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METHOD OF BOUNDARY STATES IN PROBLEMS OF INTERACTIONS OF TWO CAVITIES

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ABSTRACT

The paper deals with the formed stress-strain condition of mass resulting from the interaction of two spherical cavities taking into account physical singularity, with variations in intra-cavity pressure and dependence of strength characteristics on the distance between cavities. Singularity was taken into account due to the use of special solutions of the expansion center type when organizing the method of boundary states. The strength reserve factors were calculated. The results of the numerical-analytical solution are presented in graphical form.

Disciplinary: Engineering Mechanics (Modeling & Simulation)

1. INTRODUCTION

Today, mathematical modeling is widely used in various fields of knowledge, including geology. Construction of a high-quality mathematical model is a prerequisite for the design of such engineering structures as underground gas storage (UGS). Such models are one of the main tools for solving various geological and engineering problems.

There is a unique Unified Gas Supply System in Russia that includes the UGS system. Underground storage facilities allow guaranteeing natural gas supply to consumers regardless of season, temperature fluctuations, force majeure [1].

This article deals with the issue of modeling the interaction of two cavities, which are the UGS model at varying intra-cavity pressure, as well as the dependence of the strength characteristics of the load-bearing mass on the distance between the cavities. The stress-strain condition is constructed utilizing a sophisticated energy method - method of boundary states (MBS).

2. METHOD OF BOUNDARY STATES

The fundamental concept of MBS is the condition of the medium, which is a particular solution to the defining equations of the medium, regardless of the conditions determined at the boundary of a

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body [5]. Defining relations in the mathematical model of a homogeneous elastostatic body are presented in tensor-index notation (a point in the index means differentiation, repetition of indexes means summation) and are enclosed in Cauchy relations

$$\varepsilon_{ij} = {}^1/_2\,(u_{i,j} + u_{j,i}) \tag{1},$$

the generalized Hooke's law

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij} \tag{2},$$

the equilibrium equation

$$\sigma_{ij,j} + X_i^0 = 0 \tag{3}.$$

where u_i – displacement vector components, ε_{ij} – strain tensor components σ_{ij} – stress tensor components, θ – volumetric deformation δ_{ij} – Kronecker symbol, λ, μ – Lame parameters, X_i^0 – volumetric forces. With fixed values λ, μ the set of relations (1)-(3) is reduced to a system of Lame's equations

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ji} + X_i^0 = 0.$$

Their general solution was developed by Papkovic and Neuber and presented in the form of Arzhanykh-Slobodyansky (body forces absence case) for a limited simple-connected body

$$u_i = 4(1-\nu)B_i + x_j B_{i,j} - x_i B_{j,i} \tag{4},$$

where ν – Poisson's ratio, B_i – component of an arbitrary harmonic vector. General solutions (4) are an effective tool for building the basis of a state-space representation for a body with no singular factors [5].

The concept of the medium condition is transformed into the notions of internal ξ and boundary γ conditions if it is the case of a specific body V with a boundary ∂V

$$\xi = \{u_i, \varepsilon_{ij}, \sigma_{ij}\}, \quad \gamma = \{u_i|_{\partial V}, p_i\} \tag{5},$$

where $p_i = \sigma_{ij}|_{\partial V} \cdot \boldsymbol{n}$.

A set of all possible conditions $\xi \leftrightarrow \gamma$ forms isomorphic Hilbert spaces of internal Ξ and boundary Γ conditions with scalar products,

$$(\xi^{(k)}, \xi^{(m)})_\Xi = \int_V \sigma_{ij}^{(k)} \varepsilon_{ij}^{(m)} \partial V, \qquad (\gamma^{(k)}, \gamma^{(m)})_\Gamma = \int_{\partial V} p_i^{(k)} u_i^{(m)} \partial S,$$

which are equal due to the principle of virtual displacements

$$(\xi^{(k)}, \xi^{(m)})_\Xi = (\gamma^{(k)}, \gamma^{(m)})_\Gamma.$$

After orthogonalization, the attributes of resulting internal and boundary conditions are presented in Fourier series by elements of orthonormal bases

$$u_i = \textstyle\sum_k c_k u_i^{(k)}, \quad \sigma_{ij} = \sum_k c_k \sigma_{ij}^{(k)}, \quad \varepsilon_{ij} = \sum_k c_k \varepsilon_{ij}^{(k)},$$

$$u_i|_{\partial V} = \textstyle\sum_k c_k u_i^{(k)}|_{\partial V}, \quad p_i = \sum_k c_k p_i^{(k)}.$$

Thus, the solution to the primal problems for any linear media and bodies of arbitrary outlines is reduced to the elementary calculation of quadratures. The convergence of a series of factors c_k was proven [8].

