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SYNCHRONIZATION OF DELAYED INTEGER ORDER AND DELAYED FRACTIONAL ORDER RECURRENT NEURAL NETWORKS SYSTEM WITH ACTIVE SLIDING MODE CONTROL

Fatin Nabila Abd Latiff [1*], Wan Ainun Mior Othman [1*], N. Kumaresan [1]

[1] Institute of Mathematical Sciences, Faculty of Science, University of Malaya, Kuala Lumpur, 50603, MALAYSIA.

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ABSTRACT

Chaotic Neural Networks (CNNs) has been gaining a lot of attention and have become a hot topic from researchers with good expectation. To resolve the synchronization's problem of delayed integer order recurrent neural networks (IoDRNNASM) and delayed fractional-order recurrent neural networks (FoDRNNASM), an active sliding mode control (ASMC) scheme is introduced. Factional Lyapunov direct methodology (FLDM) is designed and is enforced to ASMC of the systems to keep the stability of the systems. To investigate the characteristics of IoDRNNASM and FoDRNNASM, we tend to enforce the method of numerical simulation by utilizing MATLAB programming to demonstrate the performance and efficiency of the results. Based on this study, the results show that the synchronization between integer-order and fractional-order will significantly occur once the recommended ASMC is introduced. This main result can provide a great advantage within the area of network security of secure communication by implement double encryption by conducting RSA encryption. We do believe that this idea can improve security and provides strong protection in secure communications.

Disciplinary: Mathematics, Computer Science (Network/Cyber Security).

1 INTRODUCTION

Fractional calculus is a mathematical theory that has been studied and applied in different fields for the past 300 years. It can trace back to Leibniz, Liouville, Letnikov, Grunwald, and Riemann [1]. Earlier, the dearth of effective techniques was the main reason for not using fractional-order calculus, but now the main reason for its exploitation is because it may be simply

*Corresponding author (W.A.M.Othman, F.N.A.Latiff) wanainun@um.edu.my, fatin_nabilaa@yahoo.com

employed in a variety of fields. Various fractional-order calculus has been discovered because of its implementation in emerging engineering vary from cryptography, secure communication to electrically couple neuron system [2]. So recently, the application of their significant and various type of recurrent neural networks (RNNs) have been widely studied by numerous researchers.

Chaotic systems synchronization has to obtain a load of recognition between analyzers and scientists from completely different sorts of research fields [3]. Since it was proposed by [4], investigation of chaotic systems has been studied intensively because of its striking possible implementation in a variety of fields like secure communication [5, 6], chemical and biological systems [7], and cryptography [5] due to sensitivity of initial condition and prediction difficulty of chaos. By now, a great deal of different chaotic synchronization methods was introduced to perform a chaos synchronization. They are PC method [4], active control [8], adaptive control [9], impulsive control [10], sliding mode control (SMC) [11], adaptive fuzzy control [12], etc. Additionally, a range of fractional-order neural networks (FoNNs) synchronization is found by researchers like Mittag-Leffler synchronization [10], Cohen-Grossberg synchronization [13], complete synchronization [14], lag synchronization [15], adaptive synchronization [13], memristive synchronization [16] and etc.

Previously, Bai and Lonngren [17] used a technique like active control to synchronize two Lorenz systems. A similar method has been employed by several researchers to synchronize different fractional-order chaotic systems [18, 19]. This is because of its inherent advantages, as it is a powerful control technique in synchronization chaotic systems [20, 21]. While delayed sliding mode control (DSMC) method has various benefits, as well as poor vulnerability to the intrusion and also the parameter perturbation, implementation restraint, and rapid response. This approach may be a discontinuous control (DC) strategy requiring the selection of a switching surface [SWS] for the defined dynamics systems as well as the creation of a DC law [22-24]. There are two stages required to design a delayed sliding mode controller (DSMC). The first move is to decide on the most effective active controller (ACC) to assist the designing of the series of ASMC series. Another step is by developing a delayed active sliding mode controller (DASMC) to achieve synchronization.

The synchronization of FoNNs with integer-order neural networks [IoNNs] is widely alleged to be efficiently practical in encryption which can increase the key area. A lot of finding has been obtained from the synchronization of FoNNs [13, 25]. Nevertheless, to the finest of the author's information, the results on the synchronization of FDRNNASM systems and IDRNNASM systems are limited and we do believe that the idea never been employed in any research. But other researchers have developed chaotic systems for the synchronization of integral order with fractional-order chaotic systems [11, 26]. From a functional point of view, it is necessary to realize that the cause of FoNN's synchronization can produce hybrid unstable transient signals that are produced before the final states. This feature will help improve communication protection, as the FoNNs have many modifiable variables compared with the IoNNs. By using this idea in secure communication, the security can be improved as it is unbreakable.

Motivated by all the analysis and literature surveys, in this paper we introduce the steps needed for the synchronization between IoDRNNASM systems with FoDRNNASM. Based on the ASMC that we build to support the existence of sliding motion, the synchronization of

IoDRNNASM systems and FoDRNNASM systems is achieved. The delayed sliding surface [DSS] and DSMC play a major role in this research to help the synchronization process.

This work, section 2 briefly summarizes the structure of the neural network. In Section 3, based on the FLDM theory, DASMC and tracking control are built to ensure the synchronization of the proposed model and the idea of synchronization between FoDRNNASM and IoDRNNASM are introduced. In Section 4, numerical simulation of the proposed model of synchronization of FoDRNNASM with IoDRNNASM is done to demonstrate the suitability and efficiency of the gained results, and some discussion is drawn with the idea for double encryption using RSA (Rivest-Shamir-Adleman) encryption. Finally, the conclusion is drawn based on our study of the proposed system.

