ISSN 2228-9860 eISSN 1906-9642 CODEN: ITJEA8

International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies

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Methodology of Local Geoid Improvement Using Least-Squares Collocation with Parameters and an Optimal Covariance Function

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Paper ID: 12A5U

Volume 12 Issue 5

Received 19 December 2020
Received in revised form 25 February 2021
Accepted 15 March 2021
Available online 23 March 2021

Keywords:
Geoid; GNSS; Least-Squares Collocation; Leveling; THAI17G; Airborne gravity data; Geoid undulation; Local geoid model; Orthometric height; Gravimetric geoid model.

Abstract

The local geoid model of Thailand, THAI17G, developed in the corroboration project between the Royal Thai Survey and Chiang Mai University, relied on terrestrial and airborne gravitational data during 2015–2017. We evaluated the model using the 100 GNSS/Levelling control stations, showing the standard deviation of height accuracy at ±5.8cm. The height determination in Thailand referred to the mean sea level at the Ko Lak vertical datum. To align THAI17G with the orthometric height determination from GNSS, we required a conversion surface that connected THAI17G to Ko Lak 1915 vertical datum using 299 GNSS/Leveling stations. This research aims to study the least-squares collocation (LSC) with parameters as the surface conversion technique for geoid model improvement. LSC with parameters is an interpolation method that properly integrates two types of data with different statistical properties, THAI17G and 299 GNSS/Levelling co-point stations of geoid undulation. The polynomial technique was used to defined tilt and bias as optional parameters for LSC at every co-points. We evaluated three types of covariance functions to be optimally used for the collocations. The study result showed that the geoid model using LSC with parameters and Gaussian (exponential) covariance function yielded the most improvement of standard deviation ±3.7cm. In comparison, the ordinary LSC and the EGM2008 provided the standard deviations ±3.9cm and ±10.5cm, respectively.

Disciplinary: Geodetic & Surveying Engineering, Spatial Information Engineering & Technology.

Cite This Article:

Phinyo, K., Dumrongchai, P., Srimanee, C. and Witchayangkoon, B. (2021). Methodology of Local Geoid Improvement Using Least-Squares Collocation with Parameters and an Optimal Covariance Function. *International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies, 12*(5), 12A5U, 1-10. http://TUENGR.COM/V12/12A5U.pdf DOI: 10.14456/ITJEMAST.2021.105

1 Introduction

The local gravimetric geoid model, THAI17G [1], was based on more than 10,000 terrestrial gravity stations and airborne gravity data covering the whole country. The gravity surveys were in precise local geoid model development, starting from 2015 to 2016 under the corroboration between the Royal Thai Survey Department (RTSD) and Chiang Mai University (CMU). THAI17G has recently played an important role in the method of orthometric height determination using GNSS technology. However, the method's height accuracy still yielded at the decimeter level when the model was evaluated with the 100 GNSS/Levelling control stations. The leveling, in Thailand, is generally based on the mean sea level at Ko Lak (1915) vertical datum, while THAI17G is in a global mean sea level. The inconsistency between THAI17G and Ko Lak 1915 deteriorated the accuracy of height determination using THAI17G and GNSS survey.

Least-squares collocation approaches are proper methods for integrating two data types with different statistical properties for generating conversion surfaces. THAI17G is a local geoid model that covered Thailand's whole country and has a one-arcminute spatial resolution. The fitting of the model to Ko Lak vertical datum is useful in the modern height determination technique, based on the GNSS method, which avoids the costly and time-consuming spirit leveling. Hence, this research aimed to study the geoid model improvement techniques to provide more height accuracy from GNSS using LSC with parameters integrated with the distributed 299 GNSS/Levelling co-points as control stations. The GNSS/Levelling stations are established and measured under FGCC1984 [2] by RTSD.

2 Theoretical Concepts

2.1 Polynomial Interpolation

This method is geometric method that based on co-points of study area with known horizontal coordinates and vertical coordinates (ellipsoidal heights and orthometric heights). Then the geoid height of unknown points will be expressed by the polynomial equation [3,4] as

$$N(x,y) = N^0(x,y) + \sum_{i=0}^{m}\sum_{j=0}^{m-i} a_{ij}x^i y^j \quad (1),$$

where $N(x,y)$ = dependent value of the polynomial (e.g. geoid height), $N^0(x,y)$ = approximate value determined by a given geoid model, (x,y) = coordinates of an interpolation point, m = order of the chosen polynomial, a_{ij}= model parameters, for i, j = 0 to m. From the Equation (1), the form of the least-squares can be written by Equation (2).