3. CONSTRUCTION OF A SPECIAL SOLUTION FOR THE CENTER OF EXPANSION

Singularity is taken into account using special solutions in the method of boundary states, namely formation of countable bases with their direct inclusion for spaces of internal and boundary conditions.

Displacement of observation point M in unbounded elastic medium from the center of expansion in point Q with intensity $\boldsymbol{\rho}\times\mathbf{P}$ is determined by Formula (6) using the Kelvin-Somigliana tensor [3]

$$\mathbf{u}(M,Q)=\frac{1-2\nu}{24\pi\mu(1-\nu)}\boldsymbol{\rho}\times\mathbf{P}\frac{\mathbf{R}}{R^3} \quad (6),$$

where $\mathbf{R}=r_M-r_Q$, $R=\left|r_M-r_Q\right|$.

Below is an example of the construction of a special solution for the basis of internal conditions described in the paper [9] for the expansion center at the point with coordinates $\left(0,0,-\frac{1}{2}\right)$ and at $\mu=1,\ \nu=\frac{1}{4}$

$$\mathbf{R}=\begin{pmatrix}x\\ y\\ {}^1\!/_2+z\end{pmatrix},\quad R=\sqrt{x^2+y^2+({}^1\!/_2+z)^2},\quad \mathbf{u}(M,Q)=\begin{pmatrix}\frac{7x}{204\pi R^3}\\ \frac{7y}{204\pi R^3}\\ \frac{7({}^1\!/_2+z)}{204\pi R^3}\end{pmatrix}.$$

Using (1), we will form the deformation tensor components

$$\hat{\varepsilon}=\begin{pmatrix}\frac{14(-8x^2+4y^2+(1+2z)^2)}{204\pi R^5} & \frac{-7xy}{68\pi R^5} & \frac{-7x(1/2+z)}{68\pi R^5}\\ \frac{-7xy}{68\pi R^5} & \frac{14(4x^2-8y^2+(1+2z)^2)}{204\pi R^5} & \frac{-7y(1/2+z)}{68\pi R^5}\\ \frac{-7x(1/2+z)}{68\pi R^5} & \frac{-7y(1/2+z)}{68\pi R^5} & \frac{28(2x^2+2y^2-(1+2z)^2)}{204\pi R^5}\end{pmatrix}.$$

By means of (2), we will construct the components of the stress tensor

$$\hat{\sigma}=\begin{pmatrix}\frac{-28(8x^2-4y^2-(1+2z)^2)}{204\pi R^5} & \frac{-7xy}{34\pi R^5} & \frac{-7x(1/2+z)}{34\pi R^5}\\ \frac{-7xy}{34\pi R^5} & \frac{28(4x^2-8y^2+(1+2z)^2)}{204\pi R^5} & \frac{-7y(1/2+z)}{34\pi R^5}\\ \frac{-7x(1/2+z)}{34\pi R^5} & \frac{-7y(1/2+z)}{34\pi R^5} & \frac{56(2x^2+2y^2-(1+2z)^2)}{204\pi R^5}\end{pmatrix}.$$

A set of expressions for $\mathbf{u}$, $\hat{\varepsilon}$, $\hat{\sigma}$, form, according to (5), element ξ of internal conditions space, stored in the computer's memory in analytical form.