2 PRELIMINARIES

The master system of FoDRNNASM is denoted by

$$D^{\alpha}\chi_i(t) = -q_i\chi_i(t) + \sum_{l=1}^{m}\sum_{a=1}^{c} u_{ikj} f_k\big(\chi_k(t)\big) + \sum_{l=1}^{m}\sum_{p=a}^{c} v_{ikj} f_k\big(\chi_k(t-\tau)\big) + \Gamma_i \quad (1).$$

Or the vector form

$$D^{\alpha}\chi(t) = -Q\chi(t) + Uf(\chi(t)) + Vf(\chi(t-\tau)) + \Gamma \quad (2),$$

where D^{α} denotes an α fractional derivative of Caputo, where $0 < \alpha < 1$. State vector denoted by $\chi(t) \in R^n$, the activation function is denoted by $f(\cdot)$. While C, U, and V represent the constant matrices, the external vector is indicated as Γ. The primary states of (1) are represented by

$$\chi_i(r) = \varphi_i(r),\ r \in [-\tau, 0],\ i = 1, 2, \cdots, n \quad (3),$$

where $\varphi_i(r) \in C([-\tau, 0], R)$, and the norm, $||\varphi|| = \sup_{r\epsilon[-\tau,0]} \sum_{i=1}^{n} \varphi_i(r)$. The output of the system (2) is denoted by $\Psi(\mathfrak{z}(t), \chi(t)) := \varXi_1\chi(t) + \varXi_2\,\chi(t-\tau)$, where $\Psi(\mathfrak{z}(t), \chi(t)) \in R^m$ is the output state vector and $\varXi_1$, $\varXi_2 \in R^{m\times c}$ are the constant matrices. The equivalent system for slaves is defined as

$$D^{\alpha}\mathfrak{z}(t) = -Ez(t) + Bg\big(\mathfrak{z}(t)\big) + Tg\big(\mathfrak{z}(t-\tau)\big) + J + \Theta(\chi(t)) + \Psi(\mathfrak{z}(t), \chi(t)) \quad (4)$$

where $z(t) \in R^n$ denotes the state vector, while the constant matrices represented by E, B, and H. The SMC that we want to design is represented as $\Theta(\chi(t)) + \Psi(\mathfrak{z}(t), \chi(t))$. The primary states of (4) are represented as

$$z_i(r) = \psi_i(r),\ r \in [-\tau, 0],\ i = 1, 2, \ldots, n,$$

where $||\psi|| = \sup_{r\epsilon[-\tau,0]} \sum_{i=1}^{n} \psi_i(r)$.

Definition 1. {Li, 2010 #28}The basis of the system (1) is unchanging if

$$||\chi(t)|| = \{\Re\big(\chi(t_0)\big)E_q(-\delta(t-t_0)^q)\}^{\sigma}. \quad (5)$$

Lemma 1. (27) "*Assume that the upper right-hand derivative of function* $\mathcal{v}$ *in Caputo's sense of order* $\alpha (0 < \alpha < 1)$ *with respect to time variable is such that for* $t \in [t_0, \infty)$, $\varphi \in PC[-\tau, 0], R^n]$, $\varsigma \in R$ *is a constant, and the following inequality*

$$D_+^\alpha \mathcal{v}(t, \emptyset(t)) \leq \varsigma \mathcal{v}(t, \emptyset(0)),$$

where t_0 denotes the initial instant, $q \in (0,1)$, $\delta > 0$, $\sigma > 0$, $\Re(0) = 0$, $\Re(\chi) \geq 0$, and $\Re(\chi)$ is locally Lipschitz on $x \in R$ with respect to Lipschitz constant $\Re_0$."

Lemma 2. *(28) "If* $f(t)$, $g(t) \in C^1 [t_0, b]$, *then*

(1) $D^{-\alpha} D^{-\beta} f(t) = D^{-\alpha-\beta} f(t), \quad \alpha, \beta \geq 0$;

(2) $D^{\alpha} D^{-\alpha} f(t) = f(t), \quad \alpha \geq 0$;

(3) $D^{-\alpha} D^{\alpha} f(t) = f(t) - \sum_{k=0}^{n-1} \frac{t^k}{k!} f^k(0), \quad \alpha \geq 0.$

(4) $D^\alpha (v_1 f(t) + v_2\, g(t)) = v_1\, D^\alpha f(t) + v_2\, D^\alpha g(t)$, v_1 *and* v_2 *are any constants.*

(5) $D^\alpha c = 0$, c *is any constant."*

3 METHODOLOGY/MATERIALS

Method to procedure ASMC is introduced which is the merging of SMC with ACC. Then we will discuss the proposed method's stability problem.

3.1 METHOD IN ASMC

A chaotic system by a nonlinear differential equation defined as follows:

$$\dot{\chi} = U_1 \chi + f_1(\chi) \qquad (6),$$

where $\chi(t)$ is the state vector, $f_1(\chi)$ is the nonlinear function and U_1 represents a self-connection weight matrix. Equation (1) is the master system. The controller $\Theta(\chi(t))$ and $\Psi(\mathfrak{z}(t), \chi(t))$ are substituted into the slave system (4), such that:

$$D^\alpha \mathfrak{z}(t) = U_2 \mathfrak{z} + f_2(\mathfrak{z}) + \Theta(\chi(t)) + \Psi(\mathfrak{z}(t), \chi(t)) \qquad (7),$$

where $\mathfrak{z}(t)$ is the slave system state vector, U_2 represents a self-connection weight matrix and $f_2(\mathfrak{z})$ is the nonlinear function. $\Phi(\chi(t)) + \Psi(\mathfrak{z}(t), \chi(t))$ is the tracking controller, and we design the tracking controller as

$$\Theta(\chi(t)) = D^\alpha \chi(t) - f(\chi(t)) \qquad (8)$$

The problem in the synchronization procedure is to design the controller $\Theta(\chi(t))$ and $\Psi(\mathfrak{z}(t), \chi(t))$ that will synchronize both system (1) and (4). We expressed the dynamics synchronization error

$$D^\alpha \mathcal{E}(t) = U_2 \mathfrak{z} + f_2(\mathfrak{z}) - U_1 x + f_1(\chi) + \Theta(\chi(t)) + \Psi(\mathfrak{z}(t), \chi(t))$$

$$= U_2 \mathcal{E} + F(\chi, \mathfrak{z}) + \Theta(\chi(t)) + \Psi(\mathfrak{z}(t), \chi(t)) \qquad (9),$$

where

$$\mathcal{E}(t) = \mathfrak{z}(t) - \chi(t)$$

$$F(\chi,\mathfrak{z}) = f_2(\mathfrak{z}) - f_1(\chi) + (U_2 - U_1)\chi$$

The objective is to design controller $\Theta(t)) + \Psi(\mathfrak{z}(t),\chi(t))$ such that

$$\lim_{t\to\infty}||\mathcal{E}(t)|| = 0$$

Based on the design technique of ACC [11, 13, 14], to remove the nonlinear part of error dynamic we used a control input of $\Theta(t)) + \Psi(\mathfrak{z}(t),\chi(t))$. The new input vector is

$$\Theta(t) + \Psi(\mathfrak{z}(t),\chi(t)) = H(t) - F(\chi,\mathfrak{z}).$$

The error system (4) is then rewritten as

$$D^{\alpha} = U_2\mathcal{E} + H(t) \qquad (10).$$

Equation (10) is the newly defined control input $H(t)$ of the dynamics error. For the control input $H(t)$, there are various possibilities to choose from. Finally, we choose SMC law as

$$H(t) = K\Theta(t) \qquad (11),$$

where $K = [k_1,k_2,k_3]^T$ is a constant gain vector and $\Theta(t)$ is the control input that fulfills:

$$\Theta(t) = \begin{cases}\Theta^{+}(t) & s(\mathcal{E}) \geq 0\\ \Theta^{-}(t) & s(\mathcal{E}) < 0\end{cases} \qquad (12).$$

Then $s = s(\mathcal{E})$ is the SWS which indicates the error. Then the dynamics error is

$$D^{\alpha}\mathcal{E} = U_2\mathcal{E} + K\Theta(t) \qquad (13).$$

Then the suitable SMC will be drawn based on the SMC theory:

3.1.1 SLIDING SURFACE

The sliding surface is determined as

$$s(\mathcal{E}) = Q\mathcal{E} \qquad (14),$$

where $Q = [q_1,q_2,q_3]$ is a constant vector.

