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MAXIMUM EMPIRICAL LIKELIHOOD AS AN ALTERNATIVE TO GENERALIZED METHOD OF MOMENTS FOR A FINITE SAMPLE CASE

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ABSTRACT

The study explored the implications of Maximum empirical likelihood (MEL) as an alternative approach to the Generalized Method of Moments (GMM) on a real economic model using finite samples. The literature frequently discussed that the MEL has good finite samples properties in contrast to GMM however, it has not been tested empirically. This study used the MEL approach to an economic model that has finite samples and endogeneity problems. For this purpose, the most famous economic model Keynesian consumption function is used using data of several countries with a small sample of thirty observations. The analysis found that elasticity estimated by MEL and GMM is significantly different and in some cases, the differences are drastically high. It is believed that such differences are due to poor finite sample properties of GMM. Thus, the study suggests that GMM provides biased and less efficient estimates in finite samples. Therefore, the researchers are recommended to use MEL rather than GMM to estimate economic models having a problem of endogeneity in a finite sample.

Disciplinary: Economics and Econometrics.

1. INTRODUCTION

Researchers often face the problem of endogeneity in the estimation of simultaneous Equation models. In the estimation of the general linear regression model, if the assumption of orthogonality condition is not satisfied then Ordinary Least Square (OLS) estimates become biased. To deal with such issues, practitioners often use the Generalized Method of Moments (GMM) approach as proposed by Hansen (1982). For instance, the determinant of the textile industry in Pakistan was examined by an exhausting GMM approach (Latif and Javid, 2016). Similarly, Kendix & Walls (2010) computed the impact of oil industry consolidation on the prices of refined products. Ngo (2006) compared the GMM with an instrumental variable approach to explore the linkage between

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bank capital and profitability.

The Maximum Likelihood approach is most commonly used in econometric modeling. It can be used only in the case when the probability distribution of the model is completely known. Though, the GMM approach does not require such conditions in the estimation of the econometric model. The GMM requires only a set of moment functions specifications according to the model. The GMM construct sample analogs of the population of orthogonality condition and set them close to zero.

The Empirical Likelihood (EL) approach is advantageous than GMM for at least two reasons. First, the EL approach provides stable asymptotic bias with the increase in a number of moment restrictions while GMM does not. Second, the GMM estimator requires a two-step procedure with an efficient weight matrix while the EL approach internally handles the weight matrix within the estimation procedure (Judge and Mittelhammer, 2003**)**. In other words, the MEL approach is a one-step estimation procedure as compared to the two-step GMM procedure that makes it free from the pre-estimation of the weight matrix. This one-step procedure makes EL a more efficient and less biased approach as compared to GMM in small samples. Since in real economic problems, the sample is small, therefore using GMM is such a scenario can provide misleading results.

However, the Maximum empirical likelihood (MEL) approach is reported as more appropriate than GMM but still, authors often use GMM. It may be due to complex computation procedures and the non-existence of MEL software packages. Though, MEL is difficult to estimate but still possible to apply in a real economic model. Therefore, this study estimated a real economic problem i.e. consumption function using both GMM and MEL approach. The purpose of this research is to provide the foundations for the implementation of the MEL approach in real economic models. Furthermore, the results of GMM and MEL are also compared to explore the differences between the two estimation procedures. Thus, the primary contribution of this research is the implementation of the MEL approach as an alternative to GMM in real economic problems having a small sample.