4. SETTING OF PROBLEMS ABOUT THE INTERACTION OF TWO SPHERICAL CAVITIES

The first primal problem of the theory of elasticity (according to Muskhelishvili's classification

[4]) is solved employing MBS in non-dimensional formulation for homogeneous elastostatic medium enclosed inside the semi-sphere of radius T and height T located on the axis z within the limits of $z \in [-T; 0]$ (Figure 1). The origin of the coordinates is located on the boundary S_4, so that Oxy plane touches the earth's surface. The coordinates of the centers of spheres with S_1 and S_2 radii $R/100$ are $(\pm c, 0, -T/4)$ with the distance between the spheres R. Under the Saint-Venant principle, the boundaries S_3 and S_4 are significantly removed from the region in question so as not to distort the pressure value directly on and near cavities S_1 and S_2.

Granite layers, which are located closer to the earth's surface, are selected for UGS modeling. Shear modulus for granite is a value ranging from 1.4×10^{10} to 4.4×10^{10} Pa, a value of 1.6×10^{10} Pa was used in the solution. The value of Poisson's ratio for granite is taken in the range from 0.10 to 0.15, a value of 0.15 was used in the solution [10].

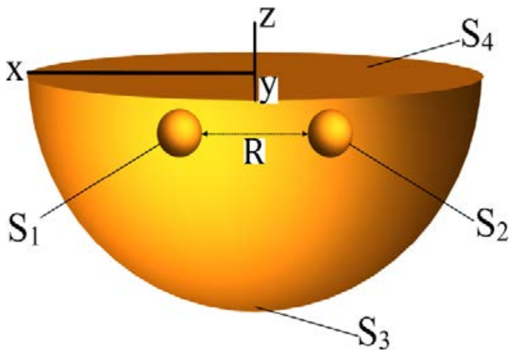

Figure 1: 3D Medium.

The boundary conditions on the border of ∂V body are of the following form:

$$\mathbf{p_1}|_{\partial V} = \begin{cases} \{\cos\varphi\cos\theta, \cos\theta\sin\varphi, \sin\theta\}, \in S_1 \\ \{0,0,0\}, \in S_2, S_3, S_4 \end{cases}$$

$$\mathbf{p_2}|_{\partial V} = \begin{cases} \{\cos\varphi\cos\theta, \cos\theta\sin\varphi, \sin\theta\}, \in S_1, S_2 \\ \{0,0,0\}, \in S_2, S_3, S_4 \end{cases}$$

Parameters:

$$c \in \left\{\frac{1}{10}, \frac{1}{20}, \frac{1}{30}, \frac{1}{40}, \frac{1}{60}, \frac{1}{80}, \frac{1}{99}\right\},$$

$$R \in \left\{\frac{9}{50}, \frac{2}{25}, \frac{7}{150}, \frac{3}{100}, \frac{1}{75}, \frac{1}{200}, \frac{1}{4950}\right\}.$$

Calculations were performed by non-dimensionalization through scales μ, T; after non-dimensionalization the value $T = 1$ was taken. The load parameter was set equal to μ (at any other value resulting in the stress and force fields change proportionally).

In the case of the first primal problem, Fourier coefficients of the splitting of target condition on the orthonormal basis $\{\xi^{(1)}, \xi^{(2)}, \cdots, \xi^{(k)}, \cdots\} \in \Xi$ of the space of internal conditions Ξ are calculated per the definition of the scalar product on the body border [6]

$$c_k = (\gamma, \gamma^{(l)}) = \int_{\partial V} p_i u_i^{(l)} dS,$$

where p_i – the components of the surface forces $u_i^{(l)}$ – axis X_i translation from the orthonormal basis of the boundary conditions space Γ.

The system of Fourier coefficients is subordinated to Bessel's inequality (the left part of Bessel's inequality that correlates Fourier coefficients with the Euclidean norm of a split element)

$$\sum_{j=1}^{n} c_j^2 \leq \|\xi\|_{\Xi}^2 = \|\gamma\|_{\Gamma}^2 \quad (7),$$

where n is the dimensionality of a truncated base.

Figure 2a graphically shows Fourier coefficients, where the coefficient number is indicated on the horizontal axis, and the value of this coefficient is indicated on the vertical axis.

