposed by Wu et al. (2009) to optimize the initial cluster obtained from similarity coefficient method and ROC.

Generally most of the abovementioned approaches are developed for binary or generalized CF problems. Only Lee-Post (2000) has incorporated the part coding system in her study.

3 Opitz Coding System

This classification and coding system was initially proposed by Opitz (1970) at Aachen

Technology University in Germany. The basic code comprises of nine digits that can be extended by additional four digits. The general interpretations of the nine digits are as indicated in Figure 1.

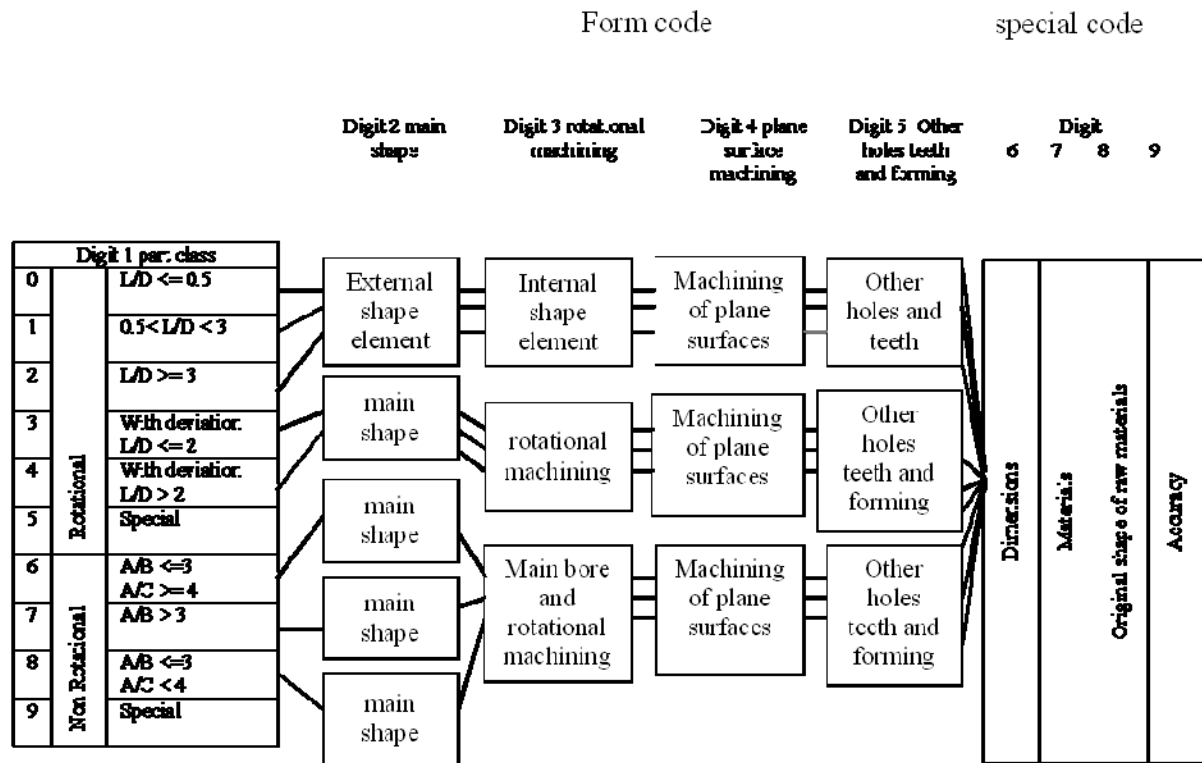


Figure 1: Opitz part coding system.

The interpretation of the first 9 digits is:

Digit 1: General shape of workpiece, otherwise called 'part-class'. This is further subdivided into rotational and non-rotational classes and further divided by size (length/diameter ratio.)

Digit 2: External shapes and relevant form. Features are recognized as stepped, conical, straight contours. Threads and grooves are also important.

Digit 3: Internal shapes. Features are solid, bored, straight or bored in stepped diameter. Threads and grooves are integral part.

Digit 4: Surface plane machining, such as internal or external curved surfaces, slots, splines.

Digit 5: Auxiliary holes and gear teeth.

Digit 6: Diameter or length of workpiece.

Digit 7: Material Used.

Digit 8: Shape of raw materials, such as round bar, sheet metal, casting, tubing etc.

Digit 9: Workpiece accuracy.