2. LITERATURE REVIEW

The asymptotic properties of the GMM estimator have great advantages in numerous settings. However, Newey & Smith, (2004) argued that the asymptotic bias in GMM may increase with the increase in a number of moment restrictions. Similarly, the finite sample properties of GMM are criticized. Several studies reported that GMM has poor finite samples properties (Altonji and Segal, 1996; Hall and Horowitz, 1996; Pagan and Robertson, 1997**)**. The authors studied the finite sample properties of GMM and found they are very different from asymptotic properties **(**Hansen et al., 1996). They examined the properties of GMM with three different methods based on the weight matrix to differentiate these estimators. In the first method, they used the identity matrix as a weighted matrix. The second way, they calculated the Iterative weight matrix where the weight matrix is continuously updating until the convergence of the parameters. The third method is a continuous updating estimator where the weight matrix is repeatedly adjusted with parameter changing in the minimization of the quadratic function. Hansen et al. (1996**)** argued that the properties of the small samples of GMM depend upon an efficient weight matrix. GMM requires pre-estimation of the weight matrix that makes it a two-step estimator. Since we have less information available due to the small sample, therefore GMM may estimate a less efficient weight matrix that ultimately affects its efficiency and biasness. Thus, there is a need for some alternative approach having good finite sample properties and stable asymptotic bias. In contrast, Owen (1988) proposed MEL as an alternative approach that improved the inference in econometric literature. Moreover, Various authors extended the use of the MEL approach to estimate generalized linear models (Kolaczyk, 1994; Owen, 1991; Qin & Lawless, 1994). Afterward, Chen and Keilegom (2009) used the MEL approach to construct the inference of the regression model. Judge and Mittelhammer (2011) used the MEL approach to estimate the regression model having an endogeneity problem. According to the authors, the MEL approach can be used when the parametric functional form for the likelihood function is not available. Therefore, it is also known as the Nonparametric Maximum Likelihood (NPML).

The various studies examined the properties of the finite samples of the MEL approach and reported the MEL approach is less biased than the GMM estimator (Imbens and Spady, 2002; Kitamura, 2001; Newey and Smith, 2004). Particularly, Imbens (1997) compared the MEL approach with standard GMM (Hansen, 1982) and iterative GMM (Hansen et al. 1996) estimator and suggested that the MEL approach is superior to GMM in finite samples.

3. METHOD

The main objective of this study is to implement the Empirical likelihood approach on the economic model that does not satisfy the condition of orthogonality and cause simultaneous biasness, especially for small samples. For this purpose, we have selected the consumption function proposed by Keynes (1936), also known as the Absolute Income Hypothesis (AIH). The structural form of the consumption function is

$$\boldsymbol{Con_t = a + b\,Inc_t + \varepsilon_t} \qquad (1),$$

$$\boldsymbol{Inc_t = Con_t + Inv_t} \qquad (2).$$

Equation (1) is known as the Keynesian consumption function. The term Con_t is household's consumption, Inc_t is per capita income and Inv_t is the investment a and b are intercept and slope respectively. Since, in aggregate demand function given in Equation (2), the coefficient of the consumption and investment are equal to unity therefore there is no need to estimate Equation (2) (Thomas, 1993). The endogeneity problem can be described by a reduced form as

$$\boldsymbol{Con_t = \frac{a}{1-b} + \frac{b}{1-b} Inv_t + \frac{\varepsilon_t}{1-b}} \qquad (3)$$

$$\boldsymbol{Inc_t = \frac{a}{1-b} + \frac{1}{1-b} Inv_t + \frac{\varepsilon_t}{1-b}} \qquad (4)$$

The reduced form of consumption system in Equations (3) and (4) shows that consumption and income are correlated with disturbance term ε_t that is the indication of the endogeneity problem. We could not estimate the consumption function 1 directly by the OLS technique because it will produce biased estimates.

To estimate Equation (1), we require instrumental variables to resolve the endogeneity issue, therefore, investment: *Inv* and Government expenditure: G (Keynes (1936) discussed that the government expenditures are exogenously determined) are appropriate instrumental variables. The Instrumental variables matrix (Z) is

$$\mathbf{Z} = [\text{ constant Investment Government expenditures}] = [\boldsymbol{ones\ Inv\ G}]$$

This system of Equations has simultaneity bias, therefore Ordinary Least Square (OLS) estimator is not suitable to estimate this model. As an alternative, most authors applied the GMM estimator to such kind of models. However, GMM does not hold good finite sample properties reported by numerous authors (Imbens & Spady, 2002; Imbens, 1997; Kitamura, 2001; Newey & Smith, 2004) in literature. Conversely, the MEL approach holds good finite sample properties and can be used to estimate economic models having the problem of simultaneity bias. This study applied the MEL approach to estimate the consumption function and compared its results with GMM.

The consumption function data is collected from the International Financial Statistics (IFS) database. The analysis is conducted for 18 different (developing and developed) countries and the sample size is taken 30 to fulfill the condition of the small samples. In consumption function, the slope coefficient of income is known as the marginal propensity to consume (MPC), with $0 < MPC < 1$ in absolute measures. Therefore, we have used the log transformation where the coefficient of income is known as elasticity of consumption with respect to income. The log transformation may avoid the problem of Heteroskedasticity.

