

SIMULATION OF SEQUENTIAL PROCESSING OF A MOVING EXTENDED OBJECT

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ABSTRACT

In this article, we propose two models for the task of conveyor processing of a moving extended object. As a typical situation of such processing, the problem of cooling the hot rolling strip on the discharge roller table to achieve a given winding temperature profile is considered. The two models under consideration correspond, firstly to Euler coordinates, in which the conveyor base is stationary, and, secondly, to Lagrange coordinates, in which the processed strip is motionless. In both cases, we describe the neighborhood structure for the model, that is, a directed graph whose vertices correspond to the state and control variables of the model. The advantages and disadvantages of each of the two models are discussed.

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1. INTRODUCTION

The main motivation for this article was the well-known problem of modeling the process of forced cooling of the hot rolling strip on the discharge roller table by means of water shower units - see, for example, [1, 2, 3]. In this article, we propose and consider an abstract and largely simplified statement of this problem. Namely, we consider a certain abstract process of sequential handling of a moving extended object (in what follows - the conveyor processing of a moving strip) by stationary devices. It is assumed that the properties of the strip are characterized by a one-dimensional random process at the entrance to the conveyor and, in the general case, the strip has self-acting. In the original problem of cooling a hot-rolling strip, a one-dimensional random process is the input temperature recorded by the pyrometer, and self-interaction corresponds to internal heat transfer processes. We describe so called neighborhood structures (see [4, 5, 6]) for two discrete models of conveyor processing of the moving strip. The first model uses Euler variables associated with a fixed conveyor base while the strip is considered a moving part of the conveyor. In the second model, the Lagrange variables associated with the moving strip are used, so the processing devices are considered as moving objects. In subsequent publications on the basis of these

structures, firstly, discrete dynamic systems will be recorded and, secondly, an algorithm for controlling the on-off modes of processing devices depending on the random input process and on the processes inside the strip will be indicated. The purpose of the control is to approximate some predefined profile of the strip properties at the exit of the conveyor.

2. Method Description: Neighborhood structures and systems

Suppose we are going to build a mathematical model of a process in the form of a discrete control system

$$\begin{cases} X^{t+1} = F(X^t, U^t) \\ W^t = C(X^t, U^t) \end{cases} \quad (1)$$

(see [7]). It is convenient, in many cases, to begin with the stage of constructing a *neighborhood structure*, see [4]-[6]. The concepts of the neighborhood structure and the corresponding neighborhood system have arisen as a generalization of the finite differences method and are especially convenient in those cases when the discrete system (1) corresponding to any particular problem is very sparse.

The neighborhood structure $\mathfrak{N}(V)$ over a finite set V is a connected digraph $\mathfrak{N} = \mathfrak{N}(V, E)$, containing vertices $V = U \sqcup X \sqcup W$ of three types: inputs U , nodes X and outputs W , while:

- each input $u \in U$ has only outgoing arcs of the form $e(u, *)$, where $* \in X \sqcup W$;
- each output $w \in W$ has only incoming arcs of the form $e(*, w)$, where $* \in U \sqcup X$;
- each node $x \in X$ has incoming and outgoing arcs;
- each node $x \in X$ can have a loop $e(x, x)$, which is considered both as incoming and as outgoing arc;
- any two nodes $x', x'' \in X$ can be connected by no more than two (oppositely directed) arcs $e(x', x'')$ and $e(x'', x')$.

The arcs of a neighborhood structure are also called connections or links. In the figures, the nodes are represented by circles, the entrances and exits by squares. All inputs U are divided into two classes: internal controlled inputs \tilde{U} and external uncontrolled inputs \hat{U} .

The neighborhood system associated with the neighborhood structure $\mathfrak{N}(V)$ consists of equations for the nodes and outputs of the structure, such that:

- for each input u_i , node x_i and output w_i there is a corresponding (scalar or vector) variable $U(i)$, $X(i)$ and $W(i)$;
- for each node x_i and each output w_i there is a corresponds (scalar or vector) equation;
- the equation for each node x_i contains only variables corresponding to the vertices from U and X , entering this node;
- the equation for each output w_i contains only variables corresponding to the vertices from U and X , entering this output.

In the general case, the equations of a neighborhood system can be explicit or implicit, static or dynamic; dynamic equations can be discrete or continuous. In this paper, it is assumed that all equations of the neighborhood system are explicit discrete-dynamic equations of the form $X^{t+1}(i) = F_i^t(*)$ (for nodes) and $W^t(i) = C_i^t(*)$ (for outputs). Depending on the presence or absence of a loop $e(x, x)$, the equation $X^{t+1}(i) = F_i^t(*)$ for the node x_i contains or does not contain the variable $X^t(i)$ in the right-hand side.