All the 9 digits are interpreted numerically (0-9). An example of square cast-iron flange is shown in Figure 2 in this context. The Opitz codes of square cast-iron flange is 65443 6070 (Ham et al., 1985). The attributes are denoted as a1-a9,

a1=6 (Non-rotational, flat component with $A/B \leq 3$, $A/C > 4$.)

a2=5 (Flat small deviations from casting.)

a3=4 (Main bores are parallel.)

a4=4 (Plane stepped surface.)

a5=3 (Drilling pattern for holes, drilled in one direction.)

a6=6 (400 mm. < length of edge \leq 600 mm.)

a7=0 (material is cast iron.)

a8=7 (Internal form: Casting.)

a9=0 (surface finish none.)

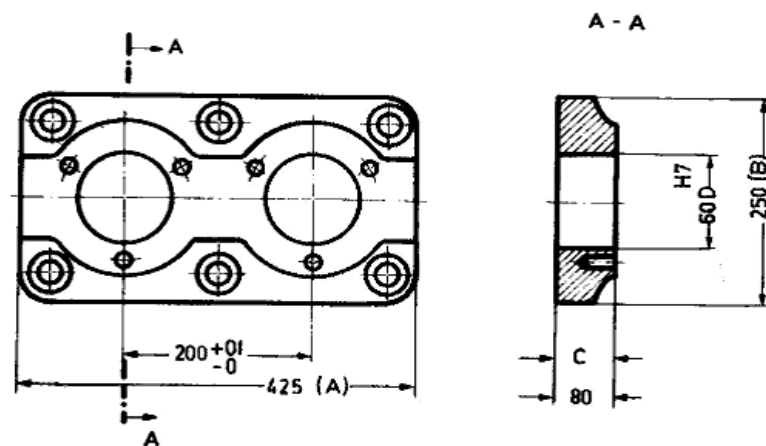


Figure 2: Square cast-iron flange.

Part grouping problem deals with categorical data in the vicinity of cellular manufacturing system. However linkage clustering methods are developed to group the observations (continuous items) rather than the categorical variables (Anderberg, 1973). The issue of variable clustering generally requires for dimension scaling. In order to utilize the linkage methods such as average linkage (ALC) or single linkage (SLCA) substantial modifications are assumed to be added (Seifoddini and Wolfe, 1986). Therefore to achieve this goal an improved similarity measure was proposed by Offodile (1992) which is appropriate for the categorical data presented by Opitz coding system. In present study Offodile's similarity measure is

utilized to avoid such drawbacks and the ALC and HPFOCS techniques are modified accordingly which further can consider the categorical data to group the parts using classification and coding system. The next section elaborates these methodologies.

4 Problem Definition and Research Methodologies

The part family formation problem stated in this research could be formulated using Opitz code as a matrix $B = [b_{ij}]$ which is further named as part-attribute incidence matrix of size $p \times n$, where p is the number of parts and n is the number of attributes of that part. b_{ij} represents the coding value (0-9) of j^{th} attribute of i^{th} part. Therefore Each row presents different part with Opitz code (column 1-9). A (5×9) example problem based on Opitz coding system is shown in Figure 3.

The solution to the problem is to form the families of parts in such a way that the sum of similarities among the each pair of parts in a same family would be maximized. A large number of similarity coefficients methods (Yin and Yasuda, 2005) and clustering techniques have been proposed for the part-machine grouping problem as reported in section 2. In this study an effective similarity metric for part grouping (Offodile, 1992) and an Average Linkage Clustering (ALC) algorithm (Seifoddini and Wolfe, 1986) is used to find the initial feasible part family which might not be the best solution to the problem. Therefore a novel heuristic method namely HPFOCS is further employed to improve the quality of solution obtained.

	a1	a2	a3	a4	a5	a6	a7	a8	a9
p1	4	4	4	0	7	3	8	9	1
p2	0	1	7	5	9	6	7	6	8
p3	5	9	3	3	1	5	5	7	7
p4	3	8	5	3	5	7	7	1	0
p5	2	0	0	2	9	8	4	2	0

Figure 3: problem #1 (5×9) matrix

4.1 Similarity Coefficient Method

The part grouping similarity metric proposed by Offodile (1992) is presented as,

$$S_{ij} = \frac{\sum_{k=1}^K S_{ijk}}{K} \quad (1)$$

Where

$$S_{ijk} = 1 - \frac{|b_{ik} - b_{jk}|}{R_k} \quad (2)$$

Where

S_{ijk} is similarity measured between part i and part j on attribute k ,

K is total number of attributes considered,

b_{ik} is part coding for part i on attribute k ,

b_{jk} is part coding for part j on attribute k ,

R_k is the range of attribute k considered over the population space of parts.