$$(h_i - H_i - N_i^0) = a_0 + a_1 x_i + a_2 y_i + higher\ order \quad (2).$$

The form of the error vector can be written by

$$v = AX - l \quad (3),$$

$$l = \begin{pmatrix} h_i & - & H_i & - & N_i^0 \\ \vdots & & \vdots & & \vdots \\ h_n & - & H_n & - & N_n^0 \end{pmatrix} = \begin{pmatrix} l_i \\ \vdots \\ l_n \end{pmatrix} \quad (4),$$

where h_i = known ellipsoidal heights, H_i = known orthometric heights, N_i^0 = known geoid heights resulting from a given geoidal model, a_o, a_1 and a_2 = unknown parameters to be estimated, (x_i, y_i) = horizontal coordinates, v = error vector, A = coefficient matrix, X = unknown parameter vector, l = observation vector, n = number of observation, i = 1 to n. The unknown parameter vector, X can be estimated by the least-squares solution

$$\hat{X} = (A^T PA)^{-1} A^T Pl \qquad (5),$$

where $\hat{X}$= estimate of parameter vector X, P = a weight matrix of the observations.

2.2 Least-Squares Collocation

Least-squares collocation [1,5,6,7,8] is a method for integrating two types of data with different statistical properties in a random process. In this study, the collocation method yields the interpolated conversion surface values using the residual values of two sources of geoid undulations, i.e., THAI17G and GNSS/Levelling co-point. Then, combining the adjusted residuals to THAI17G fits the Ko Lak vertical system. We define the residual as follows

$$e = (h_{WGS84} - H_{Kolak}) - N_{THAI17G} \qquad (6),$$

where h_{WGS84} was the geometric height in World Geodetic System 1984 (WGS84) reference ellipsoid, H_{Kolak} was the orthometric height on Kolak 1915, and $N_{THAI17G}$ was the geoid undulation from THAI17G. The residual e was in the vector form l with the signal vector s and the noise vector n as

$$l = s + n \qquad (7),$$

and

$$\tilde{s} = C_{st}(C_{tt} + C_{nn})^{-1} l \qquad (8),$$

where C_{st} was the covariance matrix between the predicted points and the observation values, C_{tt} was covariance matrix of observation values. The symbol C_{nn} represents the covariance matrix of random errors (or noises) in the residuals. The full matrix of C_{nn} was difficult to obtain because we had a limited knowledge of random errors, for instance, in gravimetric geoid noises, the leveled Kolak-1915 heights, and GNSS heights during the time of geoid modeling. For simplicity, we assume no correlation between observations. Thus, C_{nn} is defined by $\sigma_0^2 I$ as

$$C_{nn} = \sigma_0^2 I \qquad (9),$$

where σ_0^2 is a priori variance and I is an identity matrix.

2.3 Least-Squares Collocation with Parameters

Least-squares collocation with parameters [5,8] is an LSC technique that integrated the tilt and bias between two surfaces, THAI17G and GNSS/Levelling co-point at the co-point locations as

$$(h_{WGS84} - H_{Kolak}) - N_{THAI17G} = a_0 + a_1 x + a_2 y + s + n = A\beta + s + n \qquad (10),$$

with

$$\hat{\beta} = (A^T(C_{tt} + C_{nn})^{-1}A)^{-1}A^T(C_{tt} + C_{nn})^{-1}((h_{WGS84} - H_{Kolak}) - N_{THAI17G}) \quad (11)$$

where A was a coefficient matrix and (x,y) were horizontal coordinates. The vector "β" consisted of the a_0 a_1 and a_2 unknown parameters to be estimated. The unknown parameters possibly corresponded to, for instance, bias and tilts between two datums. It should be noted that Equation (9) included systematic errors in the adjustment computation.

3 Methodology

In this study, we selected the 399 RTSD GNSS/Levelling co-point as control stations, distributed in the whole area of Thailand. The leveling was in the 1st order FGCC (1984) standard. We randomly chose 100 co-point for geoid evaluations, and the remaining points were for geoid model improvement. The assessments were done in five separated areas, having different topographics as listed in Table 1 and shown in Figure 1.