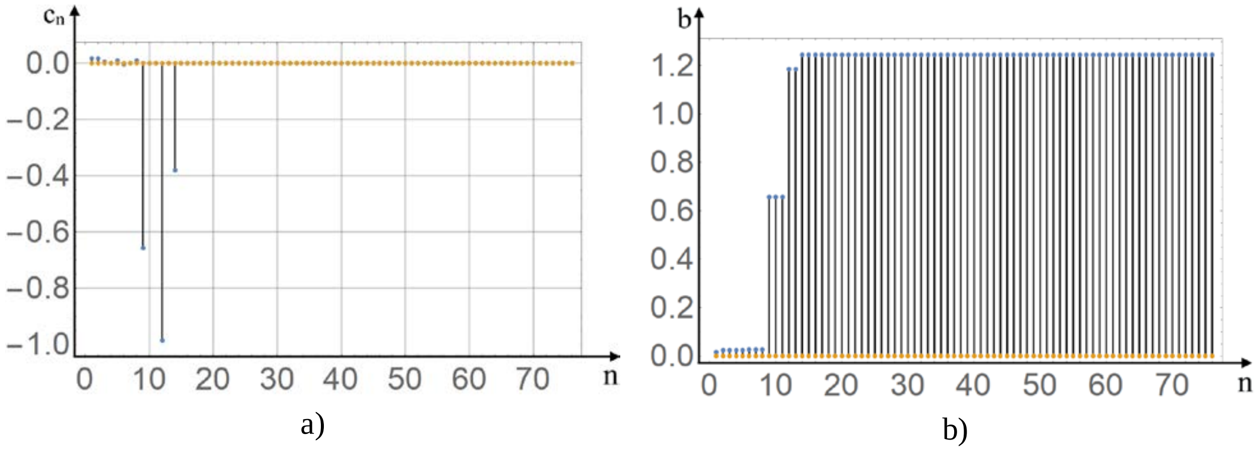

Figure 2: a) Fourier coefficients, b) saturation of Bessel's sum.

Reliability of the obtained results can be characterized by the quadratic residual of boundary conditions with the results of the surface solution, which amounted to the value of the order of 0.003, as well as the fact of saturation of Bessel's sum (left part of the Bessel inequality (7)), see Figure 2b, where the horizontal axis indicates the number of summed coefficients, and the vertical axis indicates the sum of squares of these coefficients.

The results show the explicit saturation of Bessel sum, which indicates the practically observed stability of the solution of the infinite system of linear algebraic equations. This fact is one of the indirect indicators characterizing the quality of the solution.

5. SOLUTION RESULTS IN GRAPHICAL FORM

The study revealed that stresses reach their positive maximum on the boundary, approximately at a point with coordinates relative to the center of the cavity: $r \to 0.01$; $\theta \to -0.685$; $\varphi \to 2.356$ (spherical coordinate system). The analysis of the stress condition in the cross-section $x = -0.09$ for the project with the parameters $c = 0.1$ and $\mathrm{R} = 0.18$ is presented by isolines in Figure 3.

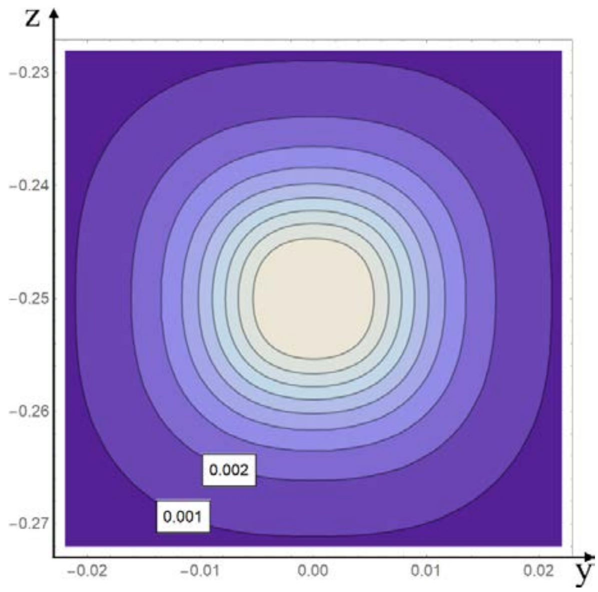

Figure 3: Distribution of stress intensity in Oyz plane.

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The characteristics that are responsible for the stress-strain condition take the form of cumbersome analytical expressions; due to their visual invisibility, they are not given here. For the sake of brevity, Figures 4 and 5 present the stress intensity isolines built in the cross-section $y = 0$.