We know that $\dot{s}(\mathcal{E}) = 0$ is an essential state for remaining on the SWS where $s(\mathcal{E}) = 0$ is the state trajectory. So, another identical control is created and the controlled system must follow these two states

$$s(\mathcal{E}) = 0$$

$$\dot{s}(\mathcal{E}) = 0 \qquad (15)$$

By using (13), (14) and (15), we obtain

$$\dot{s}(\mathcal{E}) = \frac{\partial s(\mathcal{E})}{\partial e}\dot{\mathcal{E}} = Q[U\mathcal{E} + K\Theta(t)] = 0 \qquad (16)$$

Solving (16) for $\Theta(t)$ results in the identical control $\Theta_{eq}(t)$

$$\Theta_{eq}(t) = -(QK)^{-1}CU\mathcal{E}(t) \qquad (17),$$

where the necessary condition is the existence of $(CK)^{-1}$.

Substituting for $\Theta(t)$ in (13) from $\Theta_{eq}(t)$ of (12). Then, the sliding mode state equation is

$$D^{\alpha}\mathcal{E} = [I - K(QK)^{-1}C]U\mathcal{E} \quad (18).$$

It is asymptotically stable if the system (13) has negative real parts for all eigenvalues.

3.1.2 SMC

It is assumed that the constant plus proportional rate reaching law (15-17) is used. And we can choose the reaching law as

$$\dot{s} = -q\, sgn(s) - rs \quad (19)$$

where $sgn(.)$ is the sign function. The sliding condition is decided from the value of r and q, where $r > 0$ and $q > 0$, then the sliding mode motion (SMM) occurred.

From (13) and (14), it is discovered that

$$\dot{s} = Q[U\mathcal{E} + K\Theta(t)] \quad (20).$$

Now, the control input is constructed from (19) and (20) as

$$\Theta(t) = -(QK)^{-1}[Q(rI + U)\mathcal{E}(t) + q\, sgn(s)] \quad (21).$$

3.2 SMC WITH TIME DELAY

We let the master system as

$$D^{\alpha_0}\chi(t) = -Q\chi(t) + Uf(\chi(t)) + Vf(\chi(t-\tau)) + \Gamma$$

and, we rewrite the slave system equation (4) as

$$D^{\alpha}\mathfrak{z}(t) = -Ez(t) + Bg\big(\mathfrak{z}(t)\big) + Tg\big(\mathfrak{z}(t-\tau)\big) + J + D^{\alpha_0}\chi(t) - f(\chi(t)) + \Psi(\mathfrak{z}(t), \chi(t)) \quad (22),$$

where $f\big(\chi(t)\big) = -E\chi(t) + Bg\big(\chi(t)\big) + Tg\big(\chi(t-\tau)\big)$.

The error in synchronization is denoted as $\mathcal{E}(t) = \mathfrak{z}(t) - \chi(t)$. Combining the master system (2) and the slave system (4) determines the synchronization error. And from the calculation, this is expressed as

$$D^{\alpha}\mathcal{E}(t) = -E\big(\mathfrak{z}(t) - \chi(t)\big) + Bg\big(\mathfrak{z}(t) - \chi(t)\big) + Tg\big(\mathfrak{z}(t-\tau) - \chi(t-\tau)\big) + D^{\alpha_0}\chi(t) + Q\chi(t) - Uf\big(\chi(t)\big) - Vf\big(\chi(t-\tau)\big) + (J - \Gamma) + \Psi(\mathfrak{z}(t), \chi(t)) \quad (23),$$

where $\Phi(\mathcal{E}(t)) := g(\mathfrak{z}(t)) - g(\chi(t))$, $\Phi(\mathcal{E}(t-\tau)) := g(\mathfrak{z}\,(t-\tau)) - g(\chi(t-\tau))$.

By applying Lemma 2, we have

$$\mathcal{E}(t) = \mathcal{E}(0) + D^{-\alpha}\big[-E\mathcal{E}(t) + B\Phi\big(\mathcal{E}(t)\big) + T\Phi\big(\mathcal{E}(t-\tau)\big) + D^{\alpha_0}\chi(t) + Q\chi(t) - Uf\big(\chi(t)\big) - Vf\big(\chi(t-\tau)\big) + (J - \Gamma) + \Psi\big(\mathfrak{z}(t), \chi(t)\big)\big] \quad (24)$$

Based on the synchronization error (24), the delayed active sliding surface (DASS) is represented as

$$\mathcal{s}(t) = \mathcal{E}(t) + D^{-\alpha}[\Pi(\Xi_1\mathcal{E}(t) + \Xi_2\mathcal{E}(t-\tau) - \Psi(\mathfrak{z}(t),\chi(t))]$$
$$-D^{-\alpha}\left[-Q\mathcal{E}(t) + B\Phi\left(\mathcal{E}(t)\right) + T\Phi\left(\mathcal{E}(t-\tau)\right)\right] \quad (25),$$

where $\Pi \in R^{q\times m}$ is a gain matrix.

