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MODELING THE PROCESS OF MECHANICAL OSCILLATIONS OF HELICAL SPRINGS

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ABSTRACT

The present article is devoted to the development of a mathematical model of oscillations of cylindrical helical springs under the action of external mechanical forces. The article is devoted to the equations of oscillations of spatial rods. The necessary assumptions to consider are the oscillations, Kirchhoff equilibrium equations, and the additional Clebsch equations that allow one to solve them. The system of linear differential equations and conditions allow calculating the values of linear and angular displacements of the spring, by calculating the corresponding values for individual sections. The equations model make it possible to determine the arising internal forces and moments of forces during oscillations.

1. INTRODUCTION

A theoretical study of mechanical vibrations of the screw springs is considered in [1-5]. The oscillation simulation of a helical spring with a distributed mass, stiffness and an infinite number of degrees of freedom is carried out without using approximate mathematical models in which it is replaced by an equivalent bar. Calculation of the model, in this case, is associated with computational difficulties since it is associated with solving a system of partial differential equations that describe the dynamics of a thin spatial curvilinear rod. However, numerical methods for solving equations allow us to solve the posed problem.

In the study of spatial mechanical vibrations, the following assumptions are made:

- During oscillations, the elastic moduli of the first and second kind of material for manufacturing the spring do not change;
- The force of internal friction of the spring material is not taken into account.

Assumptions are valid for fluctuations around the position of static equilibrium with amplitude, the parameters of which do not violate Hooke's law [6, 7]:

$$Q = EF\varepsilon \quad (1)$$

Where Q is the amount of internal force during vibrations; F is the cross-sectional area of the spring wire; ε - relative elongation of the wire.

The increasing influence of internal friction forces in the spring material during oscillations is not taken into account. The magnitude of these forces is much less than those of the magnitude of the internal forces increasing.

2. METHODOLOGY

To analyze oscillations, it is proposed to use the Kirchhoff equations, describing the vibrations of a thin spatial rod 1 [1]. The equations determine the linear and angular displacements of the rod, under the action of an external load in a moving (rotating) coordinate system. The origin of this coordinate system coincides with the center of gravity of cross-section of the rod, and the axes OX, OU, OZ, respectively, coincide in direction with the normal, binormal, and tangent to the axial line. Moving the coordinate system along the axis of the rod by an elementary distance ds, causes its rotation relative to OX, OU, OZ.

The Kirchhoff system of equations is written:

$$\begin{aligned}
&\frac{\partial Q_x}{\partial s} + Q_z q - Q_y r - q_x = 0,\\
&\frac{\partial Q_y}{\partial s} + Q_x r - Q_z p - q_y = 0,\\
&\frac{\partial Q_z}{\partial s} + Q_y p - Q_x q - q_z = 0,\\
&\frac{\partial M_x}{\partial s} + M_z q - M_y r - Q_y - m_x = 0,\\
&\frac{\partial M_y}{\partial s} + M_x r - M_z p + Q_x - m_y = 0,\\
&\frac{\partial M_z}{\partial s} + M_y p - M_x q - m_z = 0, \qquad (2),
\end{aligned}$$

where Q_x, Q_y, Q_z are the internal forces in the rod along the axes *OX, OY, OZ;* M_x, M_y, M_z are the moments of forces in the rod along the axes *OX, OY, OZ; p, q, r* - the projection of the curvature of the curve of the rod axis on the *OX, OU, OZ*; *s* is the length coordinate; q_x, q_y, q_x are the projected vector of the distributed load on the axis *OX, OY, OZ;* m_x, m_y, m_z are the projected vector of the distributed moment on the axes *OX, OY, OZ.*

The differential Equations (2) are the equilibrium equations for the internal forces for the spring element, the scheme of which is shown in Figure 1.

Since the mathematical analysis of the spatial rod's vibrations is a statically indefinite problem, in addition to the equilibrium Equations (2), the Clebsch equations are used, which determine the relationship between linear, angular displacements, and linear deformation.