This abovementioned similarity metric technique is utilized in this study to calculate the similarity coefficient value between pair of parts presented as rows of part-attribute incidence matrix of Figure 3. To minimize the computation in Matlab the similarity matrix is transformed into distance matrix by using equation (3),

$$d_{ij} = 1 - S_{ij} \quad (3)$$

4.2 Average Linkage Clustering (ALC) Technique

ALC technique is conceptually and mathematically simple algorithm practiced in hierarchical clustering analysis (Seifoddini and Wolfe, 1986). It delivers informative descriptions and visualization of potential data clustering structures. In present scenario ALC uses the average distances between all pairs of parts in any two part families. Since in this study similarity matrix has been transformed into distance matrix therefore the average distance between family a and another family b is defined as:

$$A_{ij} = \frac{1}{n_a \times n_b} \sum_{i=1}^{n_a} \sum_{j=1}^{n_b} d_{ij} \quad (4)$$

n_i = Number of parts in family i

A matrix Z is generated using equation (4), which is a $(p-1) \times 3$ matrix, where p is the number of parts in the original dataset. Columns 1 and 2 of the matrix contain cluster indices linked in pairs to form a binary tree. The leaf nodes are numbered from 1 to p . Leaf nodes are the singleton clusters from which all higher clusters are built.

$$Z = \begin{bmatrix} 4 & 5 & 0.2963 \\ 1 & 6 & 0.3457 \\ 2 & 7 & 0.3827 \\ 3 & 8 & 0.3920 \end{bmatrix}$$

The dendrogram could be obtained from this computation which visually indicates a tree of potential solutions as shown in Figure 4 (A trail of ALC method could be realized in Appendix).

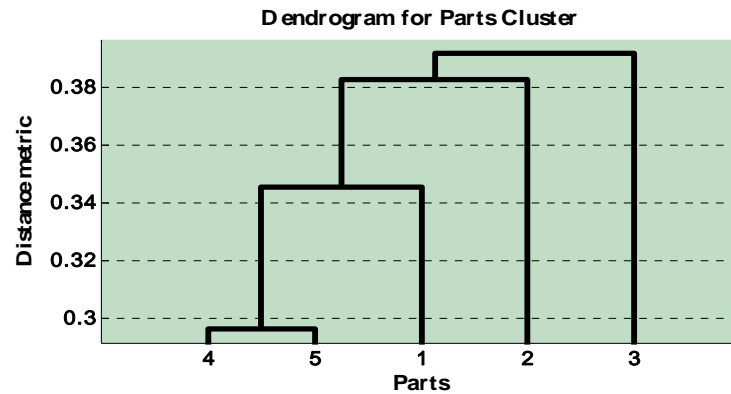


Figure 4: Dendrogram obtained for problem #1 (5×9)

The part families formed are PF1 {parts 1, 2, 4 and 5} and PF2 {part 3}. Since part family formation is an NP-complete problem which indicates, in polynomial time there could exist multiple solutions for the problem. Therefore the solution obtained from ALC approach might not be the best or near best solution and there always a scope remains to improve the goodness of the solution. Thus this article proposes a novel heuristic algorithm to serve the purpose.

4.3 HPFOCS: The Proposed Heuristic Algorithm

In this article a heuristic technique based on Opitz coding system namely HPFOCS is proposed to improve the goodness of the solution obtained via ALC method. HPFOCS takes the solution obtained from ALC method as an input which is presented as a bit string (instead of the solution matrix) of length p (p = number of parts) in order to minimize the computational effort. The initial input string for the example problem of Figure 3 could be symbolized as ‘11211’, which means assigning parts 1 to 5 to the families 1, 1, 2, 1 and 1 respectively as stated by ALC method. To understand the goodness of the solution a performance evaluation criteria is assumed to be explained.

4.4 Performance Evaluation Metric

The goodness of the solution is a measure of how well the part families are formed. The objective of part family formation is to maximize the sum of similarities of parts and to maximize the perfection percentage of the part families obtained. Therefore, maximizing the sum of similarities could be used as the evaluation criteria to assess the goodness of each solution string, which is expressed mathematically as (Lee-Post, 2000),

$$Max f = \sum_{n=1}^N S_n \quad (5)$$

Where

$$S_n = \frac{\sum_{i \in n, j \in n} S_{ij}}{0.001 + C_2^{P_n}} \quad (6)$$

$$\text{Perfection Percentage} = \frac{\sum_{n=1}^N S_n}{N} \quad (7)$$

where S_{ij} = similarity measure between part i and part j computed from equation (1)

$C_2^{P_n}$ = Number of pairwise combinations formed in part family n , and P_n is the number of parts in family n

N = Number of part families formed

In equation (6) a small value (0.001) is added with $C_2^{P_n}$ at the denominator in order to avoid the division by zero rule. The obtained solution may be deleted, kept, or marked as good on the basis of the goodness of the solution. HPFOCS procedure keeps the record of best solution encountered accordingly with the iteration count. When a new solution is obtained, the objective function of equation (5) is applied, and based on the result, it could be decided as to add the solution to the elite list, or eliminate the solution and generate a new one in the neighbourhood.