To explore the computation method of both GMM and MEL approach, let us assume a model

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \qquad (5),$$

Where $\mathbf{Y} = [\boldsymbol{Con}] = \begin{bmatrix} con_1 \\ con_2 \\ con_3 \\ \vdots \\ con_t \end{bmatrix}_{\boldsymbol{n}\times\mathbf{1}}$ $,\mathbf{X} = [\mathbf{1}\ \boldsymbol{Inc}] = \begin{bmatrix} 1 & Inc_1 \\ 1 & Inc_2 \\ 1 & Inc_3 \\ \vdots & \vdots \\ 1 & Inc_t \end{bmatrix}_{\boldsymbol{n}\times\mathbf{2}}$ $,\ \boldsymbol{\beta} = \begin{bmatrix} a \\ b \end{bmatrix}_{2\times 1}$ $and\ \boldsymbol{\epsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_t \end{bmatrix}_{\boldsymbol{n}\times\mathbf{1}}$.

This model represents the consumption function in Equation (1). On the other hand, the instrumental variables matrix is

$$\mathbf{Z} = [\mathbf{1}\ \boldsymbol{Inv}\ \boldsymbol{G}] = \begin{bmatrix} 1 & Inv_1 & G_1 \\ 1 & Inv_2 & G_2 \\ 1 & Inv_3 & G_3 \\ \vdots & \vdots & \vdots \\ 1 & Inv_t & G_t \end{bmatrix}_{\boldsymbol{n}\times\mathbf{3}}.$$

The GMM approach is used when the moment conditions are greater than the number of parameters proposed as by (Hansen, 1982). The GMM estimator is defined as

$$\widehat{\boldsymbol{\beta}}_{\mathbf{GMM}} = [\mathbf{X'Z\,W\,Z'X}]^{-1}\mathbf{X'Z\,W\,Z'Y} \qquad (6).$$

According to Hansen (1982**),** the weight matrix 'W' is equal to the inverse of the covariance of $(Z'\epsilon)$. Specifically, the weight matrix can be described as

$$\mathbf{W} = (\mathbf{Z}'\sigma^2\mathbf{I}\,\mathbf{Z})^{-1} = (\sigma^2\mathbf{Z}'\mathbf{Z})^{-1} \quad (7).$$

Though various studies have used GMM to estimate such economic problems, GMM is often criticized due to its poor finite sample characteristics. Various authors have documented that GMM is a good estimator for a large sample but provides biased estimations for a small sample. It is also notable that researchers often have small sample in estimating real economic problems. Therefore, it is desirable to use some other estimator holding good finite sample properties to solve economic problems, especially with a small sample. Judge and Mittelhammer (2011) argued that MEL is one of the estimators having such properties. Various studies have used the MEL approach to estimate linear regression models (Kolaczyk, 1994; Owen, 1991; Judge and Mittelhammer, 2011). The computation procedure of the MEL approach is provided in the following section.

3.1 MAXIMUM EMPIRICAL LIKELIHOOD (MEL) APPROACH

Judge and Mittelhammer (2011) used MEL approach when the orthogonal condition does not hold (i.e. $E(\mathbf{X}'\boldsymbol{\epsilon}) \neq 0$). The authors used instrumental variables which satisfy the condition of $(E(\mathbf{Z}'\boldsymbol{\epsilon}) = 0)$, accordingly the moment Equation is

$$E\left(\mathbf{Z}'_{[i,.]}\left(\mathbf{Y}_i - \mathbf{X}_{[i,.]}\boldsymbol{\beta}\right)\right) = 0 \quad (8).$$

Judge and Mittelhammer, (2011) applied MEL approach to resolve the problem of endogeneity with instrumental variables is given as

$$\max_p [n^{-1}\textstyle\sum_{\mathbf{i=1}}^{\mathbf{n}} \ln(\boldsymbol{p_i})$$

subject to

$$\textstyle\sum_{\mathbf{i=1}}^{\mathbf{n}} \boldsymbol{p_i} = \mathbf{1} \ \& \ \sum_{\mathbf{i=1}}^{\mathbf{n}} \boldsymbol{p_i}\left(\mathbf{Z}'_{[i,.]}\left(\mathbf{Y}_i - \mathbf{X}_{[i,.]}\boldsymbol{\beta}\right)\right) = \mathbf{0} \quad (9).$$