The combination of (24) with (25) will produce new DASS such as

$$\mathcal{s}(t) = \mathcal{E}(0) + D^{-\alpha}\left[(Q - E + \Pi\Xi_1)\mathcal{E}(t) + \Xi_2\mathcal{E}(t-\tau) + D^{\alpha_0}\chi(t) + Q\chi(t) - Uf\left(\chi(t)\right) - Vf\left(\chi(t-\tau)\right) + (J - \Gamma) + \Psi(\mathfrak{z}(t),\chi(t))\right] \quad (26).$$

Then, from (26)

$$D^{\alpha}\mathcal{s}(t) = D^{\alpha}\mathcal{E}(0) + D^{\alpha}D^{-\alpha}\left[(Q - E + \Pi\Xi_1)\mathcal{E}(t) + \Xi_2\mathcal{E}(t-\tau) + D^{\alpha_0}\chi(t) + Q\chi(t) - Uf\left(\chi(t)\right) - Vf\left(\chi(t-\tau)\right) + (J - \Gamma) + \Psi(\mathfrak{z}(t),\chi(t))\right]$$
$$= (Q - E + \Pi\Xi_1)\mathcal{E}(t) + \Xi_2\mathcal{E}(t-\tau) + D^{\alpha_0}\chi(t) + Q\chi(t) - Uf\left(\chi(t)\right) - Vf\left(\chi(t-\tau)\right) + (J - \Gamma) + \Psi(\mathfrak{z}(t),\chi(t)) \quad (27)$$

From the DASS, the theory of DASMC and its derivative must fulfill the fact that $\mathcal{s}(t) = 0$ and $\dot{\mathcal{s}}(t) = 0$. Besides, by utilizing Lemma 2, we can gain $\dot{s}(t) = D^{\alpha}D^{-\alpha}\mathcal{s}(t)$. So, $\dot{s}(t) = 0$ is equivalent to $D^{\alpha}\mathcal{s}(t) = 0$. Thus, we can define DSMC law by

$$\Psi_1\left(\mathfrak{z}(t),\chi(t)\right) = -(Q - E + \Pi\Xi_1)\mathcal{E}(t) - \Xi_2\mathcal{E}(t-\tau) - D^{\alpha_0}\chi(t) - Q\chi(t) + Uf\left(\chi(t)\right) + Vf\left(\chi(t-\tau)\right) - J + \Gamma \quad (28).$$

Then, we solved (23) and (28), then we defined delayed sliding mode dynamics (DSMD) as

$$D^{\alpha}\mathcal{E}(t) = -(Q + \Pi\Xi_1)\mathcal{E}(t) - \Xi_2\mathcal{E}(t-\tau) + B\Phi\left(\mathcal{E}(t)\right) + T\Phi\left(\mathcal{E}(t-\tau)\right) \quad (29).$$

Consequently, we can conclude that DSMD (29) is the equilibrium point of $\mathcal{E} = 0$. From the DSMC theory, an approaching law is defined such that

$$\Psi_2\left(\mathfrak{z}(t),\chi(t)\right) = -K[sgn(\mathcal{s}(t)] \quad (30),$$

where $\mathcal{s}(t) = [\zeta_1(t),\zeta_2(t),\dots,\zeta_n(t)]^T$ and $k > 0$ is the switching gain.

$$sgn(\zeta_i(t)) = \begin{cases} -1, & \zeta_i(t) < 0 \\ 0, & \zeta_i(t) = 0 \\ 1, & \zeta_i(tt) > 0 \end{cases}$$

Finally, we denote the DSMC $\Psi(\mathfrak{z}(t),\chi(t))$ by

$$\Psi\left(\mathfrak{z}(t),\chi(t)\right) = \Psi_1\left(\mathfrak{z}(t),\chi(t)\right) + \Psi_2\left(\mathfrak{z}(t),\chi(t)\right)$$
$$= -(Q - E + \Pi\Xi_1)\mathcal{E}(t) - \Xi_2\mathcal{E}(t-\tau) - D^{\alpha_0}\chi(t) - Q\chi(t) + Uf\left(\chi(t)\right) + Vf\left(\chi(t-\tau)\right) - J + \Gamma - K[sgn(\mathcal{s}(t)] \quad (31).$$

In the DSMC (31) there is a discontinuous function $sgn(\cdot)$ that may generate some harmful chattering. Then, to reduce the problems, we replace the discontinuous function $sgn(\cdot)$ by applying a continuous function $tanh(\cdot)$. Hence, the modified delayed sliding mode control (31) is denoted as

$$\Psi(\mathfrak{z}(t),\chi(t)) = \Psi_1(\mathfrak{z}(t),\chi(t)) + \Psi_2(\mathfrak{z}(t),\chi(t))$$
$$= -(Q - E + \Pi\Xi_1)\mathcal{E}(t) - \Xi_2\mathcal{E}(t-\tau) - D^{\alpha_0}\chi(t) - Q\chi(t) + Uf(\chi(t)) + Vf(\chi(t-\tau)) - J + \Gamma - K[tanh(\mathcal{s}(t))] \quad (32).$$

At present, the law of exponential convergence is alternatively practical as
$\dot{\mathcal{s}}(t) = -\varepsilon sgn(\mathcal{s}(t)) - \omega\mathcal{s}(t)$.
The system has the advantages which are short transition time if we are using a smaller value of ε and larger value of ω.

Theorem 1. (10)"*Suppose that the delayed sliding switching surface defined by (25) holds, based on the delayed sliding mode control (31), the trajectories of the synchronization error (21) can be asymptotically reached against the delayed sliding switching surface $\mathcal{s}(t) = 0$.*"

Proof. We represent the Lyapunov quadratic function as

$$v(t) = \frac{1}{2}\mathcal{s}(t)\dot{\mathcal{s}}(t) \quad (33).$$

From the Lemma 1, by computing the Caputo fractional derivative of $v(t)$ relative to t, time along the trajectories of DASS (25), one detects that

$$\dot{v}(t) = \mathcal{s}(t)\dot{\mathcal{s}}(t)$$
$$= \mathcal{s}(t)\big[(Q - E + \Pi\Xi_1)\mathcal{E}(t) + \Xi_2\mathcal{E}(t-\tau) + D^{\alpha_0}\chi(t) + Q\chi(t) - Uf(\chi(t)) - Vf(\chi(t-\tau)) + (J - \Gamma) + \Psi(\mathfrak{z}(t),\chi(t))\big]$$
$$= \mathcal{s}(t)\big[-K[sgn(\mathcal{s}(t))]\big]$$
$$= -K\cdot|\mathcal{s}(t)| \quad (34).$$

The system must converge asymptotically to $\mathcal{s}(t) = 0$ when $K > 0$. This indicates that the synchronization error (24) onto the delayed active sliding SWS is reached globally. This completes the verification.