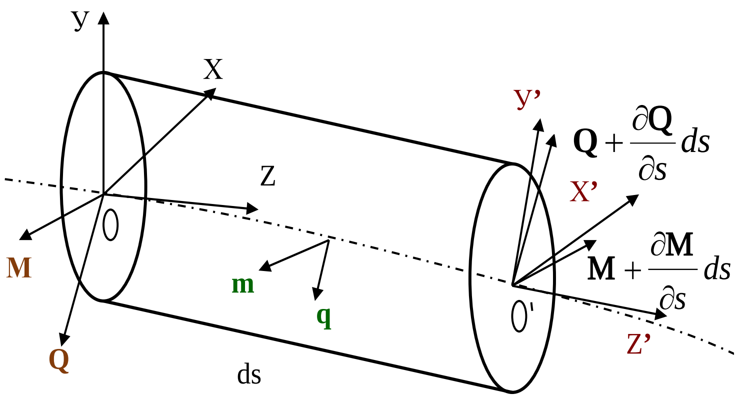

Figure 1: Diagram of a loaded element.

3. ANALYSIS

In the study of oscillations, an element is considered that changes its initial length from ds0 to ds, and occupies a new position in the space under the action of an applied external load (Figure 2).

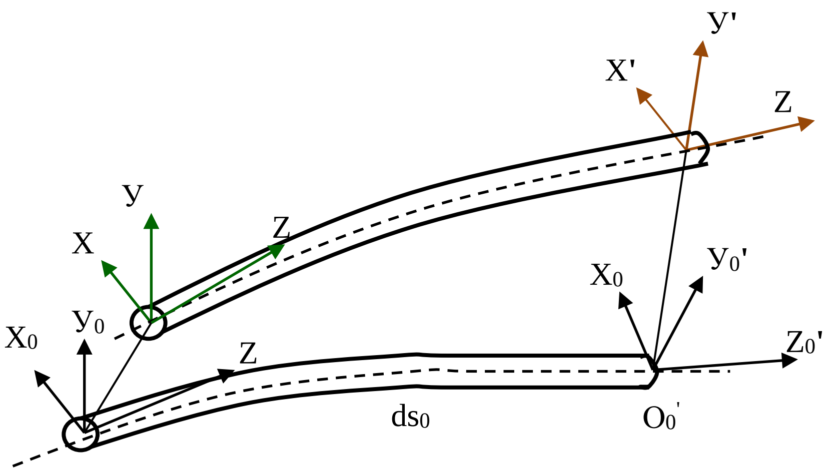

Figure 2: The deformation pattern.

Clebsch equations are written as follows:

$$\psi_y = \frac{\partial u}{\partial s} - v r_o + w q_o,$$

$$-\phi_x = \frac{\partial v}{\partial s} - w p_o + u r_o,$$

$$\varepsilon = \frac{\partial w}{\partial s} - u q_o + v p_o,$$

$$p^* = \frac{\partial \phi_x}{\partial s} + q_o \Theta_z - r_o \psi_y,$$

$$q^* = \frac{\partial \psi_y}{\partial s} + r_o \phi_x - p_o \Theta_z,$$

$$r^* = \frac{\partial \Theta_z}{\partial s} + p_o \psi_y - q_o \phi_x,$$

$$\psi_y = \frac{\partial u}{\partial s} - v r_o + w q_o,$$

$$-\phi_x = \frac{\partial v}{\partial s} - wp_o + ur_o,$$
$$\varepsilon = \frac{\partial w}{\partial s} - uq_o + vp_o,$$
$$p^* = \frac{\partial \phi_x}{\partial s} + q_o\Theta_z - r_o\psi_y,$$
$$q^* = \frac{\partial \psi_y}{\partial s} + r_o\phi_x - p_o\Theta_z,$$
$$r^* = \frac{\partial \Theta_z}{\partial s} + p_o\psi_y - q_o\phi_x \qquad (3),$$

where u, v, w are the linear displacement point O_o in the direction of the axes O_oX_o, O_oY_o, O_oZ_o; φ_x, ψ_y, Θ_z are the angles of rotation of the coordinate system OXYZ relative to the coordinate system $O_oX_oY_oZ_o$; p_o, q_o, r_o is the projections of the curvature of the un-deformed element on the axis O_oX_o, O_oY_o, O_oZ_o; p^*, q^*, r^* are the increments of the corresponding curvatures p_o, q_o, r_o during deformation.