The MEL function with Lagrange constraint is as follows

$$\ln\left(\mathbf{L_{EL}}(\boldsymbol{\beta};\mathbf{Y},\mathbf{X},\mathbf{Z})\right) = \mathbf{n}^{-1}\textstyle\sum_{\mathbf{i=1}}^{\mathbf{n}} \ln(\boldsymbol{p_i}) - \boldsymbol{\mu}(\sum_{\mathbf{i=1}}^{\mathbf{n}}(\boldsymbol{p_i}) - \mathbf{1}) - \boldsymbol{\lambda}'\sum_{\mathbf{i=1}}^{\mathbf{n}} \boldsymbol{p_i}\left(\mathbf{Z}'_{[i,.]}\left(\mathbf{Y}_i - \mathbf{X}_{[i,.]}\boldsymbol{\beta}\right)\right) \quad (10)$$

Where $\mathbf{Y}_{(\mathbf{n}\times\mathbf{1})}, \mathbf{X}_{(\mathbf{n}\times\mathbf{2})}, \mathbf{Z}_{(\mathbf{n}\times\mathbf{3})}$ $\boldsymbol{\beta}_{(\mathbf{k}\times\mathbf{1})}$and $\boldsymbol{\lambda}_{(\mathbf{k}\times\mathbf{1})}$. The $\boldsymbol{p_i} > 0\ \forall\ i$ is an implicit structure in the MEL function.

The first order optimization condition with respect to p_i, $\boldsymbol{\lambda}$ and $\boldsymbol{\beta}$ is as follows:

$$\boldsymbol{\Psi_p} = \boldsymbol{p}_{\mathbf{i}}(\boldsymbol{\beta},\boldsymbol{\lambda}) = \left(\mathbf{n}\boldsymbol{\lambda}'\left(\mathbf{Z}'_{[i,.]}\left(\mathbf{Y}_i - \mathbf{X}_{[i,.]}\boldsymbol{\beta}\right)\right) + \mathbf{1}\right)^{-1} \quad (11),$$

$$\boldsymbol{\Psi_\lambda} = \textstyle\sum_{\mathbf{i=1}}^{\mathbf{n}} \boldsymbol{p_i}\left(\mathbf{Z}'_{[i,.]}\left(\mathbf{Y}_i - \mathbf{X}_{[i,.]}\boldsymbol{\beta}\right)\right) = \mathbf{0} \quad (12),$$

$$\boldsymbol{\Psi}_{\beta} = \sum_{i=1}^{n} \boldsymbol{p}_i\left(-\boldsymbol{\lambda}'\mathbf{Z}'_{[i,.]}\mathbf{X}_{[i,.]}\right) = \mathbf{0} \tag{13}$$

We restrict Equations (12) and (13) by putting the optimal value of $\boldsymbol{p}_i$ obtain from Equation (11) and then

$$\boldsymbol{\Psi}_{\lambda} = \boldsymbol{\lambda}(\boldsymbol{\beta}) = \sum_{i=1}^{n}(\mathbf{n}\boldsymbol{\lambda}'\left(\mathbf{Z}'_{[i,.]}\left(\mathbf{Y}_i - \mathbf{X}_{[i,.]}\boldsymbol{\beta}\right)\right) + \mathbf{1})^{-1}\left(\mathbf{Z}'_{[i,.]}\left(\mathbf{Y}_i - \mathbf{X}_{[i,.]}\boldsymbol{\beta}\right)\right) = \mathbf{0} \tag{14}$$

$$\boldsymbol{\Psi}_{\beta} = \sum_{i=1}^{n}\left(\mathbf{n}\left[\left(\mathbf{Z}'_{[i,.]}\left(\mathbf{Y}_i - \mathbf{X}_{[i,.]}\boldsymbol{\beta}\right)\right) + \mathbf{1}\right]\right)^{-1}\left(-\boldsymbol{\lambda}'\mathbf{Z}'_{[i,.]}\mathbf{X}_{[i,.]}\right) = \mathbf{0} \tag{15}$$

The mathematical expression $\lambda(\beta)$ in Equation (14) is an implicit function of β, it cannot be specified in closed form (Judge & Mittelhammer, 2011). The probability weights p_i in Equation 11 that must be satisfied with the condition of $0 \leq p_i \leq 1$ which demonstrate that the values of the vectors of the coefficient of constraints λ and the parameters β principally fulfill the condition $[(\lambda'(Z'(Y - X\beta)) + 1) > 1/n]$ for each observation i (Qin and Lawless, 1994). To estimate the MEL problem, Equations 14 and 15 can be solved both; sequentially and simultaneously by using the Newton-Raphson algorithm.