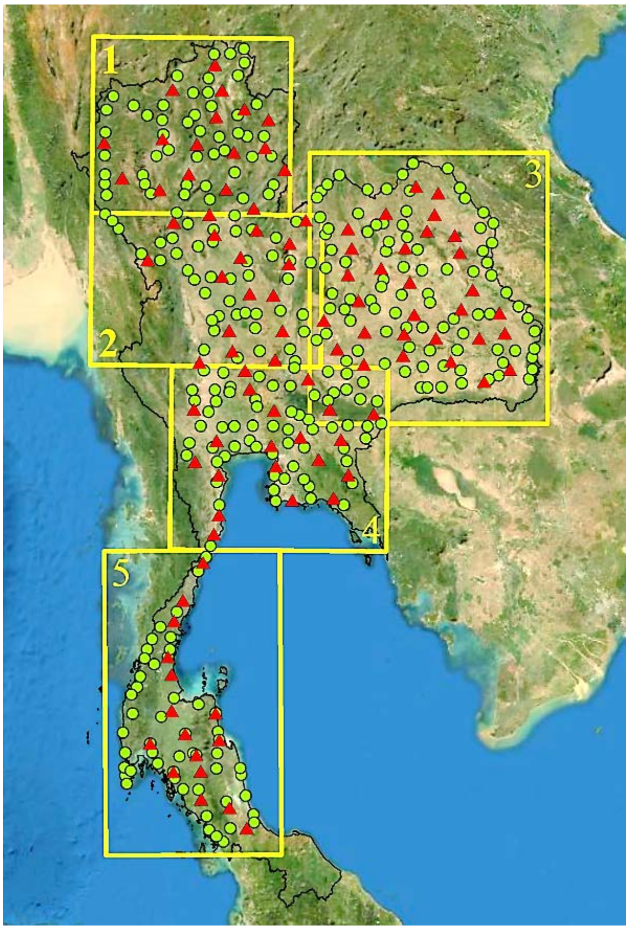

Figure 1: The 399 GNSS/leveling co-points: 299 points for geoid improvement (green dots) and 100 points for geoid evaluation (triangle red dots).

Table 1: Statistics of topographics. (Unit: m)

Area	min	max	mean	SD
1	72.919	872.245	337.432	151.062
2	14.61	893.873	168.741	203.392
3	82.371	467.195	178.263	48.118
4	0.984	468.197	55.514	88.215
5	1.694	98.595	20.283	19.221

Figure 2 showed the steps of the THAI17G improvement technique using least-squares collocations. The first step was to calculate the residuals of geoid undulations between THAI17G and GNSS/Levelling. We constructed the empirical covariance function under the general assumptions of stationary and isotopic processes. This study evaluated three covariance functions to determine the optimal function, suitable for the collocations. The covariance matrices, C_{st} and C_{tt} were derived from the covariance functions. Finally, the improved geoid models were tested by the orthometric heights at 100 checkpoints and also compared with THAI17G and EGM2008.

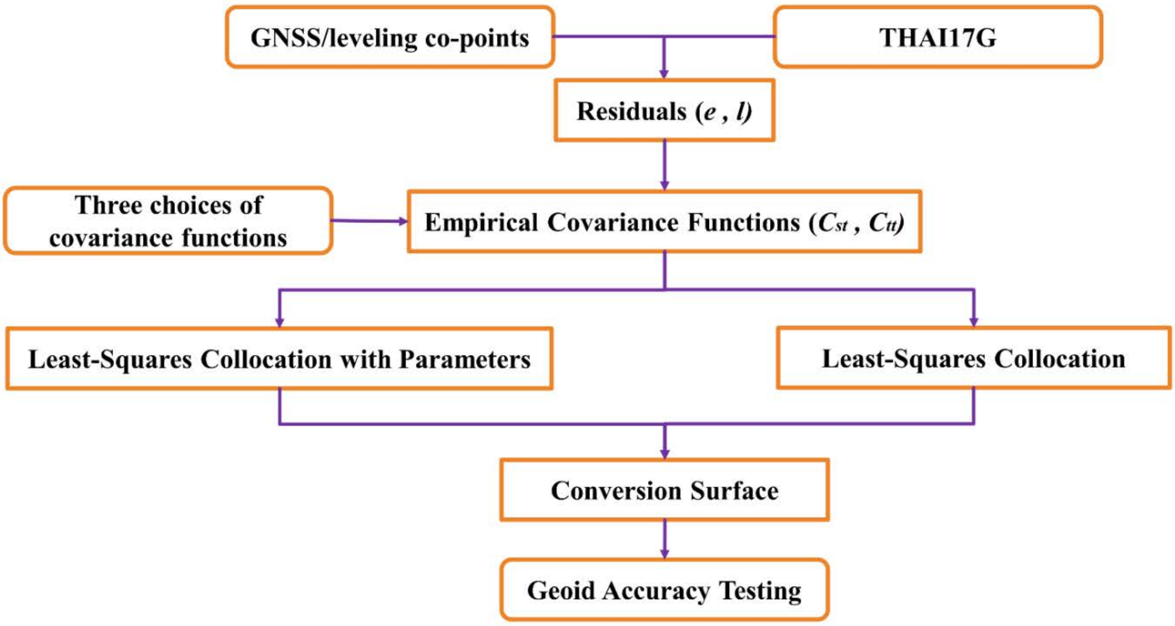

Figure 2: Flowchart of methodology.