The considered systems of equations describe oscillations in the space under which the laws of elasticity can be violated [8].

When the spring oscillates around the static equilibrium position of the curvature, the internal forces and the moments of forces are the sums of their initial values and increments [9, 10]. When considering fluctuations, the corresponding values are recorded:

$$p = p_o + p^*, \qquad q = q_o + q^*, \qquad r = r_o + r^*,$$
$$Q_x = Q_{xo} + Q_x^*, \qquad Q_y = Q_{yo} + Q_y^*, \qquad Q_z = Q_{zo} + Q_z^*$$
$$M_x = M_{xo} + M_x^*, \qquad M_y = M_{yo} + M_y^*, \qquad M_z = M_{zo} + M_z^* \qquad (4)$$

Where Q_{xo}, Q_{yo}, Q_{zo} are the values of the internal forces along the axes OX, OU, OZ in the material of the spring, which is in the position of static equilibrium; Q_x^*, Q_y^*, Q_z^* are the increments of forces Q_{xo}, Q_{yo}, Q_{zo} with fluctuations; M_{xo}, M_{yo}, M_{zo} are the values of the moments of internal forces along with the axes OX, OU, OZ in the spring material, which is in the position of static equilibrium; M_x^*, M_y^*, M_z^* are moment increments M_{xo}, M_{yo}, M_{zo} with vibrations.

With minor oscillations of the spring element, the increments of the quantities are much less than the initial values, which simplifies the system (2). The substitution in (2) of expressions (4) is carried out with the exception of products of small quantities $Q_z^*q^*$, $Q_y^*r^*$, $Q_x^*r^*$, $Q_z^*p^*$, $Q_y^*p^*$, $Q_x^*q^*$, $M_z^*q^*$, $M_y^*r^*$, $M_x^*r^*$, $M_z^*p^*$, $M_y^*p^*$, *and* $M_x^*q^*$.

The system of linear differential equations describing the process of spatial oscillations of the spring, after substituting (4) into (2) and excluding small second-order quantities, is written as:

$$\frac{\partial Q_x^*}{\partial s} + Q_{zo}q_o + Q_{zo}q^* + Q_z^*q_o - Q_{yo}r_o - Q_{yo}r^* - Q_y^*r_o - q_x = 0,$$
$$\frac{\partial Q_y^*}{\partial s} + Q_{xo}r_o + Q_{xo}r^* + Q_x^*r_o - Q_{zo}p_o - Q_{zo}p^* - Q_z^*p_o - q_y = 0,$$

$$\frac{\partial Q_z^*}{\partial s} + Q_{yo}p_o + Q_{yo}p^* + Q_y^*p_o - Q_{xo}q_o - Q_{xo}q^* - Q_x^*q_o - q_z = 0,$$

$$\frac{\partial M_x^*}{\partial s} + M_{zo}q_o + M_{zo}q^* + M_z^*q_o - M_{yo}r_o - M_{yo}r^* - M_y^*r_o - Q_{yo} - Q_y^* - m_x = 0,$$

$$\frac{\partial M_y^*}{\partial s} + M_{xo}r_o + M_{xo}r^* + M_x^*r_o - M_{zo}p_o - M_{zo}p^* - M_z^*p_o + Q_{xo} + Q_x^* - m_y = 0,$$

$$\frac{\partial M_z^*}{\partial s} + M_{yo}p_o + M_{yo}p^* + M_y^*p_o - M_{xo}q_o - M_{xo}q^* - M_x^*q_o - m_z = 0. \quad (5)$$

The system of equations, in contrast to the equations obtained by the author [2], takes into account the forces Q_x^*, Q_y^* , and the products of initial forces, moments, curvatures $Q_{zo}q_o$, $Q_{yo}r_o$, $Q_{xo}r_o$, $Q_{zo}p_o$, $Q_{yo}p_o$, $Q_{xo}q_o$, $M_{zo}q_o$, $M_{yo}r_o$, $M_{xo}r_o$, $M_{zo}p_o$, $M_{yo}p_o$, *and* $M_{xo}q_o$.