The proposed HPFOCS is presented as,

Procedure *HPFOCS* ()

Step 1. input the initial solution string ' s_0 ' obtained by ALC and set *max_iterations*

Step 2. calculate the objective value ' f ' for input string using equation (6)

Step 3. Store $f \leftarrow \text{best_objective_value}$

Step 4. $s_0 \leftarrow \text{best_solution}$

Step 5. While $i \leq \text{max_iterations}$

Step 6. do

Step 7. create initial set ' S ' of randomly generated strings ($s_i \in S, i=1,2,\dots,n$)

Step 8. for $i = 1$ to n

Step 9. calculate objective value f_i for $s_i \in S$

Step 10. compute $\delta = (f_i - f)$

Step 11. if $\delta >$ small random no. (1.0000e-006)

Step 12. $\text{best_solution} = s_i$

Step 13. $\text{best_objective_value} = f_i$

Step 14. else

Step 15. pick a part randomly and put it to another family in s_i (interchange the positions of two elements of s_i with a small probability p_x)

Step 16. repeat step 9 to 15
 Step 17. accept the arrangement
 Step 18. else eliminate the solution string
 Step 19. $i = i+1$
 Output: *part families configuration with highest sum of similarity value*

Convergence analyses are almost equivalent for all the problem datasets. Problem #6 of size 30×9 is selected as an example to illustrate the convergence curve during iterations of the heuristic technique (Figure 5). For the first iteration the objective function f attained a value of 5.315. Since the computer program is designed to maximize the objective function with the iteration counts therefore at 20th iteration it attained the value of 5.482, an increase of 3.14%. At 80th iteration it attained the value of 5.652, an increase of 6.34%. The final optimal solution is obtained during the 126th iteration having the objective value of 5.7496, an increase of 8.18%. Based on the exhaustive experimentation for all the datasets reported in this article, it is observed that the objective value is increased with the iteration counts till it reaches the best objective value at some iteration and thereafter the objective value continues to remain constant even though the number of iterations is increased. Since the proposed heuristic gives the same pattern of convergence for all the tested problems therefore the convergence property is proved. For the example problem the heuristic approach is executed for 133 iterations. The size of the generated solutions set considered is 300. This proposed algorithm took 18.3122 CPU seconds to attain the best solution which proves its computational efficiency.

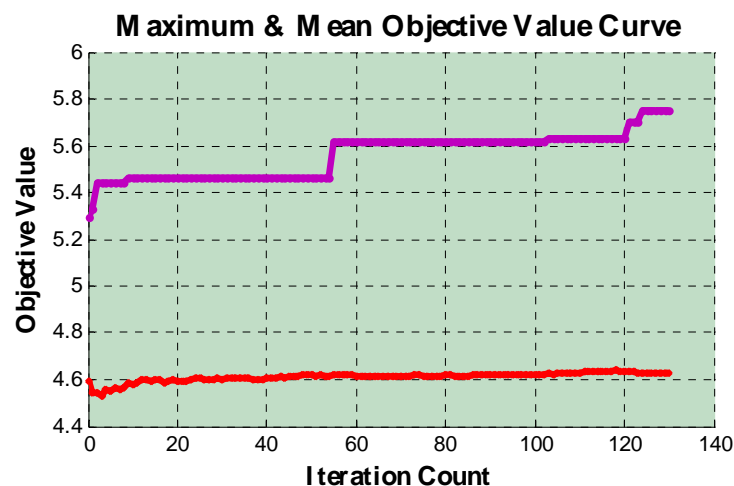


Figure 5: Convergence analysis curve of problem #6 (30×9)

5 Results and Discussion

The proposed technique is tested on six different problem datasets of size 5×9 to 30×9 , which are generated in Matlab environment using Opitz coding system. Problem datasets are provided in Figure 3 and Figure 6 to Figure 10. The HPFOCS and ALC algorithms are coded in Matlab 7.0 and executed on PIV Core 2 Duo laptop computer. The results are compared and shown in Table 1.