$$\boldsymbol{\Omega} = \boldsymbol{\Psi}_{\lambda}{}^{2} + \boldsymbol{\Psi}_{\beta}{}^{2} \tag{16}$$

Equation 16, $\boldsymbol{\Omega}$ can be optimized numerically (minimize the squared Euclidean norm for necessary conditions) by using Equations 14 and 15. The MEL problem can be solved by the sequential procedure as, where first 14 is solved for $\boldsymbol{\lambda}$ for a given value of $\boldsymbol{\beta}$ (initial guess $\mathbf{0}$ as the starting point of $\boldsymbol{\beta}$) and then solve 15 for $\boldsymbol{\beta}$ at given the previous value of $\boldsymbol{\lambda}$ to minimize the square Euclidean norm. We continue the process until the convergence achieves. We have used the linear transformation discussed in (Mittelhammer et al., 2000; Qin & Lawless, 1994) to transform the overdetermined model to just a determined model that can estimate the unique values of unknown parameters of one step MEL approach.

4. RESULT AND DISCUSSION

In this section, we have reported the results and their interpretation of the analysis. All coefficients shown in table 1 are from the results of consumption function estimated by GMM and MEL approach for several countries. The results of MEL and GMM to estimate consumption function are provided in Table 1. In this Table, α represents the intercept while β is the measurement of the elasticity of consumption with respect to income. Values given in parenthesis are the t-statistics. The results of MEL are showing that the impact on income on consumption is significant for all countries except in the case of Germany. The t-statistics of consumption in the case of Germany is 1.6301 showing insignificant effect. Conversely, the highest value of t-statistics (34.1184) of b is found in the case of Hungary. Results of MEL also showed that the elasticity of consumption w.r.t income ranges from 0.7929 (China) to 1.1101 (Denmark).

Table 1: Result of Consumption Function Estimated by GMM and MEL

Countries	Ln (Consumption) = α + β Ln (Income)+ e					
	Maximum Empirical Likelihood (MEL) Estimates			Generalized Method of Moment (GMM) Estimates		
	$\hat{\alpha}$	$\hat{\beta}$	R^2	$\hat{\alpha}$	$\hat{\beta}$	R^2
Sri Lanka	0.9410 (0.6997)	0.9074 (9.1952)	0.999	0.2504 (0.1259	0.9583 (6.4337)	0.999
Spain	0.7457 (0.3760)	0.8789 (4.6004)	0.997	-0.1728 (-0.1090)	0.9703 (6.0925)	0.999
Pakistan	0.4632 (0.3635)	0.9020 (5.1949)	0.992	-0.2146 (-0.1815)	0.9906 (6.2893)	0.998
Norway	0.2837 (0.1896)	0.8464 (4.0184)	0.998	-0.1718 (-0.0795)	0.9153 (2.9167)	0.998
Italy	-0.5071 (-1.2651)	0.9949 (16.9601)	0.999	-0.4995 (-0.5104)	0.9938 (6.4026)	0.999
Ireland	0.4267 (0.4622)	0.9031 (9.5402)	0.998	0.0733 (0.0472)	0.9404 (5.7164)	0.999
India	0.8146 (0.5591)	0.8660 (5.7388)	0.996	0.1028 (0.0697)	0.9418 (5.9166)	0.999
Hungary	-0.6312 (-2.5723)	1.0231 (34.1184)	0.999	-0.5898 (-0.4824)	1.0184 (7.1307)	0.998
Greece	-0.6391 (-1.5393)	1.0250 (19.8596)	0.999	-0.6324 (-0.5788)	1.0243 (8.3178)	0.999
Germany	0.7255 (0.1923)	0.8206 (1.6301)	0.985	-0.9732 (-0.3805)	1.0493 (3.0747)	0.999
France	0.3264 (0.1965)	0.8926 (4.5497)	0.990	-0.4848 (-0.2602)	0.9913 (4.3434)	0.999
Finland	-0.5406 (-2.9108)	0.9823 (27.7550)	0.999	-0.5330 (-0.4287)	0.9810 (4.4161)	0.999
Fiji	0.4281 (0.3051)	0.8780 (4.7293)	0.988	0.3976 (0.1750)	0.8821 (2.9465)	0.988
Egypt	0.5412 (0.2144)	0.9249 (4.4769)	0.998	-0.7116 (-0.3941)	1.0315 (6.8427)	0.999
Denmark	0.3810 (0.1315)	1.1101 (2.0263)	0.993	1.6114 (0.7497)	0.8731 (2.1317)	0.998
Hong Kong	0.5022 (0.2828)	1.0677 (3.1828)	0.992	0.5257 (0.3952)	1.0632 (4.2049)	0.992
China	0.9694 (0.4358)	0.7929 (3.2017)	0.979	-0.1116 (-0.0970)	0.9187 (6.7383)	0.998
Canada	0.0911 (0.0558)	0.9067 (3.7276)	0.998	-0.5973 (-0.2244)	1.0050 (2.5231)	0.999