Let us explain the appearance of the superscript t in the right-hand sides of the equations. It is

usually assumed that the neighborhood structure completely determines the set of variables on the right-hand sides of the equations, namely, this set corresponds to the existing arcs. However, in the case when the system is not stationary, the set of arcs and, therefore, the set of variables may depend on time. In our definition of a neighborhood structure, a change in arcs over time is not provided. The reason is that for neighborhood structures there is a natural analogue of the transition to an extended phase space, which allows us to always consider only structures with constant arcs, i.e., as in the our definition. Unfortunately, after extension, the neighborhood structure often becomes too complex. This applies also to our problem, regardless of the choice of coordinate system, Euler or Lagrange. To simplify the exposition and drawings, we choose a compromise option when the structure does not expand, and instead, for each vertex, all arcs activated at any moment of time are considered. In order to write down the equations of a non-stationary neighborhood system in this case, we need additional information about arcs activated at each given moment of time. We reflect this in the presence of the variable t in F_i^t and C_i^t .

3. NEIGHBORHOOD STRUCTURE FOR MODEL IN EULER COORDINATES

We call the Euler's model for conveyor processing a model in which the vertices and variables of the neighborhood structure are associated to the fixed conveyor base.

The neighborhood structure corresponding to the Euler model of conveyor processing contains $n + 1 = 1 + n_1 + n_2 + n_3$ nodes x_0, x_1, \dots, x_n (discretization of the conveyor base) with variables $X(i)$, $i = 0, 1, \dots, n$, one external input \hat{u} with a variable \hat{U}^t , $t = 0, 1, \dots, N$, one output w with a variable W^t , $t = n, n + 1, \dots, n + N$, and n_2 control inputs $\tilde{u}_{n_1+1}, \dots, \tilde{u}_{n_1+n_2}$ with variables $\tilde{U}^t(i)$, $i = n_1 + 1, \dots, n_1 + n_2$. The initial node x_0 is connected to the input \hat{u} that generates and transmits a time-dependent input state $X^t(0) = \hat{U}^t$, $t = 0, 1, \dots, N$. Next, $n_1 + 1$ is the number of starting nodes when the strip is not processed. The next n_2 nodes, from $n_1 + 1$ to $n_1 + n_2$, are the band processing nodes. In the final n_3 nodes, from $n_2 + 1$ to $n = n_1 + n_2 + n_3$, the strip is not processed, as in the initial nodes. Above the band processing nodes there are n_2 controlled inputs $\tilde{u}_{n_1+1}, \dots, \tilde{u}_{n_1+n_2}$ (which explains the rule of numbering rule for controlled inputs). In the case of cooling the hot rolling strip, the indicated nodes and controlled inputs correspond to the initial section of the discharge roller table, the section with cooling units, the final section of the roller table until the winding unit and the block of cooling units themselves. Below, in Fig. 1, the case is shown when $n_1 = n_2 = n_3 = 2$.

The passage of the strip along the conveyor takes $n + N$ discrete time moments $t = 1, \dots, n + N$, each time moment corresponds to a shift of the strip by one node to the right. At the initial moment $t = 0$, the starting point of the strip located in the node x_0 , at the moment $t = n + N$, the ending point of the strip located in the node x_n . Through each node, the strip passes for $N+1$ time moments. In the general case, n and N can be any: $n \leq N$ or $n \geq N$.

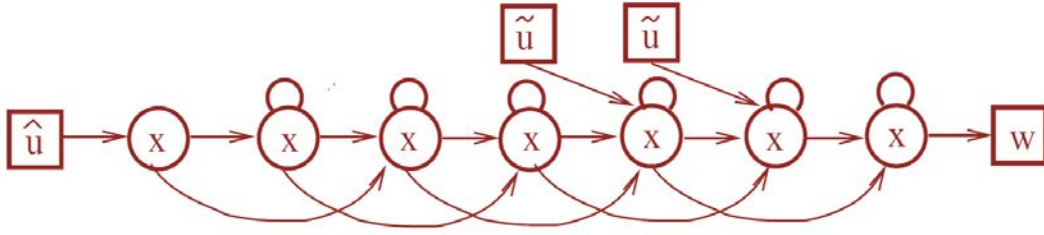


Figure 1: Neighborhood structure for Euler model

The presence of the zero-node is convenient for synchronization with discrete time: at the zero time moment, the beginning of the strip is in the zero node. The direction of the arrows from the controlled inputs down and to the right is associated with the movement of the strip, that is, the input \tilde{u}_i acts as a result of the movement not on the node x_i , but on the node x_{i+1} . In addition, the movement of the strip is reflected in the structure of the arcs between the nodes: the action of a certain element of the strip on its neighbors left and right (i.e., the self-action of the strip) in the Euler model corresponds to a loop and incoming arrows from two neighbors to the left. Thus, this non-trivial circumstance is associated with the use of the Euler coordinate system.

In the model adapted for the problem of cooling the rolling strip, one time moment corresponds to a shift by a distance Δ between the centers of the shower units, the total length of the strip is $N \cdot \Delta$ and, as a rule, $N \gg n$.