Table 1 depicts that for all the problems the HPFOCS approach outperforms the ALC technique in terms of sum of similarity values. The part families obtained by both the approaches are also shown. According to Lee-Post (2000), maximum similarity value would produce better quality of solution, i.e. part families. Therefore Table 1 demonstrates that part families obtained by HPFOCS is better than the solutions produced by ALC technique. For all the solutions obtained by the HPFOCS and ALC method, the perfection percentage is depicted in Table 2 Which authenticates the dominance of HPFOCS over ALC algorithm. HPFOCS has depicted a 105.6% improvement over the worst result exhibited by ALC method for problem #1, is also computationally efficient as it took not more than 20 CPU seconds (Table 1) to solve the largest dataset of size (30×9). Figure 11 further presents a clear pictorial view of the level of enhancements shown by HPFOCS over ALC technique. Hence the proposed heuristic approach is established as an effective part grouping method and could be utilized further for more complex and real life problems.

	a1	a2	a3	a4	a5	a6	a7	a8	a9
p1	4	7	1	3	7	2	0	5	0
p2	7	4	2	5	0	3	5	7	2
p3	2	2	3	4	4	5	2	0	6
p4	3	1	3	6	6	2	3	5	7
p5	2	2	0	1	8	7	2	2	8
p6	7	5	7	6	8	6	8	6	0
p7	4	5	8	9	1	1	6	3	7
p8	1	5	5	3	5	4	0	3	7
p9	3	6	0	3	6	8	1	4	8
p10	0	0	3	4	5	4	7	1	2

Figure 6: Problem #2 (10×9) matrix.

6 Conclusions

A novel heuristic technique, namely HPFOCS is proposed and implemented in this article to form part families by exploiting the Opitz part coding system. Six different test datasets ranging from (5×9) to (30×9) are tested using the aforementioned technique. Due to the

NP-complete nature of the reported problems this HPFOCS method is effective in attaining improved solution and corresponding part families. The proposed heuristic method is compared successfully with ALC algorithm. The objective function utilized in this study is to maximize the sum of similarities among parts in all the part families formed. As shown in Table 1 HPFOCS algorithm has outperformed the ALC technique in all instances in terms of the solution quality. Table 1 further reports that this novel heuristic approach is more effective and less complex in terms of computational efforts. This study has assumed identical weightage for every attribute, however in formation of part families some attributes could be more important than some other attributes. Therefore future work could be done by considering fractional weightage of the attributes and could further be extended by considering operational time and sequence of each part to form more efficient and robust part families.

	a1	a2	a3	a4	a5	a6	a7	a8	a9
p1	4	7	6	4	6	7	3	5	6
p2	1	0	3	3	0	0	1	1	8
p3	2	1	5	3	1	0	6	1	6
p4	0	5	0	0	6	3	1	6	2
p5	6	0	3	2	9	5	6	7	7
p6	7	4	7	5	8	2	8	1	7
p7	3	2	5	3	8	7	6	5	1
p8	0	2	8	3	5	7	3	2	9
p9	3	0	9	6	1	9	7	8	5
p10	5	2	9	4	0	6	4	6	1
p11	6	3	8	6	4	6	3	0	8
p12	7	9	3	7	3	2	6	8	7
p13	5	4	7	5	7	6	4	6	6
p14	3	6	3	7	7	0	1	5	4
p15	2	6	3	8	9	9	5	5	4

Figure 7: problem #3 (15×9) matrix

	a1	a2	a3	a4	a5	a6	a7	a8	a9
p1	4	4	6	2	2	4	9	1	6
p2	6	3	7	7	1	9	5	6	0
p3	1	0	9	2	7	2	9	1	5
p4	7	6	0	0	3	1	1	1	2
p5	6	2	3	8	0	9	5	0	5
p6	8	7	2	5	2	8	1	8	7
p7	1	8	0	8	5	4	3	3	4
p8	5	2	5	8	8	8	5	7	3
p9	7	7	5	4	9	3	6	8	1
p10	0	3	0	8	3	1	1	5	2
p11	3	1	2	3	6	7	2	5	5
p12	3	9	4	2	7	7	9	6	7
p13	8	8	7	1	3	7	3	0	9
p14	0	4	0	8	2	3	4	2	8
p15	9	2	2	4	2	2	9	3	9
p16	7	1	3	2	1	1	5	5	1
p17	7	1	0	9	5	4	8	5	9
p18	3	2	0	9	0	1	4	4	2
p19	0	1	2	9	2	7	6	7	2
p20	3	2	8	9	9	4	8	6	3