In (5), the projections of the vectors q, m on the *OX*, *OU*, *OZ* axes, with a periodic external load, are respectively equal to the sum of the distributed inertial forces or moments:

$$q_x = \rho F \frac{\partial^2 u}{\partial t^2} + q_{внx},$$

$$q_y = \rho F \frac{\partial^2 v}{\partial t^2} + q_{вну},$$

$$q_z = \rho F \frac{\partial^2 w}{\partial t^2} + q_{внz},$$

$$m_x = \rho J_x \frac{\partial^2 \phi_x}{\partial t^2} + m_{внx},$$

$$m_y = \rho J_y \frac{\partial^2 \psi_y}{\partial t^2} + m_{вну},$$

$$m_z = \rho J_z \frac{\partial^2 \Theta_z}{\partial t^2} + m_{внz}, \quad (6)$$

where J_x, J_y, J_z are the moments of inertia of the cross-section of the wire spring along with the *OX*, *OУ*, *OZ* axes*;* $q_{xвн}$, $q_{yвн}$, $q_{zвн}$, $m_{xвн}$, $m_{yвн}$, $m_{zвн}$ are the intensity of external forces and moments along with the axes *OX*, *OU*, *OZ*.

Substituting the Equations (6) into (5), and combining them with the Clebsch equations allows us to obtain the system:

$$\frac{\partial Q_x^*}{\partial s} + Q_{zo}q_o + Q_{zo}q^* + Q_z^*q_o - Q_{yo}r_o - Q_{yo}r^* - Q_y^*r_o = \rho F \frac{\partial^2 u}{\partial t^2} + q_{внx},$$

$$\frac{\partial Q_y^*}{\partial s} + Q_{xo}r_o + Q_{xo}r^* + Q_x^*r_o - Q_{zo}p_o - Q_{zo}p^* - Q_z^*p_o = \rho F \frac{\partial^2 v}{\partial t^2} + q_{вну},$$

$$\frac{\partial Q_z^*}{\partial s} + Q_{yo}p_o + Q_{yo}p^* + Q_y^*p_o - Q_{xo}q_o - Q_{xo}q^* - Q_x^*q_o = \rho F \frac{\partial^2 w}{\partial t^2} + q_{внz},$$

$$\frac{\partial M_x^*}{\partial s} + M_{zo}q_o + M_{zo}q^* + M_z^*q_o - M_{yo}r_o - M_{yo}r^* - M_y^*r_o - Q_{yo} - Q_y^*$$
$$= \rho J_x \frac{\partial^2 \phi_x}{\partial t^2} + m_{внx},$$

$$\frac{\partial M_y^*}{\partial s} + M_{xo} r_o + M_{xo} r^* + M_x^* r_o - M_{zo} p_o - M_{zo} p^* - M_z^* p_o + Q_{xo} + Q_x^*$$
$$= \rho J_y \frac{\partial^2 \psi_y}{\partial t^2} + m_{вну},$$
$$\frac{\partial M_z^*}{\partial s} + M_{yo} p_o + M_{yo} p^* + M_y^* p_o - M_{xo} q_o - M_{xo} q^* - M_x^* q_o = \rho J_z \frac{\partial^2 \Theta_z}{\partial t^2} + m_{внz},$$
$$\psi_y = \frac{\partial u}{\partial s} - v r_o + w q_o,$$
$$-\phi_x = \frac{\partial v}{\partial s} - w p_o + u r_o,$$
$$\varepsilon = \frac{\partial w}{\partial s} - u q_o + v p_o,$$
$$p^* = \frac{\partial \phi_x}{\partial s} + q_o \Theta_z - r_o \psi_y,$$
$$q^* = \frac{\partial \psi_y}{\partial s} + r_o \phi_x - p_o \Theta_z,$$
$$r^* = \frac{\partial \Theta_z}{\partial s} + p_o \psi_y - q_o \phi_x. \quad (7)$$

Equations (7) allow us to determine the linear and angular displacements of the spring relative to the position of static equilibrium under the action of a periodic external load, the values of the internal forces arising and the moments of forces.