An external input (random process) \hat{U}^t sets the state of the zero node: $X^t(0) = \hat{U}^t$, $t = 0, 1, \dots, N$. Further, the output of the model at time t is given by the equation $W^t = X^t(n)$, while the vector (profile) of the properties of the strip at the output corresponds to the time moments $t = n, n + 1, \dots, n + N$, that is, it is a vector of dimension $N + 1$:

$$W = [X^n(n), X^{n+1}(n), \dots, X^{n+N}(n)] \quad (2).$$

The equation for the state of a node in the general case has the form

$$X^{t+1}(i) = F_i^t(X^t(i), X^t(i-1), X^t(i-2), U^t(i-1)) \quad (3),$$

but it should be noted that, depending on the relationship between i and t , there may be no control and some of the states on the right side, and, moreover, the equation itself may be absent. For example, at $t = 0$ and $t = n + N$, the system will consist of only one equation.

According to [6], the neighborhood structure allows us to write down a formal system of equations, so called “metasystem”, associated with the structure. We already mentioned the reason why we cannot do this in our case: the neighborhood structure that is fully adequate to the dynamic problem under consideration is not stationary, that is, it depends on time. Such a structure must be described in an extended phase space, see the remarks above in section 2 and also in Section 3.1.5 of the book [6]. An alternative option that we have chosen is that we consider a stationary neighborhood structure containing vertices and arcs for all moments of time, keeping in mind that at each particular moment in time, only part of the vertices and arcs are used to record the system. A full description of the system will be given in future publications.

4. NEIGHBORHOOD STRUCTURE FOR MODEL IN LAGRANGE COORDINATES

We call the Lagrange model of conveyor processing a model in which the vertices and

variables are associated with the moving strip. As in the case of the Euler model, the neighborhood structure that is completely adequate to the dynamic problem is not stationary. Same as before, to simplify the construction, we choose an alternative option when all vertices and connections are present simultaneously in the structure (for all time instants), but at the same time, only part of them are used to record the system at any particular moment in time. A detailed description of the system is postponed until the next publication, but some rules for including or excluding vertices and links depending on time will be indicated below.

Further we will use the notation n_1, n_2, n_3 and N , introduced in the previous paragraph. The neighborhood structure corresponding to the Lagrange model contains N nodes with variables $X^t(i), i = 1, \dots, N$, corresponding to the discretization of the strip (not the conveyor, as in the Euler model), one external input with the variable $\hat{U}^t, t = 0, 1, \dots, N$, one output with the variable $W^t, t = n, n + 1, \dots, n + N$, and n_2 control inputs with variables $\tilde{U}^t(i), i = n_1 + 1, \dots, n_1 + n_2$.

The external input \hat{U}^t acts on all nodes, not simultaneously but sequentially: at time moment t on the node $X^t(i), i = t$. Each of the control inputs $\tilde{U}^t(i)$ can act (or, depending on the control program, do not act) on each of the nodes, not simultaneously, but sequentially: at time $t = n_1 + i, \dots, n_1 + i + N$ on the node $X^t(i), i = t$. Figure 2, the case is shown when $N = 5$ and $n_2 = 1$.

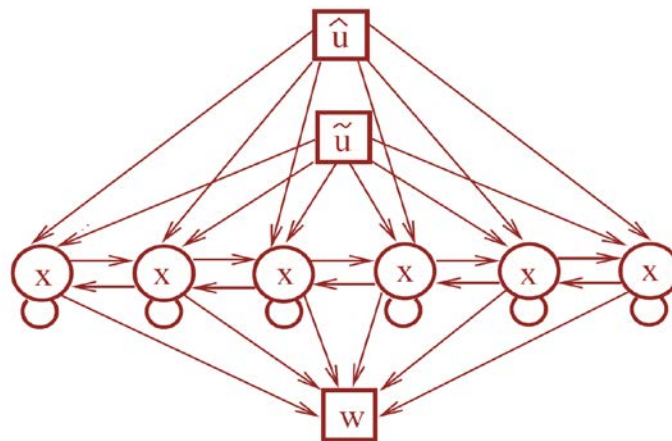


Figure 2: Neighborhood structure for Lagrange model.

In this case, the reverse numbering of nodes is convenient, from right to left: the beginning of the strip corresponds to the rightmost node. The constructed neighborhood structure does not provide information on the number of nodes of inactive parts of the conveyor n_1 and n_2 ; this information will only be contained in the corresponding neighborhood system. Note that the neighborhood structure constructed earlier for the Euler model did not contain information about the strip length N .

5. CONCLUSION

Two neighborhood structures are constructed for the conveyor processing of a moving extended object, corresponding to the use of Euler and Lagrange variables. Neighborhood structures adequate to the task are non-stationary. To simplify the constructions, an alternative option was chosen when the structure is stationary, but the neighborhood system is not uniquely restored by the structure. The connection of the discussed structures with the task of cooling the hot rolling strip to achieve the desired temperature profile of the winding is described.

6. AVAILABILITY OF DATA AND MATERIAL

Data can be made available by contacting the corresponding authors

7. ACKNOWLEDGMENT

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