Figure 8: problem #4 (20×9) matrix

	a1	a2	a3	a4	a5	a6	a7	a8	a9
p1	1	8	6	7	3	0	8	5	8
p2	6	2	6	0	4	0	8	8	8
p3	4	3	2	2	3	1	4	4	4
p4	2	3	0	9	9	8	6	8	5
p5	8	8	1	1	4	1	7	7	4
p6	5	9	4	1	9	6	7	1	7
p7	6	7	6	4	8	4	5	9	3
p8	6	3	1	1	1	3	8	4	7
p9	5	3	5	9	9	4	6	8	5
p10	3	3	9	5	8	5	8	1	0
p11	5	6	8	8	4	0	9	6	1
p12	6	0	7	4	7	0	4	8	8
p13	8	2	9	0	9	6	8	9	0
p14	3	4	9	2	0	4	8	2	8
p15	4	3	1	4	2	8	7	9	4
p16	7	1	6	0	9	4	1	0	4
p17	0	4	9	0	0	7	3	4	6
p18	8	3	2	2	9	2	3	7	2
p19	4	7	4	9	6	2	8	4	0
p20	9	3	7	8	7	9	6	1	8
p21	5	0	5	4	8	9	2	7	3
p22	9	2	3	0	9	5	0	4	9
p23	2	6	5	6	8	0	0	6	3
p24	5	0	2	0	5	0	3	8	4
p25	2	0	7	4	8	9	5	5	3

Figure 9: problem #5 (25×9) matrix

	a1	a2	a3	a4	a5	a6	a7	a8	a9
p1	3	5	9	0	5	6	8	9	1
p2	4	5	6	1	6	3	2	2	2
p3	6	1	7	7	7	5	7	0	2
p4	0	5	0	1	0	3	5	6	7
p5	0	3	9	4	7	6	5	3	1
p6	3	2	0	0	5	1	8	9	0
p7	4	8	0	1	5	8	7	7	8
p8	6	8	4	6	0	4	5	5	8
p9	7	7	5	3	1	1	4	3	5
p10	5	1	9	9	5	0	2	3	4
p11	6	8	0	9	9	5	8	4	9
p12	0	5	0	7	3	2	5	2	6
p13	9	1	3	3	5	3	7	1	8
p14	0	3	4	7	4	5	7	1	1
p15	6	4	5	7	5	4	7	4	1
p16	4	2	6	0	1	6	8	3	7
p17	4	3	7	3	7	6	3	3	5
p18	7	8	3	0	3	1	6	1	7
p19	9	6	2	8	7	8	4	8	2
p20	6	0	5	1	4	2	7	8	6
p21	4	6	0	8	9	6	6	1	4
p22	2	6	1	0	1	4	7	4	3
p23	8	8	6	7	0	7	0	0	5
p24	8	7	5	2	3	8	1	2	6
p25	3	4	7	2	5	6	9	5	2
p26	3	3	5	7	5	8	3	0	2
p27	9	2	2	0	9	9	2	3	1
p28	8	7	6	0	5	8	1	3	5
p29	1	4	7	4	0	2	1	9	0
p30	1	3	6	0	8	4	0	0	8

Figure 10: problem #6 (30×9) matrix

Table 1: Performance comparison between HPFOCS and ALC algorithms

#	Problem datasets	Number of part families	Part Families		Sum of similarities $\sum S_n$		CPU time for HPFOCS (in sec.)
			ALC	HPFOCS	ALC	HPFOCS	
1	5×9	2	(1,2,4,5), (3)	(1,2,3), (4,5)	0.6439	1.3242	2.2926
2	10×9	3	(2,6), (7), (1,3,4,5,8,9,10)	(3,8), (5), (1,2,4,6,7,9,10)	1.4349	2.3061	5.6873
3	15×9	4	(1,5,6,7,8,9,10,11,13,14,15), (2,3), (4), (12)	(4,5,6,7,10,12), (2,3), (1,13), (8,9,11,14,15),	1.543	3.0055	8.3886
4	20×9	5	(9), (2,5,8,19,20), (3,11,12), (4,10,14,15,16,17,18), (6,13)	(2,3,16,20), (5,6,7,12,13), (4,8,9,11,14,19), (10,18), (15,17)	2.8212	3.5257	11.2117
5	25×9	7	(2,3,5,8,12,18,24), (10,13), (20), (4,7,9,15,21,23,25), (6,16,22), (1,11,19), (14,17)	(10,19), (1,11), (7,18,21), (4,14,16,17,23,25), (3,8), (2,9,12,20), (5,6,13,15,22,24)	4.4835	4.9931	15.7976
6	30×9	8	(6), (12,22), (30), (7,11,21), (1,2,3,5,10,14,15,17,25,26), (8,9,13,16,18,20,23,24,28), (19,27), (29)	(10,17,22), (24,28), (3,25), (4,5,8,12,14,21,23), (2,26,27), (1,6,11,13,16,20,29,30), (9,18), (7,15,19)	3.6922	5.7496	18.3122