Darker layers in Figures 4 and 5 correspond to higher compression (zero level lies inside the cavities). Stresses σ_{xx} reach their maximum at the lateral poles of spherical cavities, stresses tend to zero with distance along Z axis. It should be noted that in axial stress character (σ_{zz}) a section with zero stress in the medium between the cavities is formed, stresses decrease with distance along X axis. Shear stresses σ_{xz} near the cavity are determined by the degree of pressure inside the cavity.

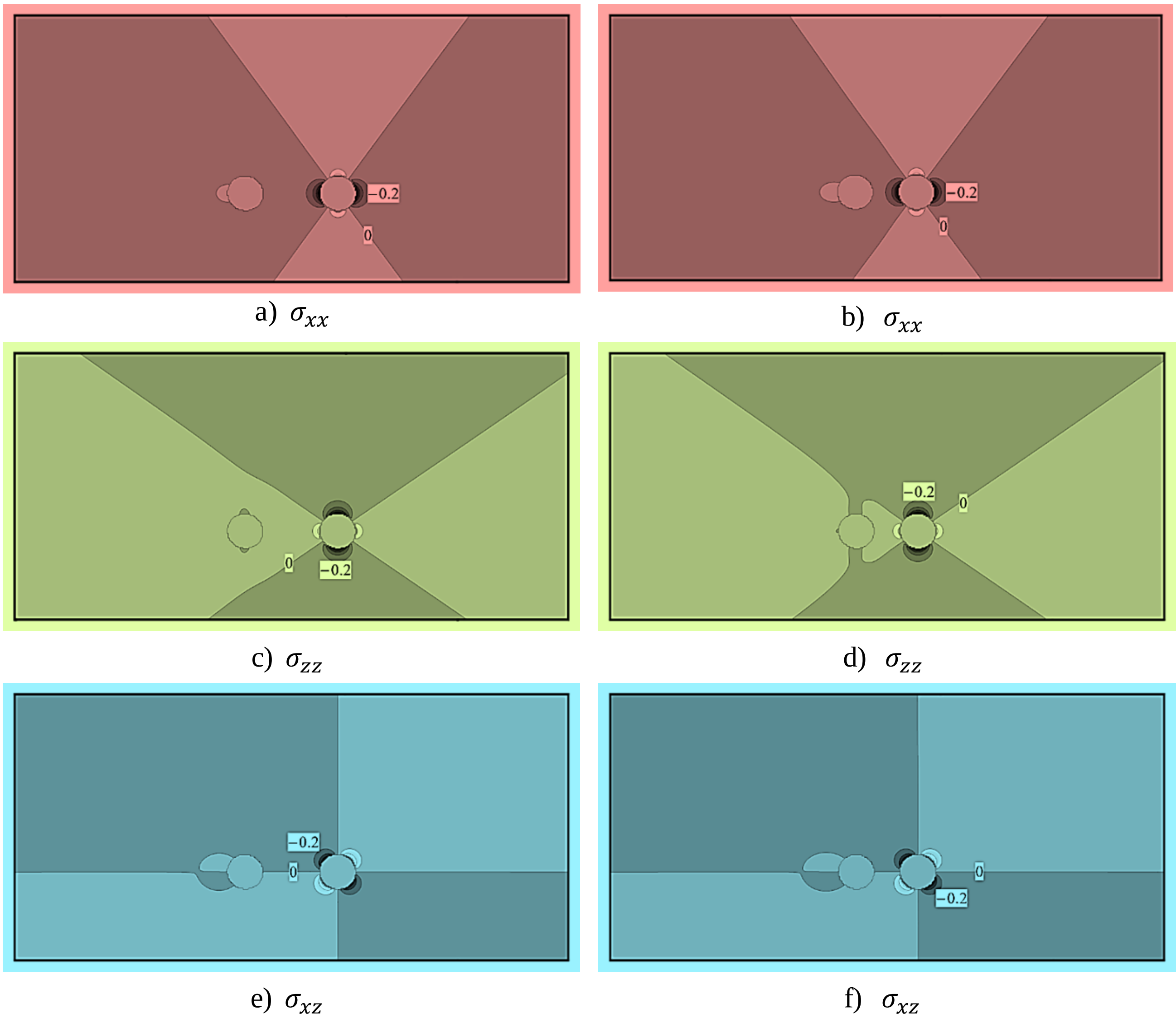

Figure 4: Stress isolines for $\mathbf{p_1}|_{\partial V}$ (pressure in the single cavity)

a) c) e) at $c = \frac{1}{40}$ and $R = \frac{3}{100}$,

b) d) f) at $c = \frac{1}{60}$ and $R = \frac{1}{75}$.