Theorem 2: (10)"*Suppose that (H) holds. Assume that there exist positive constants m_i and li such that $m_i = max\{|m_i^-|,|m_i^+|\}$, $l_i = max\{|l_i^-|,|l_i^+|\}$, and constant matrices Π, Ξ_1 and Ξ_2 such that $\Pi = (\pi_{jl})_{a\times m}$, $\Xi_1 = (\varphi_{li})_{m\times a}$, $\Xi_2 = (\vartheta_{li})_{m\times a}$ and* "

$$\begin{cases} \gamma_1 := \min_{1\le i\le c}\left[q_i - \sum_{l=1}^{m}\left(\sum_{a=1}^{c}\left(|\pi_{jl}\varphi_{li}|\right)\right) - \sum_{l=1}^{m}\sum_{a=1}^{c}\left(|b_{ji}|m_i\right)\right] > 0 \\ \gamma_2 := \max_{1\le i\le c}\left[\sum_{l=1}^{m}\left(\sum_{a=1}^{c}\left(|\pi_{jl}\vartheta_{li}|\right)\right) + \sum_{l=1}^{m}\sum_{a=1}^{c}\left(|h_{ji}|l_i\right)\right] > 0 \\ \gamma_1 - \gamma_2 > 0 \end{cases} \quad (35).$$

Then the original equation of (29) is stable.

Proof. We transformed the delayed synchronization error system of (28) as

$$D^{\alpha}\mathcal{E}(t) = -q_i x_i(t) + \sum_{l=1}^{m}\sum_{a=1}^{c}[\pi_{jl}\varphi_{li}]\mathcal{E}_j(t) + \sum_{l=1}^{m}\sum_{a=1}^{c}[\pi_{jl}\vartheta_{li}]\mathcal{E}_j(t-\tau) + \sum_{l=1}^{m}\sum_{a=1}^{c} b_{ipl}\Phi\mathcal{E}_j(t) + \sum_{l=1}^{m}\sum_{a=1}^{c} h_{ipl}\Phi\mathcal{E}_j(t-\tau) \quad (36).$$

We defined the Lyapunov function candidate by

$$D^{\alpha}v\big(t,\mathcal{E}(t)\big) = \sum_{i=1}^{c} D_+^{\alpha}|\mathcal{E}_i(t)| \leq \sum_{i=1}^{c} sgn(\mathcal{E}_i(t))D_+^{\alpha}\mathcal{E}_i(t)$$

$$\leq -\sum_{i=1}^{c} Q_i|\mathcal{E}_i(t)| + \sum_{i=1}^{c}\left[\sum_{l=1}^{m}\left(\sum_{a=1}^{c}\pi_{jl}\varphi_{li}\right)|\mathcal{E}_j(t)|\right] + \sum_{i=1}^{c}\left[\sum_{l=1}^{m}\left(\sum_{a=1}^{c}\pi_{jl}\vartheta_{li}\right)|\mathcal{E}_j(t-\tau)|\right] + \sum_{i=1}^{c}\left[\sum_{l=1}^{m}\left(\sum_{a=1}^{c}|b_{ikl}|\right)m_j|\mathcal{E}_j(t)|\right] + \sum_{i=1}^{c}\left[\sum_{l=1}^{m}\left(\sum_{a=1}^{c}|h_{ikl}|\right)l_j|\mathcal{E}_j(t-\tau)|\right]$$

$$= -\sum_{i=1}^{c} Q_i|\mathcal{E}_i(t)| + \sum_{i=1}^{c}\left[\sum_{l=1}^{m}\left(\sum_{a=1}^{c}\pi_{jl}\varphi_{li}\right)\right]|\mathcal{E}_j(t)| + \sum_{i=1}^{c}\left[\sum_{l=1}^{m}\left(\sum_{a=1}^{c}\pi_{jl}\vartheta_{li}\right)\right]|\mathcal{E}_j(t-\tau)| + \sum_{i=1}^{c}\left[\sum_{l=1}^{m}\left(\sum_{a=1}^{c}|b_{ipl}|\right)m_j\right]|\mathcal{E}_j(t)| + \sum_{i=1}^{c}\left[\sum_{l=1}^{m}\left(\sum_{a=1}^{c}|h_{ipl}|\right)l_j\right]|\mathcal{E}_j(t-\tau)|$$

$$= -\sum_{i=1}^{c}\left[Q_i - \sum_{l=1}^{m}\left(\sum_{a=1}^{c}|\pi_{jl}\varphi_{li}|\right) - \sum_{l=1}^{m}\left(\sum_{a=1}^{c}|b_{ipl}|\right)m_j\right]|\mathcal{E}_j(t)| + \sum_{i=1}^{q}\left[C_i - \sum_{l=1}^{m}\left(\sum_{a=1}^{c}|\pi_{jl}\vartheta_{li}|\right) - \sum_{l=1}^{m}\left(\sum_{a=1}^{c}|h_{ipl}|\right)l_j\right]|\mathcal{E}_j(t-\tau)|$$

$$\leq \min_{1\leq i\leq c}\left[Q_i - \sum_{l=1}^{m}\left(\sum_{a=1}^{c}(|\pi_{jl}\varphi_{li}|)\right) - \sum_{l=1}^{m}\sum_{a=1}^{c}(|b_{ji}|m_i)\right] - \sum_{i=1}^{c}|\mathcal{E}_j(t)| + \max_{1\leq i\leq c}\left[\sum_{l=1}^{m}\left(\sum_{a=1}^{c}(|\pi_{jl}\vartheta_{li}|)\right) + \sum_{l=1}^{m}\sum_{a=1}^{c}(|h_{ji}|l_i)\right]\sum_{i=1}^{c}|\mathcal{E}_j(t-\tau)|$$

$$\leq \gamma_1 V\big(t,\mathcal{E}(t)\big) + \gamma_2 \sup_{t-\tau\leq s\leq t} V\big(s,\mathcal{E}(s)\big) \quad (37).$$

As whichever result $\mathcal{E}_i(t)$ of error system (36) that fulfill the Razumikhin condition, one has

$$\sup_{t-\tau\leq s\leq t} v\big(s,\mathcal{E}(s)\big) \leq v\big(t,\mathcal{E}(t)\big) \quad (38).$$

Then, based on (37), (38) and **Theorem 2**, it is assumed that there is a constant $\delta > 0$, one has

$$\begin{cases} D_+^{\alpha}v\big(t,\mathcal{E}(t)\big) \leq -(\gamma_1-\gamma_2)v\big(t,\mathcal{E}(t)\big) \\ \gamma_1-\gamma_2 \geq \delta \end{cases} \quad (39),$$

and from (39), one observes that

$$D_+^{\alpha}v\big(t,\mathcal{E}(t)\big) \leq \delta v\big(t,\mathcal{E}(t)\big) \quad (40).$$

Then from (40) and Lemma 1,

$$v\big(t,\mathcal{E}(t)\big) \leq v(0)E_{\alpha}(-\delta t^{\alpha}) \quad (41).$$