Table 2: Comparing the perfection percentage achieved by HPFOCS and ALC algorithm.

Problem #	No. of part families formed(N)	Percent perfection, $\frac{\sum S_n}{N}$	
		ALC	HPFOCS
1	2	32.2	66.21
2	3	47.83	76.87
3	4	38.57	75.14
4	5	56.42	70.51
5	7	64.05	71.33
6	8	46.15	71.87

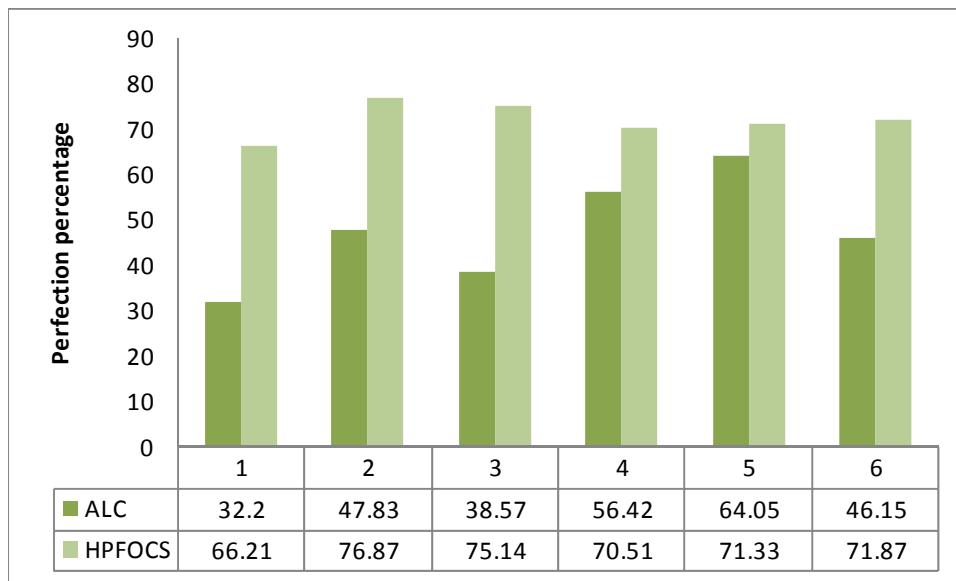


Figure 11: Improvement shown by HPFOCS in terms of Solution quality.

7 Appendix

A numerical example of ALC approach

The following information is given.

(i) Information about parts (relevant part attributes as explained in this article):

Part1	4	4	4	0	7	3	8	9	1
Part2	0	1	7	5	9	6	7	6	8
Part3	5	9	3	3	1	5	5	7	7
Part4	3	8	5	3	5	7	7	1	0
Part5	2	0	0	2	9	8	4	2	0

(ii) Information about significant part attributes

when,

Attribute 1 is numeric, $R_1=9$; Attribute 2 is numeric, $R_2=9$

Attribute 3 is numeric, $R_3=9$; Attribute 4 is numeric, $R_4=9$

Attribute 5 is numeric, $R_5 = 9$; Attribute 5 is numeric, $R_6 = 9$;
 Attribute 5 is numeric, $R_7 = 9$; Attribute 5 is numeric, $R_8 = 9$;
 Attribute 5 is numeric, $R_9 = 9$;

Step 1: Calculated pairwise similarity coefficients (S_{ij}) from equation (1).

$$\begin{aligned} S_{12} &= 0.6173 \\ S_{13} &= 0.6420 & S_{23} &= 0.6049 \\ S_{14} &= 0.6914 & S_{24} &= 0.6049 & S_{34} &= 0.6790 \\ S_{15} &= 0.6173 & S_{25} &= 0.6296 & S_{35} &= 0.5062 & S_{45} &= 0.7037 \end{aligned}$$