3.1 The Geoid Undulation Residual Calculation

The relation of combining THAI17G geoid undulation (N), WGS84 ellipsoidal height (h_{WGS84}), and Kolak-1915 orthometric height (H_{Kolak}) errors was defined by forming the residual vector *l* as

$$l_{(nx1)} = \begin{pmatrix} h_1 & - & H_1 & - & N_1^{THAI17G} \\ \vdots & & \vdots & & \vdots \\ h_n & - & H_n & - & N_n^{THAI17G} \end{pmatrix} = \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix} \quad (12).$$

The residuals indicated the differences between geoid heights from THAI17G and the local mean sea level, referred to Ko Lak vertical datum at "*n*" GNSS/Levelling co-point (*n*=299).

$$e = (h_{WGS84} - H_{Kolak}) - N_{THAI17G} \quad (13).$$

However, there possibly existed datum inconsistencies, for instance, bias and tilts. We considered such systematic errors as unknown parameters to be estimated, according to Equations (1) and (2).

3.2 Empirical Covariance Function

The covariance functions of $C_{st}(.)$ and $C_{tt}(.)$, which were the elements of C_{st} and C_{tt}, could be computed using a covariance function that fitted the empirical covariance function of residuals. In this study, we used three models : (1) Gaussian (exponential) covariance function, (2) Gaussian (2-exponential) covariance function and (3) the 2nd order covariance function [1,7,9], as shown in Table 2.

Table 2: shows three model of covariance function.

Model	Covariance function
I	Gaussian (exponential) covariance function : $C(s) = C_0\, exp\left(-\frac{S}{L}\right)$
II	Gaussian (2-exponential) covariance function : $C(s) = C_0\, exp\left(-\frac{S^2}{L^2}\right)$
III	the 2nd order Markov covariance function : $C(s) = C_0\left(1+\frac{S}{\alpha}\right) exp\left(-\frac{S}{\alpha}\right)$

The parameter C_0 was a variance of observations to be estimated, S was the displacement between points, and L was a correlation length of the residuals. We determined all parameters, i.e., C_0, L, and α, by fitting the models into the empirical covariance function of the residuals. Finally, the covariance matrices in Equation (8) were derived from the models.

3.3 Least-Squares Collocation

The ordinary least-squares collocation in Equation (8) required zero-mean data [6]. The residuals in Equation (6) generally contained long and medium wavelength errors in geoid models. They could be treated as the unknown parameters estimated through a simple least-squares adjustment. We then removed such bias and tilt from the residuals before the collocation procedure. However, the height quantities might contain systematic errors that might not be absorbed by Equation (2) [5,8]. The collocation might provide unsatisfying results. An alternative approach to improving result quality was applying least-squares collocation with parameters that combined a systematic part and a random part (i.e., $A\beta$ and s, respectively), as shown in Equation (10). The approach simultaneously yielded predicted signals and estimated parameters according to Equations (8) and (11), respectively. Using least-squares collocations provided the conversion surfaces with the estimated parameters included. We obtained the improved THAI17G models fitting to Ko Lak 1915 datum.