So,

$$\|\mathcal{E}(t)\| = \|\mathfrak{z}_i(t) - x(t)\|$$

$$= \sum_{i=1}^{n} \|\mathfrak{z}_i(t) - x(t)\|$$

$$\leq \|\Psi_o - \emptyset_0\| E_\alpha(-\delta t^\alpha) \tag{42}$$

It is concluded that the equilibrium point stabilization is stable if $\mathcal{E} = 0$. This is proof that as $\|\mathcal{E}(t)\| \to 0$ $(t \to +\infty)$, the DSMD (29) is stable. This sums up the proof. ■

4 RESULTS AND FINDINGS

In this part, to validate the viability and efficiency of the gained outcomes, the suggested idea with numerical examples is revealed. As we know the master systems are in the form of integer order, so the value for α_0 is 1. Suppose that the following are the master IoDRNNASM systems and the slave FoDRNNASM systems with the proposed outcomes.

$$D^{\alpha_0}\chi(t) = -Qx(t) + Uf(\chi(t)) + Vf(\chi(t-\tau)) + \Gamma$$

$$D^{\alpha}\mathfrak{z}(t) = -E\mathfrak{z}(t) + Bg\big(\mathfrak{z}(t)\big) + Tg\big(\mathfrak{z}(t-\tau)\big) + J + D^{\alpha_0}\chi(t) - f(\chi(t)) + \Psi(\mathfrak{z}(t), \chi(t))$$

$$\Psi\big(\mathfrak{z}(t), \chi(t)\big) := \Xi_1\chi(t) + \Xi_2\chi(t-\tau),$$

where $\alpha = 0.98$, $\tau = 0.5$, $f(\chi(t)) = tanh(\chi(t))$, $g(\mathfrak{z}(t)) = tanh(\mathfrak{z}(t))$, $\Xi_1 = (0, 1, 0)$, $\Xi_2 = (0, 1, 0)$, $\Gamma = (0, 0, 0)^T$, $J = (0, 0, 0)^T$,

$$Q = \begin{bmatrix} -9.5 & 0 & 0 \\ 0 & -10.5 & 0 \\ 0 & 0 & -3.7 \end{bmatrix}, \quad U = \begin{bmatrix} 2 & 0.5 & 5.5 \\ 0.5 & 0.5 & 5.1 \\ 0.5 & 1 & -5.5 \end{bmatrix}, \quad V = \begin{bmatrix} 7 & 7 & 4.1 \\ 1 & 1 & 2.5 \\ 0.1 & -10.1 & 4.5 \end{bmatrix}$$

$$E = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -7.7 \end{bmatrix}, \quad B = \begin{bmatrix} 0.01 & 0.3 & 7.5 \\ 0.01 & 0.5 & 7.5 \\ 0.01 & 3 & -5.5 \end{bmatrix}, \quad T = \begin{bmatrix} 0.3 & 5.5 & 5.5 \\ 0.5 & 2.5 & 5.5 \\ 0.1 & -1.5 & 1.5 \end{bmatrix}$$

The numerical simulation technique is introduced here using the MATLAB software. The results using this software are demonstrated by the graphical presentation. For this, IoDRNNASM controls the master system while FoDRNNASM controls the slave system. The findings are seen with the initial state of the master system as $(\chi_1(0), \chi_2(0), \chi_3(0)) = (0.1, 0.3, -0.1)$ while the initial state of slave system is $(\mathfrak{z}_1(0), \mathfrak{z}_2(0), \mathfrak{z}_3(0)) = (0.1, 1.2, 0.1)$.

The results of synchronization of IoDRNNASM and FoDRNNASM without the activation of the controller are shown in Figure 1 is a graphical display with the dependent variables x, y, and z and time t as an independent variable function with the order $\alpha_0 = 1$ and $\alpha = 0.98$.

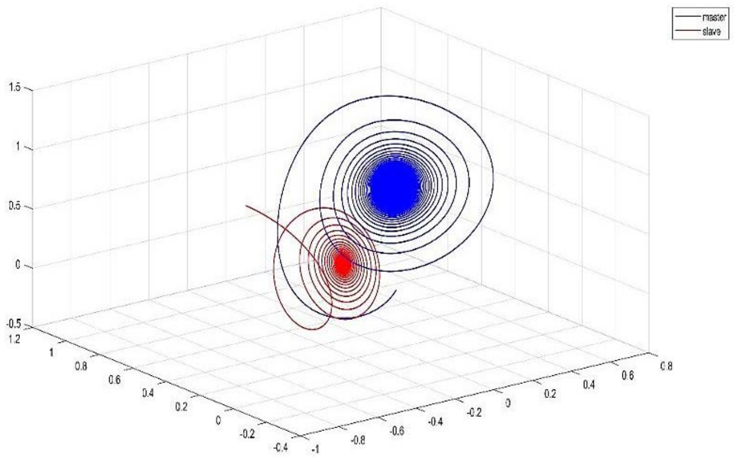

Figure 1: Chaotic attractor of IoDRNNASM system (2) and FoDRNNASM system (4) without control input.

The results of the simulation of the synchronization of IoDRNNASM and FoDRNNASM systems with the activation of the controller are shown in Figure 2 in x-y state trajectories.

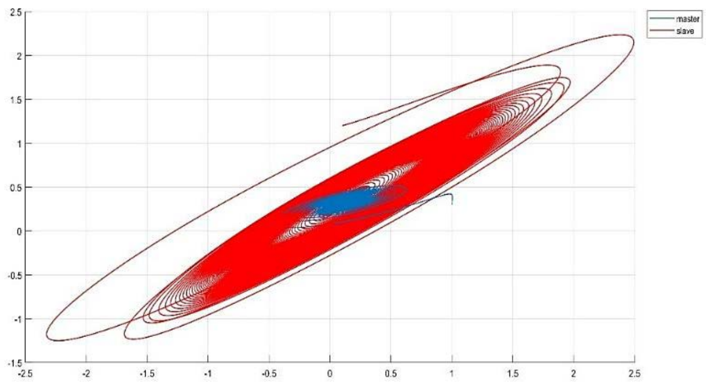

Figure 2: The x-y state trajectories of IoDRNNASM system (2) and FoDRNNASM system (4) with control input.