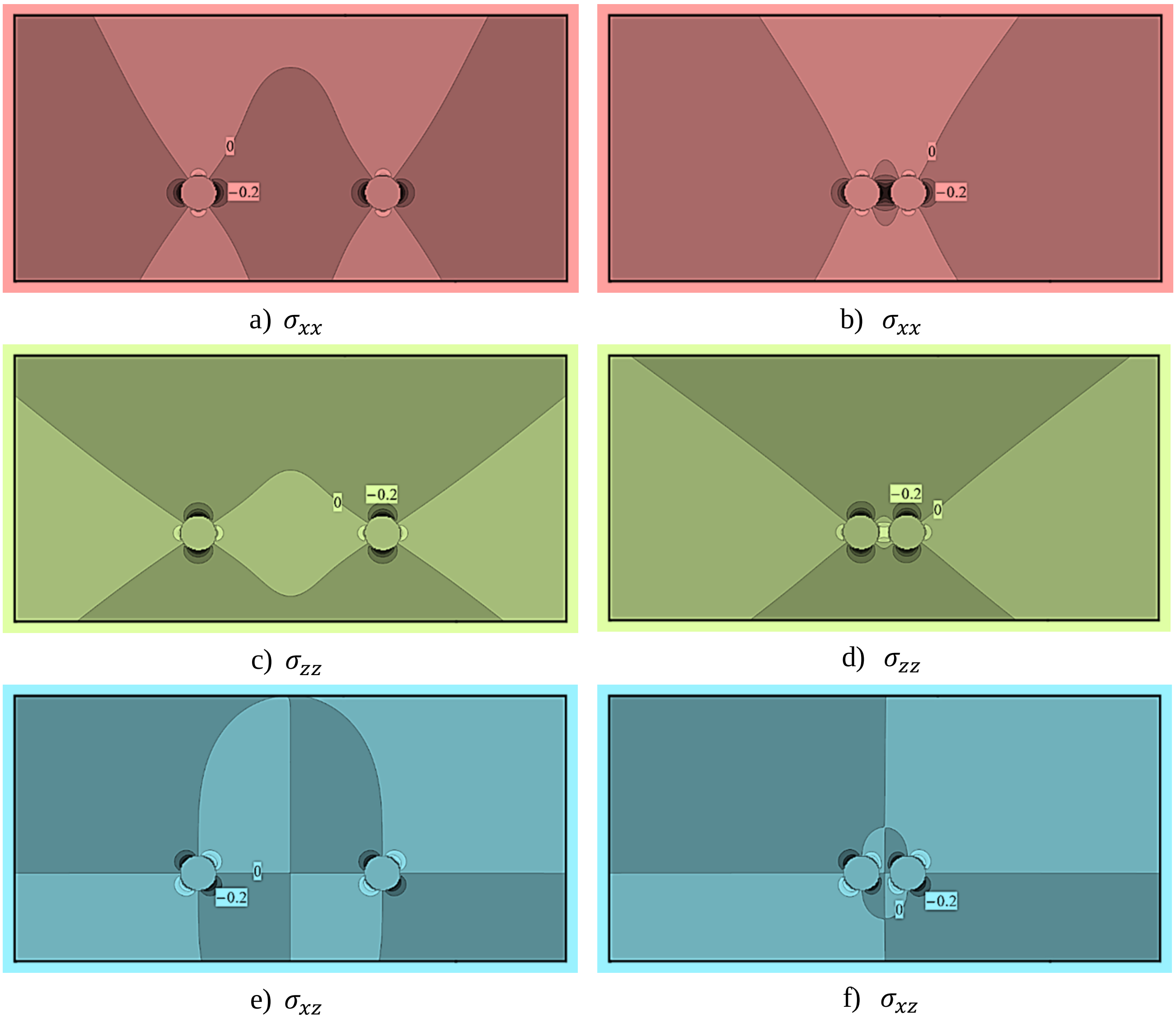

Figure 5: Stress isolines for $\mathbf{p_2}|_{\partial V}$ (pressure in both cavities)
a) c) e) at $c = \frac{1}{20}$ and $R = \frac{2}{25}$,
b) d) f) at $c = \frac{1}{80}$ and $R = \frac{1}{200}$.