4 Results and Discussion

We manually detected all residuals at 399 GNSS/leveling co-points for possible outliers. For simplicity, we chose the a priori variance (σ_0^2) in Equation (9) to be 1. The estimated empirical covariance function was plotted in the green dotted line in Figure 3. The parameter C_0 of three covariance functions in Table 1 were estimated to be 0.014 m^2. At this step, our significant and time-consuming task was to determine optimal covariance functions for each model. We repeated parameter computations for three models by trial and error to achieve the best fit models to the empirical covariance function. The correlation lengths were approximately 25 km., 21 km., and 10 km. for models I, II, and III, respectively. According to Equations (8) and (11), three models were analyzed in least-squares collocation approaches at one-arcminute spatial resolution. We computed the predicted signals, $\tilde{s}$, and then added mean biases and estimated parameters back to obtain conversion surfaces (h_{WGS84} - H_{KoLak} - $N_{THAI17G}$). Adding the conversion surfaces to $N_{THAI17G}$, yielded the improved THAI17G geoid models. We then evaluated the geoids at 100 GNSS/leveling co-points, distribute locations in Thailand's area. The discrepancies results at 95% confidence interval were summarized in Tables 3 and 4. We found that using the Gaussian (exponential) covariance function (Model I) performed at most and suitable for estimating C_{st} and C_{tt}. The standard deviations were 0.039m and 0.037m for LSC and LSC with parameters, respectively. It was also clear that LSC with parameters, having the mean of 0.009m, could absorb systematic errors superior to LSC, having the mean value of 0.014m. For the least-squares collocation with parameters, all the parameters, i.e., a_0, a_1, and a_2, were simultaneously estimated in the adjustment system. The trend surface was computed on a one-arc minute grid and had the values of a_0 = 0.901m, a_1 = -3.654x10^{-8}, and a_2 = +1.274x10^{-8}. These significant parameters showed that THAI17G might contain long and medium wavelength errors. A significant tilt (-0.036 ppm) occurred in an east-west direction, while a north-south tilt (+0.013 ppm) was much smaller.

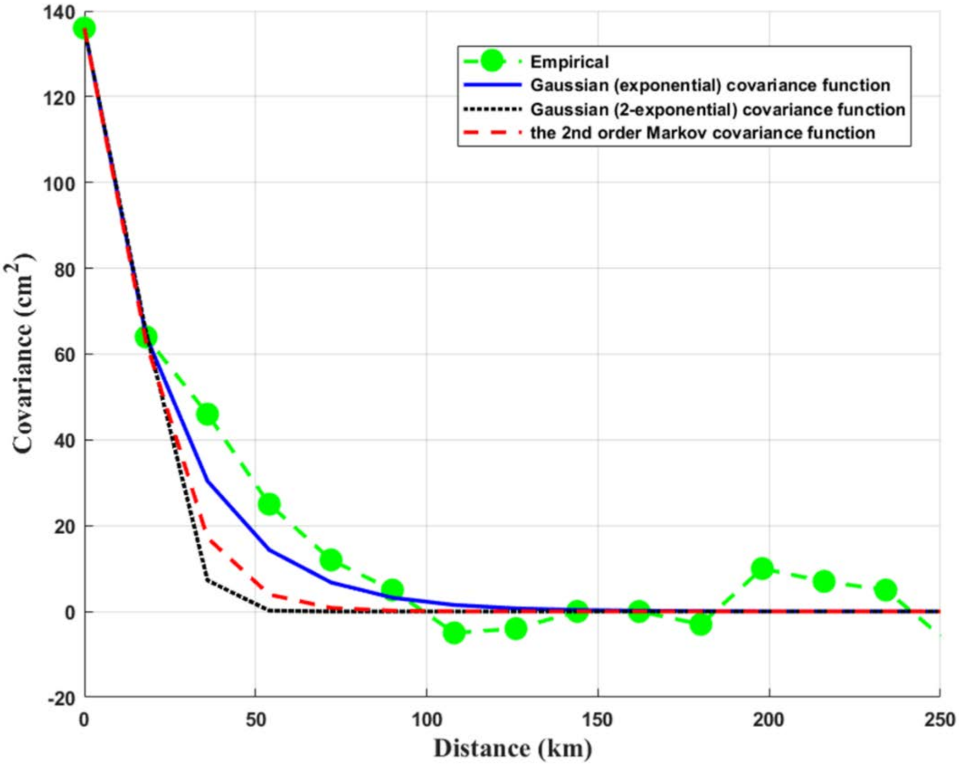

Figure 3: The empirical covariance function of observations

Table 3 Statistics of discrepancies for the improved geoid using least-squares collocation at 100 GNSS/leveling checkpoints, 95% confidence interval. (Unit: m)

Model	Min	Max	Mean	SD
I	-0.075	0.092	0.014	±0.039
II	-0.080	0.125	0.016	±0.049
III	-0.078	0.123	0.015	±0.045

Table 4 Statistics of discrepancies for the improved geoid using least-squares collocation with parameter at 100 GNSS/leveling checkpoints, 95% confidence interval. (Unit: m)

Model	Min	Max	Mean	SD
I	-0.075	0.089	0.009	±0.037
II	-0.077	0.104	0.013	±0.045
III	-0.077	0.097	0.011	±0.041