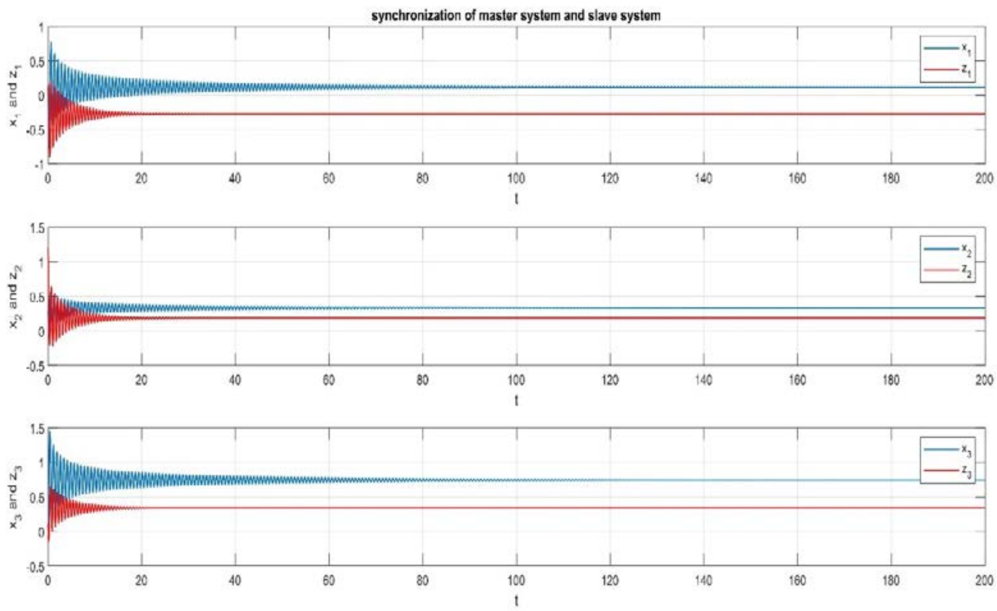

Figure 3: The IoDRNNASM and FoDRNNASM systems of state trajectories of $\chi_{1(t)}, \mathfrak{z}_{1(t)}, \chi_{2(t)}, \mathfrak{z}_{2(t)}, \chi_{3(t)}$ *and* $\mathfrak{z}_{3(t)}$ without control input.

*Corresponding author (W.A.M.Othman, F.N.A.Latiff) wanainun@um.edu.my, fatin_nabilaa@yahoo.com Volume 11 No.12 ISSN 2228-9860 eISSN 1906-9642 CODEN: ITJEA8 Paper ID:11A12D http://TUENGR.COM/V11A/11A12D.pdf DOI: 10.14456/ITJEMAST.2020.229

Figure 3 shows the state trajectories of synchronization of IoDRNNASM system with initial conditions $(0.1, 0.3, -0.1)$ and FoDRNNASM system with initial conditions $(0.1, 1.2, 0.1)$ with controller activation for the simulation time 200s.

Figure 4 shows the state trajectories of synchronization of IoDRNNASM system with initial conditions $(0.1, 0.3, -0.1)$ and FoDRNNASM system with initial conditions $(0.1, 1.2, 0.1)$ without controller activation for the simulation time 200s.

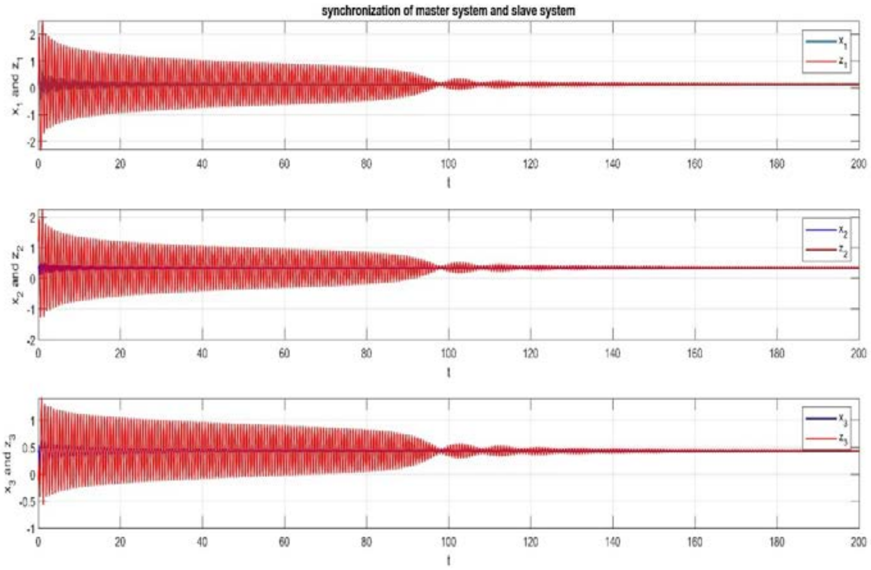

Figure 4: The IoDRNNASM and FoDRNNASM systems of state trajectories of $\chi_{1(t)}, \mathfrak{z}_{1(t)}, \chi_{2(t)}, \mathfrak{z}_{2(t)}, \chi_{3(t)}$ *and* $\mathfrak{z}_{3(t)}$ with control input

In Figure 5, it performed a comparison of the synchronization errors of IoDRNNASM and FoDRNNASM systems. From the simulation, the state error does not converge to zero as the control is not activated.

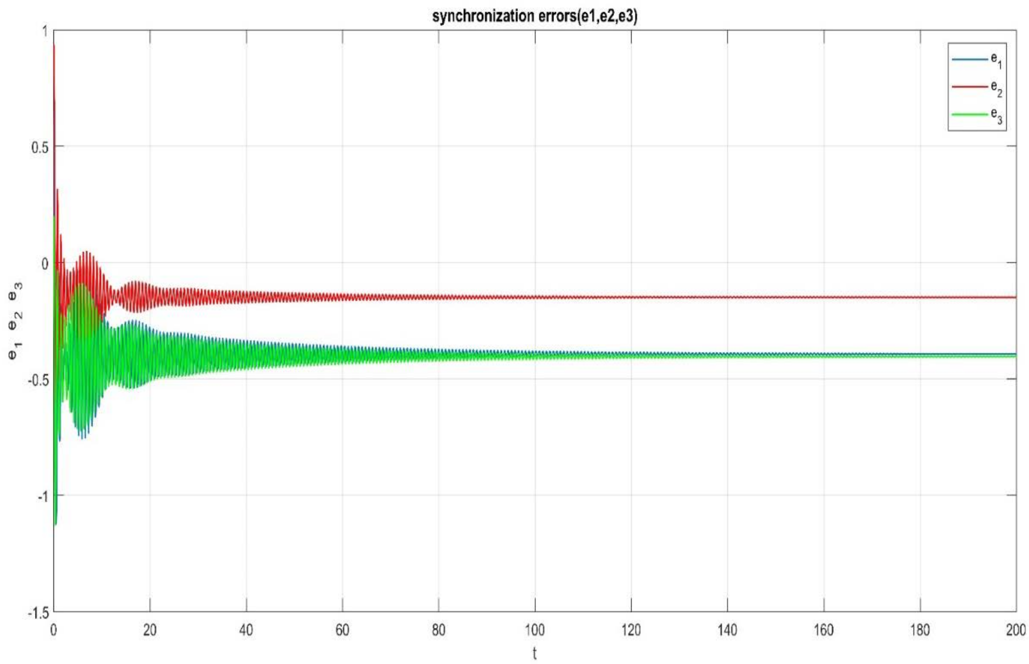

Figure 5: The error trajectories of IoDRNNASM system (2) and FoDRNNASM system (4) without control input.