*Author's calculation
**t-values in the parenthesis

In most cases, the elasticity is around 90%. This indicates that in most cases consumption is less sensitive to income. However, the β four countries are more than 1 indicating more sensitivity of consumption towards income. The MEL R^2 for all the countries is around 99%.

On the other hand, the results of GMM showed significance β for all the countries. The t-statistics of β range from 2.13 (Denmark) to 8.32 (Greece). In most cases, t-statistics is around 6. Similarly, the β's of all the countries range from 0.87 (Denmark) to 1.063 (Hong Kong). The results of GMM also showed that β's of most of the countries is around 1. Six countries are showing β greater than 1 in the case of GMM. Similarly, only two countries have the elasticities of less than 0.90. At last, the R^2 of GMM for all the countries is around 99%.

Comparing the results of MEL and GMM can provide some useful implications. A comparison of β estimated by MEL and GMM for all the countries are in Figure 1. Results show that in most cases elasticity of MEL and GMM are near to 0.90 and 1 respectively. This shows that the estimation of MEL is different from GMM. Figure 1 shows that MEL and GMM estimated similar is in the case of six countries i.e. Fiji, Finland, Greece, Hong Kong, Hungry and Italy. For other countries, both the estimators provided different results. It is found that the elasticity of Denmark (0.8731) by GMM is minimum while the same country showed the highest elasticity (1.1101) when the MEL estimation method is applied. Even in the case of Denmark and Germany the differences are high. Various studies reported that the MEL approach is less biased than the GMM estimator (Imbens and Spady, 2002; Kitamura, 2001; Newey and Smith, 2004). Therefore, the results of this study show that the estimates of the MEL approach are very different from GMM estimators due to its good small sample properties.

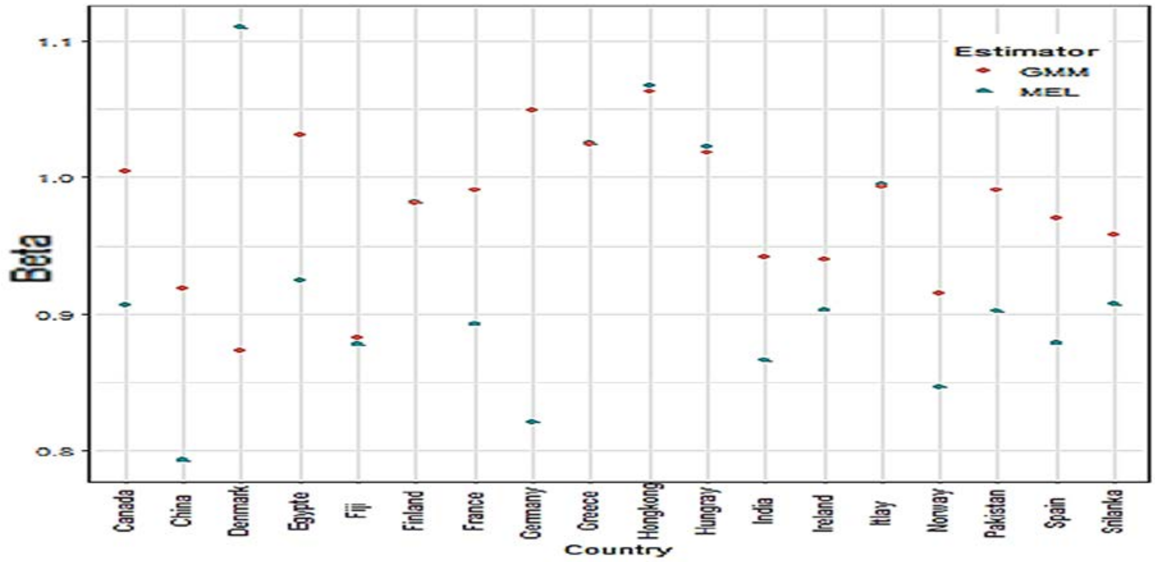

Figure 1: Estimator and estimates of endogenous regressor.