The formulas that determine the values of the curvature of the spring in the static equilibrium state are written as:

$$p_o = 0, \qquad q_o = \frac{2\cos^2\alpha}{D}, \qquad r_o = \frac{\sin 2\alpha}{D}. \quad (8).$$

Internal forces and moments of forces for a fixed spring, taking into account the smallness of the forces that have arisen after its winding during manufacture, are determined by:

$$Q_{xo} = 0, \qquad Q_{yo} = P\cos\alpha, \qquad Q_{zo} = P\sin\alpha, \quad (9)$$

$$M_{xo} = 0, \qquad M_{yo} = \frac{PD}{2}\sin\alpha, \qquad M_{zo} = \frac{PD}{2}\cos\alpha \quad (10)$$

The moments of the inertia J_x, J_u of the cross section of the wire spring with constant diameter are the equatorial moments of inertia J of this section. The moment of inertia J_z is the polar moment of inertia Jo of the cross-section:

$$J_z = J_o = \frac{\pi d^4}{32}. \quad (11)$$

The resulting increments of the vector of internal momentum M with oscillations along the axes of the rotating system OX, OU, OZ (Figure 1) are proportional to the increment of the curvature vector:

$$M_x^* = EJp^*, \qquad M_y^* = EJq^*, \qquad M_z^* = GJ_o r^* \quad (12)$$

where G is the modulus of the elasticity of the second kind.

The increase in the internal force Q_z arising from oscillations is found according to Hooke's law.

The length of the wire L, from which the spring is twisted, is determined by the formula:

$$L = \frac{\pi Di}{\cos\alpha} \quad (13).$$

Distributed, periodic perturbation load acting along the axis of the OY of the rotating coordinate system leads to the appearance of an external force on the spring element:

$$\partial Q_{y вн} = F_{vn} \sin(\frac{2\pi}{T}t)\partial s, \quad (14)$$

where F_{vn} is the amplitude value of the external force distributed along the axis of the wire; T is the period of oscillation force.

The value of the intensity of the distributed load, in this case, on the considered section is found by the formula:

$$q_{y вн} = F_{vn} \sin(\frac{2\pi}{T}t). \quad (15),$$

where $q_{x вн}$, $q_{z вн}$, $m_{x вн}$, $m_{y вн}$, $m_{z вн}$ values from the action of the external distributed load on the spring along the axis of the shelter of the rotating coordinate system are equal zero.

System (7), taking into account (8), (9), (10), (12), (15), is written as:

$$\frac{\partial Q_x^*}{\partial s} + Q_{zo}q_o + Q_{zo}q^* + Q_z^* q_o - Q_{yo}r_o - Q_{yo}r^* - Q_y^* r_o = \rho F \frac{\partial^2 u}{\partial t^2} + q_{внx},$$

$$\frac{\partial Q_y^*}{\partial s} + Q_x^* r_o - Q_z^* p_o = \rho F \frac{\partial^2 v}{\partial t^2} + F \sin(2\Pi t/T),$$

$$EF\frac{\partial \varepsilon}{\partial s} + Q_{yo}p^* - Q_x^* q_o = \rho F \frac{\partial^2 w}{\partial t^2},$$

$$EJ\frac{\partial p_x^*}{\partial s} + M_{zo}q_o + M_{zo}q^* + M_z^* q_o - M_{yo}r_o - M_{yo}r^* - M_y^* r_o - Q_{yo} - Q_y^* = \rho J_x \frac{\partial^2 \phi_x}{\partial t^2},$$

$$EJ\frac{\partial q_y^*}{\partial s} + (EJr - M_{zo})p * + Q_x^* = \rho J_y \frac{\partial^2 \psi_y}{\partial t^2}$$

$$GJ_0\frac{\partial r_z^*}{\partial s} + (M_{yo}p_o - EJr_0)p^* + Q_x^* = \rho J_z \frac{\partial^2 \Theta_z}{\partial t^2} + m_{внz},$$

$$\psi_y = \frac{\partial u}{\partial s} - vr_o + wq_o,$$

$$-\phi_x = \frac{\partial v}{\partial s} - wp_o + ur_o,$$

$$\varepsilon = \frac{\partial w}{\partial s} - uq_o + vp_o,$$

$$p^* = \frac{\partial \phi_x}{\partial s} + q_o\Theta_z - r_o\psi_y,$$

$$q^* = \frac{\partial \psi_y}{\partial s} + r_o\phi_x - p_o\Theta_z,$$

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$$r^* = \frac{\partial \Theta_z}{\partial s} + p_o \psi_y - q_o \phi_x. \tag{16}$$