The procedure of non-dimensionalization of the typical value of tensile strength for granite, which is in the range from 1.5×10^8 to 2.7×10^8 Pa, was carried out, the solution used the maximum value of 2.7×10^8 Pa: $\sigma_T = \sigma_T^0 / \mu_0$. Stress intensity is calculated according to the Huber-Mises elastic limit condition theory [7]

$$\sigma_i = \frac{p}{\sqrt{2}} \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2)},$$

where $p \in \{0.000625, \cdots, 0.0125\}$.

This force range is the non-dimensionalized equivalent of the actual force range from 10×10^6 to 200×10^6 Pa. In this regard, the highest value of stress intensity in region V, is of interest

$$\sigma_{max}(p, R) = \max_{x \epsilon V} \sigma_i(x, p, R).$$

For this purpose, the safety factor (the ratio of maximum intensity to tensile strength) is calculated as a function of center-to-center distance at different pressure values

$$F = {}^{\sigma_T}/_{\sigma_{max}(p,R)}.$$

Dependence of F on the variable dimensional parameter p, measured in MPa, is shown in Figure 6-7 as a diagram, where each column corresponds to one non-dimensional parameter of center-to-center distance R, left to right from the larger to smaller, the coordinates of the cavity centers on the axis X are shown in brackets.

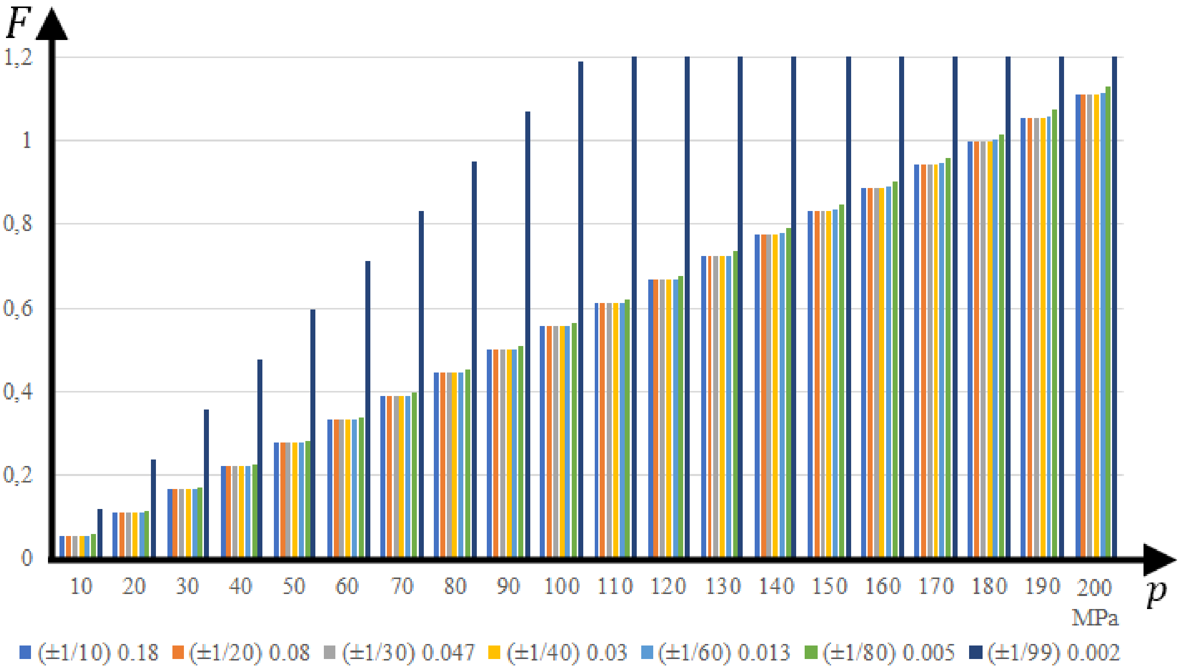

Figure 6: Safety factor for $\mathbf{p_1}|_{\partial V}$ (pressure in a single cavity).

