MODERN TECHNOLOGIES AND APPROACHES TO THE TRIANGULATION ALGORITHMS

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ABSTRACT

Features of organizing the automation systems for engineering calculations are considered. Program realization for the creation of triangular networks, in particular, the generator of networks based on a Delaunay method of creating triangulation is analyzed. Methods of creation of volume networks are also considered. Conformal networks with many-sided cells are presented. Triangulation methods, their essence, merits and demerits, algorithms of creating Delaunay triangulation are described.


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1. INTRODUCTION

At a solution to modern scientific and engineering tasks, mathematical modeling is widely applied (Bayneva, 2019). There are ready systems of automation of engineering calculations (CAE – Computer-aided engineering) for the analysis and modeling of physical processes on a computer. As a rule, it is the big closed commercial project which is developed for decades groups in several hundred specialists. The majority of the modern CAE complexes consist of a set of libraries for the graphic interface, visualization, creation of estimated networks, sampling of differential equations and a solution of systems of the linear and nonlinear equations.

From the CAE system, users’ points of view provide the convenient graphic interface for a task of parameters and statements of the problem, carrying out calculations, display and the analysis of results. The algorithms and methods used for a solution of a specific objective often are hidden from the user.

Important criteria when choosing specific libraries from the point of view of the developer are reliability, simplicity, and cost or a possibility of free use.
2. METHODS OF CREATING TRIANGULATION NETWORK

The methods of creation of networks, such as method of displays, method of the advanced front, methods of sweeping (sweeping) and pawing (paving) of area, methods existing today on the basis of octal trees (octree), are applicable or for rather narrow class of areas, or demand manual intervention in process of creation of a network.

There are two main methods of creation of a triangular network on the plane: a method of the advanced front and a method of creating triangulation Delaunay. Both methods can be used for the creation of a conformal triangular network in the set polygon at the set of tops without the addition of new tops.

There is a set of program realization for the creation of triangular networks. Let's note one of the most known reliable generators of triangular networks, the generator of networks Triangle developed by J.R. Shevchuk (1996). This generator of networks is based on a method of creating triangulation Delaunay and allows building conformal triangulations Delaunay, a triangulation Delaunay with restrictions, and triangular networks with high quality. The border of the area is set discretely; there is no opportunity to completely control the desirable size of elements in the area. The generator can use arithmetics with any accuracy for an exception of errors of rounding at calculations, for example, at the calculation of the sign of the determinant.

Usually, the process of creation of a tetrahedral network is divided into two stages:

1) creation of a superficial triangulation on area border;
2) creation of a tetrahedral network in the area.

In different tasks, the geometrical forms of models are had different complexity. For example, in the case of modeling of physical processes in a homogeneous environment, it is possible to use the area of a simple form in the form of a sphere or a cube.

In engineering, tasks are widely applied by a CAD to the creation of geometrical models of products. Also, the application of 3D scanning for receiving computer models of a real-life object is not excluded. The universal approach does not exist: in different tasks this or that way of a task of the geometry of the model is convenient.

At the creation of a volume tetrahedral network important property of a method is the possibility of preservation of the set network trace on the border. Unlike two-dimensional triangular networks, in a three-dimensional case, there are such many-sided areas for which it is impossible to construct a conformal tetrahedral network with the set trace on the border without the addition of new tops.

At the creation of tetrahedral networks, the methods based on the same ideas as in a two-dimensional case can be used. The classical method of creation of a tetramerization Delaunay builds a network for a convex cover of points and does not guarantee the preservation of the border of the area. Several methods for preservation of borders are offered: local modifications of a network, crushing of a network, and creation of a tetramerization Delaunay with restrictions (CDT – Constrained Delaunay triangulations) (Borouchaki et al, 1995). The algorithm of the advanced front, on the other hand, has no problems with the preservation of the set border as it and begins with it. However, this algorithm can face such a configuration of the front which he is not able to advance. In literature also the hybrid methods based on a combination of the ideas of methods of the advanced front and a tetramerization Delaunay meet (Yang et al, 2005).
In work the combination of two methods is used (Baynev & Fedosin, 2018a; Baynev & Fedosin, 2018b; Baynev & Fedosin, 2019a; Baynev & Fedosin, 2019b). The main method – a method of the advanced front, with its help is under construction for the most part of a network. As an additional method, the simplified version of the method offered by Borouchaki et al. (1995) is applied.

In 3D modeling different types of settlement networks are used: tetrahedral, hexahedral, prismatic (Figure 1), type networks an octal tree (Figure 2); application of hybrid networks with different types of cells is possible. As a rule, all similar types of networks get to a class of conformal networks with many-sided cells.

**Figure 1**: Examples of networks
(a) hexahedron, (b) triangular prismatic, (c) tetrahedral

**Figure 2**: Type network octal tree: (a) construction process; (b) appearance.

### 3. RESULT AND DISCUSSION

The main numerical algorithms of a solution of applied tasks include an important element – splitting area (surface) in which any processes, on very small standard cells (curvilinear triangles, quadrangles, tetrahedrons, prisms, parallelepipeds, etc.) are investigated. The set of such cells is called a differential network, and their tops – network nodes. Cells of a network and their nodes are a basis on which there is a replacement of the mathematical models describing the studied processes by the system of the algebraic
equations which numerical solution gives an approximate value of necessary characteristics of the studied phenomena. The diverse requirements imposed on differential networks (small deformation of cells, coherence with borders of physical geometry and with features of physical quantities), due process of their creation by a complex mathematical problem.

The process of splitting a surface of objects into polygons is called a tessellation. It is necessary to make at first programmatically a tessellation of source objects, and then to transfer the received polygonal areas for further processing. In practice approximation of a surface by triangular elements (triangles) – a triangulation, therefore, the triangulable network turns out is most often made. Wide use of triangles is explained by the following reasons: the triangle is the simplest polygon which tops unambiguously set an edge; the computing complexity of algorithms of splitting into triangles is significantly less than when using other polygons; implementation of procedures of rendering is simplest for the area limited to a triangle; for a triangle tree, it is easy to define the next the neighbor, having with it the general edges; any surface can be approximated with a necessary accuracy a network of triangles; accuracy of approximation is defined by quantity of triangles and way of their choice.

![Diagram of triangulation methods]

**Figure 3:** Classification of methods by the principle of construction.

Various methods of triangulation, by the principle of construction which can be broken into two classes, are for practical purposes used: direct and iterative (Figure 3). On scope of application it is possible to select two schemes of a triangulation (Figure 3): two-dimensional when the flat polygon becomes covered by a set of triangles (Figure 4(a)); three-dimensional which can use as approximation elements as flat triangles (Figure 4(b)), and tetrahedrons (Figure 4(c)). In the first case, the surface of the model becomes covered by a set of four triangles connected and not blocked among them. In the second case, the internal volume of a three-dimensional body breaks into a set of 8 tetrahedrons which are not blocked among them.
Figure 4: Schemes of a triangulation: (a) 2D triangulation triangles, (b) 3D triangulation triangles, (c) 3D triangulation tetrahedrons.

4. CONCLUSION

Considering algorithms of triangulation are an integral part practically of all 3D software products - schedules works on their improvement and the creation of new algorithms is intensively conducted. It is caused by the instability of a number of known algorithms and the unsatisfactory time of their work on real data sets.

5. AVAILABILITY OF DATA AND MATERIAL

Information used and generated from this work is available by contacting the corresponding author.

6. REFERENCES


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