A system of linear differential equations (16) is proposed in order to simplify further calculations to consider in vector form:

$$\frac{\partial Y}{\partial s} + AY = B\frac{\partial^2 Y}{\partial t^2} + C, \tag{17}$$

where **Y** is the vector of displacements, curvatures, and internal forces; **A** is interconnection matrix; **B** is a matrix of inertia forces; **C** is the vector of external and initial forces and moments. Vectors **Y** and **C** are written as

$$\mathbf{Y} = \begin{bmatrix} Q_x^* \\ Q_y^* \\ \varepsilon \\ p^* \\ q^* \\ r^* \\ u \\ v \\ w \\ \varphi_x \\ \psi_y \\ \Theta_z \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} Q_{yo} r_o - Q_{zo} q_o \\ F_{vn} \sin(\frac{2\pi}{T} t) \\ 0 \\ \dfrac{Q_{yo} - M_{zo} q_o + M_{yo} r_o}{EJ} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The matrix **A** has a size of 12×12. The coefficients of the matrix are determined by the expressions:

$a_{1\,2} = -r_o$; $a_{1\,3} = EFq_o$; $a_{1\,5} = Q_{zo}$; $a_{1\,6} = -Q_{yo}$;

$a_{2\,1} = r_o$; $a_{2\,4} = -Q_{zo}$; $a_{3\,1} = -q_o/EF$; $a_{3\,4} = Q_{yo}/EF$;

$a_{4\,2} = -1/EJ$; $a_{4\,5} = (M_{zo} - EJr_o)/EJ$; $a_{4\,6} = (GJ_oq_o - M_{yo})/EJ$;

$a_{5\,1} = 1/EJ$; $a_{5\,4} = (EJr_o - M_{zo})/EJ$; $a_{6\,4} = (M_{yo} - EJq_o)/GJ_o$;

$a_{7\,8} = -r_o$; $a_{7\,9} = q_o$; $a_{7\,11} = -1$; $a_{8\,7} = r_o$; $a_{8\,10} = 1$;

$a_{9\,3} = -1$; $a_{9\,7} = -q_o$; $a_{10\,4} = -1$; $a_{10\,11} = -r_o$; $a_{10\,12} = q_o$;

$a_{11\,5} = -1$; $a_{11\,10} = r_o$; $a_{12\,6} = -1$; $a_{12\,10} = -q_o$.

The matrix **B** also has a size of 12×12. The coefficients of the matrix are equal to:

$b_{1\,7} = \rho F$; $b_{2\,8} = \rho F$; $b_{3\,9} = \rho/E$; $b_{4\,10} = \rho/E$; $b_{5\,11} = \rho/E$; $b_{6\,12} = \rho/G$.

The corresponding undefined coefficients of the matrices **A** and **B** are equal to 0.

When solving system (17), the initial and boundary conditions are determined, which depend on the method of fixing the spring and its position at the initial moment of time. Hard end fixing and static balance allows you to write these conditions in the following form:

$$\mathbf{Y}(0, t) = 0, \qquad \mathbf{Y}(L, t) = 0, \tag{18}$$

$$\mathbf{Y}(s, 0) = 0, \qquad \frac{\partial \boldsymbol{Y}(s, \mathbf{0})}{\partial t} = 0. \tag{19}$$

4. CONCLUSION

The system of linear differential equations and conditions (18), (19) will allow calculating the values of linear and angular displacements of the spring, by calculating the corresponding values for individual sections. The equations model make it possible to determine the arising internal forces and moments of forces during oscillations.

This paper shows the high complexity of solving the problem of the mechanical vibrations of cylindrical helical springs. The calculation of oscillations can be done by representing the spring as an equivalent bar, but the accuracy of determining the parameters of deviations under the influence of external forces is approximate. The solution of the system of differential equations of vibrations is a difficult task. The transition to the vector form of writing oscillations equations allows us to simplify the form of the record and will allow the appliance of the finite element method to solve the considered system.

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