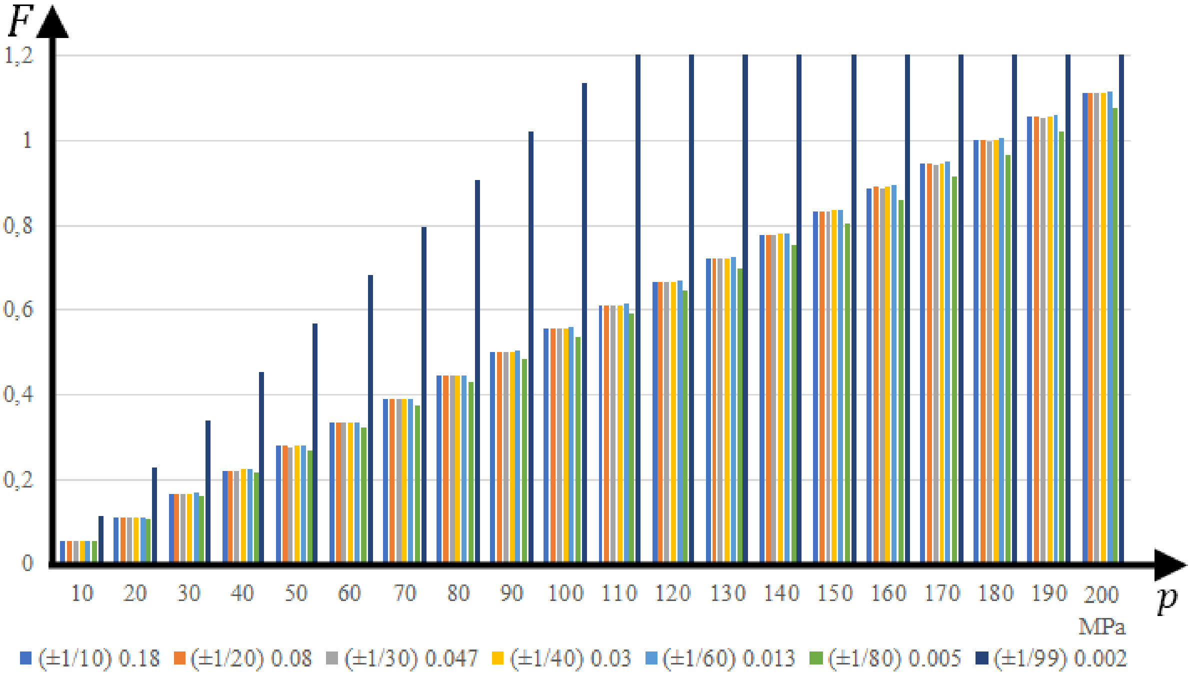

Figure 7: Safety factor for $\mathbf{p_2}|_{\partial V}$ (pressure in both cavities).

Any pair of force values and the parameter of center-to-center distance lying below the straight line corresponding to the value of the safety factor (equal to 1), will keep the complex of engineering structures in operating condition. The above values will lead to the destruction of the complex, respectively. This information allows design engineers to assign geometric and physical parameters, providing a suitable safety factor.

6. CONCLUSION

From this research, there are many key findings. The method of numerical-analytical construction of stress-strain condition of the structure has been developed based on BCM, including physical singularity: center of expansions. The solution of a series of problems on UGS modeling has been created in the varying parameter of center-to-center distance for different pressure values utilizing BCM. Safety coefficients have been calculated and recommendations have been formed for design engineers to ensure sufficient strength of the system consisting of two storage facilities. The maximum value of stresses in the solution with pressure in a single cavity is reached earlier than in the solution with pressure in both cavities.

7. AVAILABILITY OF DATA AND MATERIAL

Information can be made available by contacting the corresponding author.

8. ACKNOWLEDGEMENT

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