Similarly, Figure 2 compares the t-statistics computed by MEL and GMM. The graph is showing that MEL estimated very high t-statistics as compared to GMM in the case of Finland, Greece, Hungry, Ireland and Italy. However, in the case of Germany, Hong Kong, and China the estimation of t-statistics by MEL is less than GMM. It is also notable that for Germany, the t-statistics estimated by GMM is 3.0747 (significant) while the same statistic estimated by MEL is 1.6301 (insignificant).

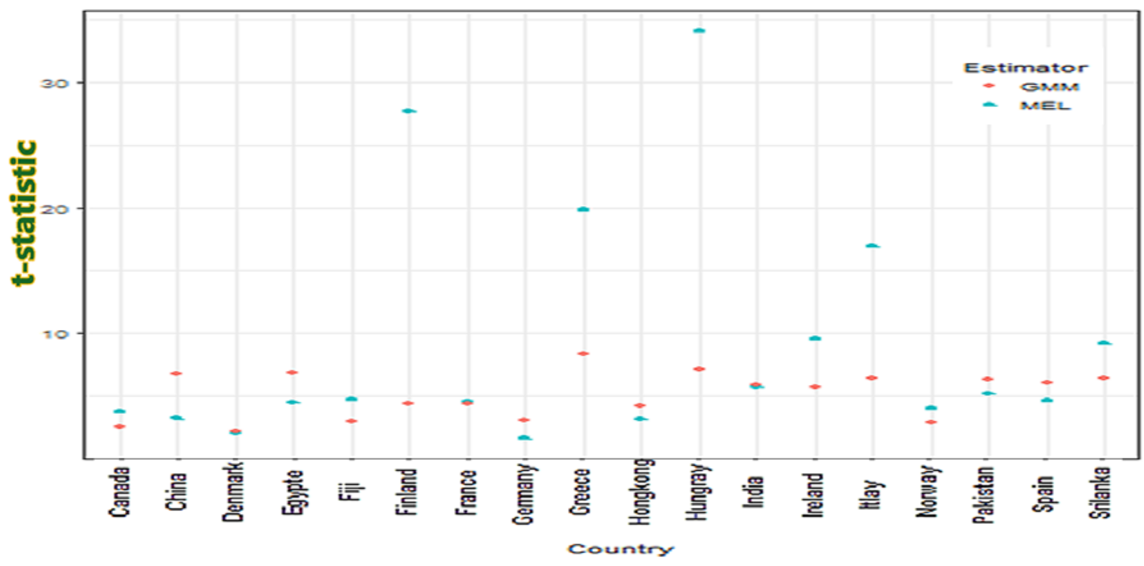

Figure 2: Estimators and significance with t-statistics.

These results and arguments are alluding towards the superiority of MEL over GMM in a small sample. Imbens (1997) compared the MEL approach with the standard GMM approach and found that the MEL approach has the least bias and lower MSE than GMM.

5. CONCLUSION

This study explored the implications of MEL and GMM in estimating a real economic problem. The Keynesian consumption function was selected due to the existence of the endogeneity problem and having a finite sample. The literature shows that GMM has poor finite sample properties whereas MEL assumed to be superior then GMM. This study primarily compared the results of MEL with GMM. Based on analysis; the study concluded that elasticities estimated by MEL and GMM are significantly different and in some of the cases differences are drastically high. This study also finds that in most of the cases both MEL and GMM estimates showed highly different t-statistics. Both estimators showed the same level of goodness of fit however there are differences in standard errors of parameters estimated by MEL and GMM. It is believed that such differences are due to poor finite sample properties of GMM. Thus, the study suggests that GMM provides biased and less efficient estimates in finite samples. Therefore, it is highly recommended to use MEL rather than GMM to estimate economic models having an endogeneity problem in a finite sample to obtain the best estimates.

6. AVAILABILITY OF DATA AND MATERIAL

Data can be made available by contacting the corresponding author.

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