Table 5 Statistics of model comparing at 100 GNSS/leveling checkpoints, 95% confidence interval. (Unit: m)

Model	Min	Max	Mean	STD
EGM2008 (2190)	0.604	1.098	0.860	±0.105
THAI17G	0.808	1.037	0.919	±0.058
LSC	-0.075	0.092	0.014	±0.039
LSC with Parameter	-0.075	0.089	0.009	±0.037

Table 6 Statistics of model comparing at GNSS/leveling checkpoints were divided the study area into five areas, 95% confidence interval. (Unit: m)

Model	Min	Max	Mean	SD
Area 1 (18 GNSS/leveling checkpoints)				
EGM2008 (2190)	0.706	1.098	0.920	±0.105
THAI17G	0.839	1.037	0.970	±0.053
LSC	-0.064	0.089	0.040	±0.045
LSC with Parameter	-0.075	0.070	0.024	±0.043
Area 2 (16 GNSS/leveling checkpoints)				
EGM2008 (2190)	0.731	1.034	0.847	±0.092
THAI17G	0.819	0.971	0.901	±0.050
LSC	-0.057	0.068	-0.004	±0.036
LSC with Parameter	-0.068	0.062	-0.013	±0.036
Area 3 (30 GNSS/leveling checkpoints)				
EGM2008 (2190)	0.562	1.079	0.862	±0.116
THAI17G	0.808	0.995	0.895	±0.051
LSC	-0.075	0.120	0.009	±0.047
LSC with Parameter	-0.070	0.123	0.012	±0.045
Area 4 (20 GNSS/leveling checkpoints)				
EGM2008 (2190)	0.774	0.964	0.869	±0.064
THAI17G	0.836	1.004	0.916	±0.050
LSC	-0.026	0.082	0.016	±0.031
LSC with Parameter	-0.034	0.076	0.013	±0.031
Area 5 (16 GNSS/leveling checkpoints)				
EGM2008 (2190)	0.616	0.987	0.809	±0.119
THAI17G	0.836	1.023	0.936	±0.051
LSC	-0.050	0.059	0.019	±0.031
LSC with Parameter	-0.056	0.058	0.017	±0.032

In Table 5, the least-squares collocation with parameters performed best fit to Ko Lak 1915 with the station deviation value of ±0.037m. whereas ±0.039m. for the ordinary least-squares collocation. Both cases showed better results than EGM2008 at the maximum degree and order 2160 and THAI17G, having the standard deviations of ±0.105m. and ±0.058m., respectively. The least-squares collocation with parameters improved the accuracy of THAI17G by 36 percent. These statistic results confirmed the best performance of least-squares collocation with parameters among the others.

We investigated whether the least-squares collocation with parameters locally performed well. The improved geoid models were tested in smaller areas where some of the 100 GNSS/leveling co-points were located, as seen in Figure 1. The results showed that LSC with parameters produced a geoid fitting to Ko Lak 1915 in Area's 1 and 3, the standard deviations of ±0.043m. versus ±0.045m. and ±0.045m. versus ±0.047m., respectively. These results indicated that LSC with parameters slightly improved the accuracy of the geoid model in mountainous areas (see also Table 1). However, there were no significant differences between LSC and LSC with parameters Area’s 2, 4, and 5, whose terrains were moderate, flat, and narrow, respectively. We require more numerical investigations by extending parameters in Equations (1) and (10) for further works.

5 Conclusion

This research aimed to study the least-squares collocation with unknown parameters for local geoid improvement. Such a stochastic process technique combined systematic part and random part. This study used the 399 GNSS/leveling co-points in the first-order leveling and GNSS horizontal networks in Thailand, 299 co-points for constructing conversion surfaces, and 100 co-points for geoid accuracy assessments. We investigated three covariance functions and achieved Gaussian (exponential) covariance function as the optimal one, beneficial to the collocation. Overall, we found that LSC with parameters significantly improved geoid models' accuracy and provided the standard deviation of ±0.037m., whereas LSC had ±0.039m. Both collocations performed better than THAI17G and EGM2008. However, testing in five areas having different topographies showed no significant differences between LSC and LSC with parameters in flat areas. To improve the performance of LSC with parameters, we needed more numerical investigations and parameters reflecting more local datum inconsistencies between local geoid models and Ko Lak 1915 vertical datum.

6 Availability of Data and Material

Data can be made available by contacting the corresponding author.

7 Acknowledgement

The authors wish to acknowledge the Royal Thai Survey Department for providing relevant data sets for this research.

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