The convergence of state error in the finite time is proven in Figure 6. It showed that all the systems converge to zero as the controller is activated

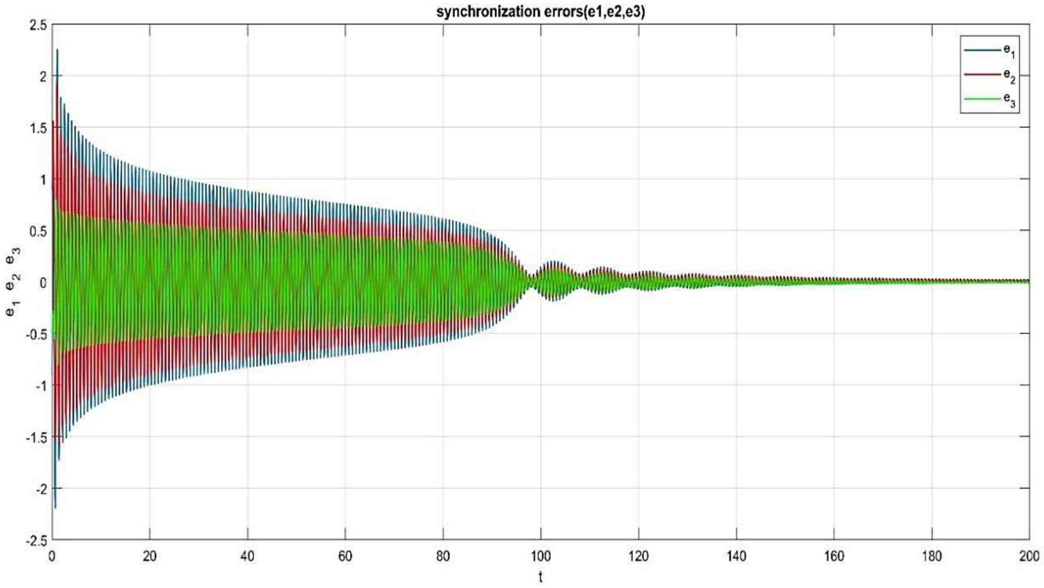

Figure 6: The error trajectories of IoDRNNASM system (2) and FoDRNNASM system (4) with control input

5 DISCUSSION

Chaotic synchronization has suffered from modulation data through the public channel by hackers. A variety of approaches and procedures has been established to hide the secrecy of the message data or information by consuming chaotic signals. But numerous studies have shown that the record of the current approaches is untrustworthy in the security problem. Based on the above problems, to improve the security strength of the cryptosystem that will provide powerful security, we believed that we can introduce a new idea which is the combination of NNs synchronization with the RSA algorithm and we called this combination as double encryption. The idea is shown in Figure 7.

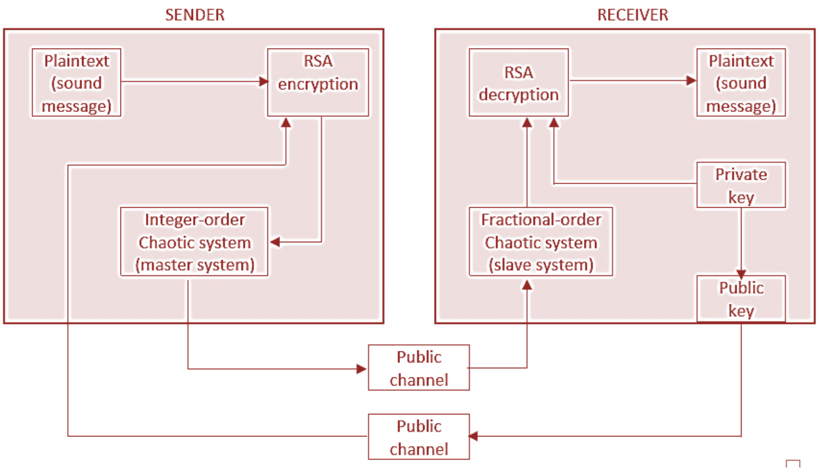

Figure 7: Double encryption IoDRNNASM and FoDRNNASM system.

6 CONCLUSION

Within this paper, we analyzed the synchronization of IoDRNNASM systems and FoDRNNASM systems. Based on the DASMC, the theory of DAMC, the FLDM is introduced to prove the synchronization efficiency. We have proved that the synchronization of IoDRNNASM systems and FoDRNNASM systems can occur when the proposed controller is activated and the result showed that the state error is converged to zero. Further research may experiment with synchronization the FoRNNs with non-identical orders.

7 AVAILABILITY OF DATA AND MATERIAL

Data can be made available by contacting the corresponding authors

8 ACKNOWLEDGEMENT

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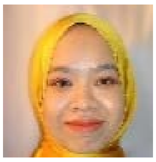

Fatin Nabila Binti Abd Latiff is a Ph.D. student at Institute of Mathematical Sciences, University of Malaya. She received a Bachelor of Sciences (Mathematics) from University of Malaya and Master of Sciences (Mathematics) from University of Malaya. Her research interests include Chaos Cryptography and Secure Communication.

Dr.Wan Ainun obtained is an Associate Professor and Head of the Centre of Mathematical and Statistical Analysis, University of Malaya. Her B.Sc. and M.Sc. degrees are from University of Charlotte, North Carolina and North Carolina State University. She got her PhD in the area of Computer Aided Geometric Design. Her research interests are Cryptography, Geometric Modelling and Differential Equations.

Dr.N. Kumaresan is a Senior Lecturer at Institute of Mathematical Sciences, University of Malaya. He got a B.S.(Mathematics) from Madras University, an M.S. (Mathematics) from Bharathidasan University, an M.Phil. (Mathematics) from Madurai Kamaraj University, a PhD (Mathematics) from Gandhigram Rural Universtiy. His research involves Applied Mathematics in the areas of Optimal Control Theory, Differential Equation & Applications, Neural Networks, Ant Colony Programming and Genetic Programming, Fuzzy systems and Neuro-Fuzzy Systems.