S_{12} is determined as follows:-

$$S_{121} = 1 - \frac{|b_{11} - b_{21}|}{R_1} = 1 - \frac{|4-0|}{9} = 0.5556$$

$$S_{122} = 1 - \frac{|b_{12} - b_{22}|}{R_2} = 1 - \frac{|4-1|}{9} = 0.6667$$

$$S_{123} = 1 - \frac{|b_{13} - b_{23}|}{R_3} = 1 - \frac{|4-7|}{9} = 0.6667$$

$$S_{124} = 1 - \frac{|b_{14} - b_{24}|}{R_4} = 1 - \frac{|0-5|}{9} = 0.4444$$

$$S_{125} = 1 - \frac{|b_{15} - b_{25}|}{R_5} = 1 - \frac{|7-9|}{9} = 0.7778$$

$$S_{126} = 1 - \frac{|b_{16} - b_{26}|}{R_6} = 1 - \frac{|3-6|}{9} = 0.6667$$

$$S_{127} = 1 - \frac{|b_{17} - b_{27}|}{R_7} = 1 - \frac{|8-7|}{9} = 0.8889$$

$$S_{128} = 1 - \frac{|b_{18} - b_{28}|}{R_8} = 1 - \frac{|9-6|}{9} = 0.6667$$

$$S_{129} = 1 - \frac{|b_{19} - b_{29}|}{R_9} = 1 - \frac{|1-8|}{9} = 0.2222$$

$$\text{Hence } S_{12} = \frac{(0.5556+0.6667+0.6667+0.4444+0.7778+0.6667+0.8889+0.6667+0.2222)}{9} = 0.6173$$

All other S_{ij} are determined in the same method.

Step 2: Calculate pairwise distances (d_{ij}) using equation (3)

$$\begin{aligned} d_{12} &= 0.3827 & d_{23} &= 0.3951 \\ d_{13} &= 0.3580 & d_{24} &= 0.3951 & d_{34} &= 0.3210 \\ d_{14} &= 0.3086 & d_{25} &= 0.3704 & d_{35} &= 0.4938 & d_{45} &= 0.2963 \\ d_{15} &= 0.3827 & & & & & & \end{aligned}$$

Step 3: Form initial part family.

Because d_{45} is the smallest distance, therefore PF1 is formed with parts 4 and 5 as its members.

Step 4: Calculate average distance A_{ij}

$$\begin{aligned} A_{(4,5)1} &= 0.3457 \\ A_{(4,5)2} &= 0.3828 & A_{12} &= d_{12} = 0.3827 \\ A_{(4,5)3} &= 0.4074 & A_{13} &= d_{13} = 0.3580 & A_{23} &= d_{23} = 0.3951 \end{aligned}$$

$A_{(4,5)1}$ is obtained as,

$$A_{(4,5)1} = \frac{d_{14} + d_{15}}{2} = \frac{0.3086 + 0.3827}{2} = 0.3457$$

Other $A_{(4,5)j}$ are calculated in the similar manner. Thereafter $A_{(4,5)1}$ has the lowest value and part 1 is introduced in family PF1.

Step 5: Repeat step 4 for part 2 and 3.

$$A_{(1,4,5)2} = 0.3827$$

$$A_{(1,4,5)3} = 0.3889 \quad A_{23} = d_{23} = 0.3951$$

$A_{(1,4,5)2}$ has the lowest value therefore part 2 is grouped in family PF1. At the end only part 3 remains for the 2nd family PF2.

Step 6: Thus all the parts are grouped. PF1 contains {parts 1, 2, 4, 5} and PF2 contains {Part 3}. Stop.

Sum of similarities is achieved as,

$$S_I = \frac{S_{12}+S_{14}+S_{15}+S_{24}+S_{25}+S_{45}}{c_2^4} = \frac{0.6173+0.6914+0.6173+0.6049+0.6296+0.7037}{6} = 0.6439$$

A numerical example of HPFOCS approach

The initial solution string is ‘22122’ obtained from ALC approach. If the size of initial solution set is fixed to 5 and generated randomly and if HPFOCS executes for one iteration,

#	Input strings	sum of similarities (f values)
1	2 1 2 2 2	0.6398
2	2 2 2 2 1	0.6398
3	1 2 2 2 2	0.6213
4	2 2 2 1 1	1.3242
5	2 1 1 1 2	1.2461

For string #4 f value is highest which is 1.3242 and $f(3)-f(0) = 0.6803$ which is larger than small number $1.0000e-006$ therefore ‘22211’ is accepted as the best solution obtained in first iteration and if the algorithm stops ‘22211’ is the final solution.

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